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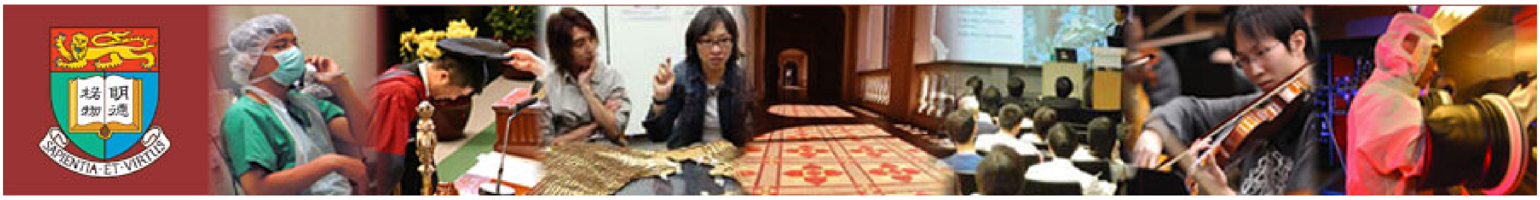

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BINOMIAL OPTION PRICING MODELS FOR REAL ESTATE DEVELOPMENT

Classical real option theory has progressed some way towards identifying main real option categories like the option to delay or defer investment, the time to build option, the option to alter scale, the abandonment option, the switch option, growth option and compound options (see Trigiorgis, 1996). These categories cover much flexibility in economic activities, also in real estate. Theoretical studies like Titman (Titman, 1985), Williams (Williams, 1991, Williams, 1997), Capozza and Li (Capozza and Li, 1994)and others have investigated flexibility in real estate development, including the option to delay development, time to build option, the option to alter land density, the option to switch land use and others. Quigg (1993) empirically confirms the value of the option to delay in land development in the US, while for the UK (Sing and Patel, 2001a, Sing, 2001, Sing and Patel, 2001b), Japan (Yamazaki, 2001) and Hong Kong (Chiang et al., 2006) show that waiting value is significant in land price and the real option method is an effective tool to evaluate the pricing of vacant land. Practical application of real options theory in real estate development is however rare, although important texts (Geltner, 2007) have included real option theory as a core part of land development decision-making. Lucius (2001) suggests it may be due to "theoretical difficulties", "mathematical complexity", identification of real option categories and other problems. In theory, real option analysis can be applied to describe and model various categories of flexibility, but such theoretical categories are also often problematic when industry-specific problems are analyzed, as theory typically assumes that options occur in a perfect, institution-free world.

This paper develops real option pricing methods following the binomial option pricing framework to assess the value of real estate development projects, while taking into account practical details and firm factors. With further development and taking into account specific corporate, regulatory and institutional requirements, the models can be developed and used in practical business applications. We commence with analysis of the institutional environment surrounding the projects that are being evaluated. In reality real options are not well-defined and explicit as theory indicates, and many variables have to be identified from details of the projects and contracts comprising the transactions. The contracts often function to determine whether any real options exist, which party owns those options and how many options each party owns. Such details have been largely ignored, with few studies including them in research into real option theory (Rose, 1998, Giaccotto et al., 2007)[1]. Transaction details and industry-specific details also introduce different characteristics into real options pricing across industries, although in theory such options may fall into similar categories. In real estate development, for example, several parties are involved in the process including developers, the Government[2] and contractors, while in countries that manage their land as a publicly owned resource through leasehold typically adds a layer of participation and complexity. For example, in the Hong Kong institutional context, two types of flexibility, the timing of payment of the land conversion fee for lease modification to change land use, and constrained time flexibility to start actual construction (affecting the option to wait), are owned by the developer but only in limited form (Yao, 2006). This paper follows the idea that to apply real option theory we firstly have to identify the exact flexibility allowed in the contractual arrangements which define the transaction to identify exactly which rights the developer holds, in order to value accurately the real options embedded in the development project. By examining institutions and the regulatory arrangements set by the Hong Kong Government our paper proposes three distinct categories of real estate development projects that typically occur in this jurisdiction, but which also cover most land development cases in most cities with well-developed and -regulated markets, including in Asia, Europe, the US, Australasia, and also to some extent also emerging markets such as China.

Limited insight into practical details of land lease contracts may not be the only problem in defining real options in real estate development projects; there are also firm-level considerations that affect the execution of real estate development projects. Further, one real estate project may not be clearly isolated from others in the firm. There could be interactions

[1] For example, Rose (1998) identifies two real options in contractual agreements in a large-scale metropolitan road infrastructure project: one is the right to terminate the "Concession Period" by the Government given some identified circumstance; the other is an option to defer payment of the concession fee by the development company. Also some arrangements in practice may rule out real options that seem apparent in theory. For example, Giaccotto, Goldberg and Hegde (2007) argue that the cancellation options theoretically embedded in consumer automobile lease contracts are not valuable for lessees because an early-termination penalty is levied.

[2] In the Hong Kong land leasehold systems government plays multiple roles in the development, as a party to negotiate the land lease contract with the developers, as a regulator to execute the town planning and urban growth control ordinances, as the tax-collector to levy the land premium, often leaving only narrowly defined and reduced real options for the developers.

between multiple projects as developers make investment decisions from the firm's perspective rather than the project perspective only (as is mostly assumed in the theoretical real options analysis literature). This paper introduces two categories of interactions, direct and indirect, in real estate development. Direct interaction refers to the effects in multiple projects within the firm. In real estate research into interactions is rare, and will be returned to below. Generally, indirect interactions are considered more common when referring to resource sharing and capital budgeting by multiple projects in the firm, but only few studies have investigated the effects of financial constraints or capital budgeting on the exercise of real options (Boyle and Guthrie, 2003, Hirth and Uhrig-Homburg, 2010, Hirth and Viswanatha, 2011).

This paper develops real option models for real estate development to address gaps between theoretical real option analysis research and the practical application of real option analysis in real estate development generally, but also from within the context of a company that is evaluating a development opportunity. We develop the models within the broad institutional and regulatory framework that define real options in land development in Hong Kong, in order to assess the viability of the models within this land leasehold jurisdiction, and thereby also to obtain insight into the potential complexity that may be encountered when attempting to generalize models to freehold (and mixed) land management systems and also across jurisdictions. Further, we relax the implicit assumption of real option theory that the option holder is always able to exercise the option without constraint and/or additional cost. We evaluate real estate development projects from within the firm perspective and incorporate firm factors like interactions among projects, financial constraints and financing costs. Results from our numerical analyses show that project value (more specifically land value, or real option value) is influenced by the policies and regulations that restrict embedded flexibility, as may be expected. The penalty for delaying effectively forces the developer to immediately execute its project, but it still leaves some valuable flexibility to the developer. It is also shown that developers can gain extra benefits from increased property value or decreased cost through the effects of interaction between assets in its portfolio, which act to lower the investment thresholds for projects and make early development more profitable. It is also shown that financial constraints, which increase the cost of capital, decrease project value and increase investment thresholds, and when the firm holds a portfolio of real options and has only limited resources to exercise them, the capital reallocation among these real options would improve firm performance as the capital can be more efficiently used from the firm perspective.

Overall, our paper differs from previous studies in the application of real option theory in that it studies individual real options, but from a within-firm asset (including real options) portfolio perspective. The paper is arranged as follows. Section two introduces the background of real estate development using three typical cases under the Hong Kong land leasehold system, and also introduces related literature in real estate development. Real option models with institutional details, direct interactions, financial cost and portfolio aspect are constructed in section three. Section four uses numerical examples to solve the real option models and derives some implications not observed in traditional theory. Section five concludes the paper.

2. Background and Related Literature Review

Public land management systems can be classified broadly into two categories: freehold and leasehold. While there are distinct differences such as priorities which affect the exercise of land rights, generally both systems are constrained by public policy, and regulatory and planning constraints which affect property value and development activities (Riddiough, 1997, Mayer and Somerville, 2000, Cunningham, 2007). In many countries including China, Singapore, Malaysia, Hong Kong and some cities in the US and Australia, land is finally owned by the state (Government) and alienated by long land leases, with land markets effectively being markets in leasehold rights. Under leasehold Government plays the principal role in creating any flexibility that may be contained in land development projects through land lease terms and conditions, and with regulatory arrangements most typically given effect through urban planning and legal requirements. Within the broad framework of leasehold in Hong Kong, real estate companies typically have three ways to obtain undeveloped land for development opportunities, which in turn generate three typical development project responses: first, it may purchase land from the market; second, it may draw on its own s land bank; and thirdly, it may acquire land from public auctions. The models for these three cases that we outline below (and develop further in Section 3) follow typical regulatory and institutional constraints in the Hong Kong context, such as typical time allowed for separate stages in the various processes. In Section 3 we also include typical numbers for relevant constraints where possible, but these obviously have to be particular to every jurisdiction practical applications. The cases outlined and developed may be viewed as generic in that they parallel generally typical opportunities in most well-functioning land markets

The *first case* is where the company buys land from the market. Here the land has existing rights for development, and the company then determines the appropriate timing to start the construction. Because the land use choice and density have been regulated in lease contracts and urban planning regulations, the developer holds the flexibility to wait as part of its investment decision. The *second case* is where a company releases land from its own land bank to develop. A typical example of this case is where a company may have acquired land with a long lease term earlier at low cost at the rural-urban fringe (possibly agricultural or village land), with a view to develop in future when the market state and urban and infrastructure development makes it attractive. In such cases land use changes have to be applied for, and profitability depends to a large extent on land use changes and development rights allowed in the process of conversion, and the "premium" payable to Government for allocating the new rights (similar to "land betterment taxes" in many other jurisdictions). Once land conversion rights have been granted, a new lease has been issued and the conversion premium paid, actual development may commence, but it is typically also subject to overall time constraints. Although extension of time to develop is not a right, the Government normally allows it in good faith circumstances and charges a sliding scale penalty for each year the developer requests to delay the development. Here the company's situation is equivalent to the exercise of an American call option before the original lease expires, as the company has bought the land (i.e. the lease) and has been able to determine the timing to convert the land use (Geltner, 1989). The flexibility in this development case is to time conversion of land use and possibly pay the penalty to defer the development, which effectively changes the value of flexibility so achieved into a compound option. The *third case* is associated with public auctions. The company competes with other developers in a public auction to acquire land that is released by the Government. Similar to the development constraint in the second case, other than the usual density and other urban planning constraints, the land bought at auction has constraints on time allowed to complete the development, as well as penalties if the developer applies to relax the constraint (and it is allowed). Essentially, the great majority of real estate developments in the Hong Kong land leasehold system fall into the three categories outlined above.

Turning to interactions between a firm's existing assets and its potential new projects and how such interactions and associated constraints may affect the firm's investment choices, it can be stated with certainty that project/asset interactions is an under-theorized and under-researched subject in the real options field. Some studies of interactions have however been conducted in industries like petroleum (Dias, 2006; Smith & Thomson, 2008), in manufacturing (Kutikula, 1998; Kutikula & Trigiorgis, 1994; Brosch, 2008)[3], and activities like research and development (Vassolo, Anand & Folta, 2004; Girotra, Terwhiesch & Ulrich, 2007). Similar to other industries, how interaction between assets in place and projects affect real estate project value generally, or how any real options associated with projects interact in combination within the firm is an undeveloped field, and the few principles uncovered elsewhere are difficult to generalize into the real estate sector. Some studies have investigated emerging "externalities" due to the co-location of properties[4]and economies of agglomeration generated by multiple properties in one area (Geltner, 2007). Positive externalities may emerge if the real estate projects owned by firms agglomerate or co-locate, in which case fixed cost may be reduced, some common inputs may be shared and complementary land uses may be capitalized upon (Rauch, 1993). Internal agglomeration economies may be achieved with large-scale developments and increase property value (Thorsnes, 2000), while external co-location of multiple properties may generate similar positive synergies. Some empirical studies confirmed the positive effects of interactions on property values (Van Cao and Cory, 1982, Burnell, 1985)[5]. Such externally achieved agglomeration economies are not easily

[3] In the petroleum industry, interactions studied include those among exploration activities including learning and synergy effects which are to solve geological uncertainty and reduce infrastructure cost. In R&D activities, projects that address similar markets interact with each other in a competitive relationship.. In manufacturing, the interactions between several product lines are seen to be realized through the operations of switching that determines optimal product lines and modes for the system according to cash flow and switching cost.

[4] We use "interactions" to note research in other industries, but for present purposes treat "interactions" and "externalities" as generating similar effects.

[5] We concentrate for present purposes on interactions between real assets, and ignore what may be termed strategic interactions as analyzed in the option-game framework by Grenadier (1996), where competitive interaction between developers may cause negative effects, including overbuilding and erosion of individual developer's option value of delaying development.

achieved, but in high-density, high-rise Asian cities such as Tokyo, Hong Kong and Singapore many undocumented cases can be identified where external economies are achieved by co-locating new development with existing company assets.[6] It is in any case clear that all these interactions are relevant when a project is evaluated and developed by a company with various existing properties and multiple projects. While previous studies show that interaction between properties and land uses can have positive interaction effects on values, real estate development evaluation/appraisal models, and in particular real option analyses in real estate development opportunities rarely attempt to incorporate the value that might be generated by these interactions. Only a small number of research studies which consider these facets of real estate development/investment decisions have been identified (Leung and Hui, 2002, Dong, 2004, Huang, 2006).

Even if projects do not interact directly with others, it is to be expected that they will nevertheless be subject to capital budgeting constraints and competition for limited firm resources, because they are considered at the firm level. Unlike financial options in perfect markets, the firm may not have access to risk free capital to exercise its options. The studies in financial constraints and financial hierarchy have found that the use of different sources of capital affect a firm's investment opportunities and may even cause delay or cancellation of investment (Fazzari et al., 1988, Whited, 1992, Cleary, 1999, Campello et al., 2010). In addition to construction cost for real estate projects, financial constraints and cost should be incorporated into option exercise cost by relaxing the classic assumption in real option theory that the real option is stand-alone and all equity funded. In this respect Boyle and Guthrie (2003) showed that the endogenous financial constraint would affect the timing and value of an investment option, while Hirth and Uhrig-Homburg (2010) introduced costly external financing into their real option model. Alternatively, there is also the opportunity cost for the capital to be invested in the construction, if it can be used for other projects in the firm (Meier et al., 2001). This actually causes the firm to accelerate its project if it anticipates more profitable future projects (Han and Park, 2008). Our paper, following these studies, incorporates financial constraints, varying capital cost and opportunity cost within the firm, and shows how these

[6] A recent case in Hong Kong is the development by the same company of Pacific Place III, an office tower located adjacent but not directly contiguous to the large mixed-use Pacific Place complex).

factors affect investment timing and project value in an accessible form. Detailed reviews on interactions, financing cost and constraints can be found in Shen (Shen, 2012).

3. Real Option Models for Real Estate Development with Institutional Details and Interactions

This section introduces three real option models for real estate development and describes the practical cases of real estate development as outlined earlier, while also taking into account the intuitions outlined in Section 2. Section 3.2 develops models to incorporate the factors of direct interaction, financing cost and opportunity cost in project/land valuation. All the real option models are in the binomial form following the framework of Cox, Ross and Rubinstein (Cox et al., 1979). We construct a binomial model with multiple options to model the development process. The value of underlying property evolves as:

$$dP = (r - \delta)Pdt + \sigma d$$
(1)

P_t is the unit price of potential property at time *t* when the land premium has paid and the developer is able to start the construction, δ is the rental yield for the property, *u* and *d* are up and down factors in binomial model.

$$u = e^{\sigma\sqrt{\Delta t}};\ \ d = 1/u$$

The probability for up movement is:

$$q = (e^{(r-\delta)\sqrt{\Delta t}} - d)/(u - d)$$

V_t is the value for the construction opportunity and also the real land value.

3.1 Real Estate Development with Institutional Details

Here the three real option models are constructed to price the land or development opportunity incorporating institutional details. We consider the practical aspects that influence the development payoffs like the regulated construction start, and penalty to defer the construction. Also we view the real estate development as an American call option with expiration period and construction period.

Land Development Model for land bought from the Market

The developer buys and develops/redevelops land from the market ("market land") according to existing development rights, as is typical of some older urban land leases. This market transaction requires no conversion formalities unless the developer applies for an intensification of rights, in which case the project defaults to a subset of Case 2. The developer has full flexibility to determine the timing to start and complete construction. The development can thus be viewed as an American call option with the "taking time to build" characteristic. The term to expiration of this option is the remaining length of land lease *h* minus the construction periods τ, as *h*-τ. Using a backward dynamic programming technique suggested by Trigeogis (Trigeorgis, 1996), the value of this development opportunity (or market land) can be priced at time *t* as[7],

max V_t, subject to[8,9],

$$V(i,s) = \max\left[\frac{P(i,s)}{(1+\delta)^{\tau}} - I, e^{-r\sqrt{\Delta t}}(V(i+1,2s-1)\times q + V(i+1,2s)\times(1-q))\right](2) \qquad (2)$$

$$i = t+1,\dots,T-1, s = 1,\dots,2^{t-1}$$

$$V(T,s) = \max\left[\frac{P(i,s)}{(1+\delta)^{\tau}} - I\right]$$

(3)

$$T = h-\tau, s = 1,\dots,2^{T-1}$$

This model is close to the Samuelson/McKean perpetual American option model for land (see Samuelson (Samuelson, 1965)), and differs only in that it is in discrete form and has a limited expiration period. The developer would exercise the American call option earlier than its expiration because there is foregone profit, the rental income, if it waits for one more period. The early exercise decision rule is derived by following the studies of Han and Park (Han and

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7 In each decision node, the developer would compare the payoff from immediate exercise of the option and delaying the exercise to next period. The binomial form of option pricing allows inclusion of various flexibility to adjust the payoff in each decision node.

8 Because of the time to build in property development, the current value of the underlying property should be divided by the rental yield. See the discussion in Geltner (2007) (Page 11 of Appendix 27).

9 The construction costs are invested sequentially throughout the construction process, and thus the costs here are the present value of the flow of construction cost, similar to the argument in Grenadier (1995) (page 109 and footnote 19).

Park, 2008). For projects falling into the case of market land, the thresholds for early exercise at year *t* is[10]:

$$P_{(T-n)} \geq \frac{e^{r}-e^{-(n-1)r}}{e^{r}-e^{r-n\delta}} e^{\delta\tau} I$$
(4)

$t = T - n, n = 0,1,2, \ldots, T$

The Land Conversion Model

The land conversion model describes the process where the company starts the property development from the stage of acquiring farmland. This whole process can be divided into three sequential stages as acquisition, application and construction. At first, the company purchases the farmland for some consideration and prepares to apply for converting it into developable/commercial land, as it cannot be developed without rights granted by Government. Most companies would hold the raw land for an extended time, observe market information, and then decide to apply for land use change at an appropriate time before the land lease expires[11]. To value this opportunity, we start from the construction stage with the technique of backward recursion. In this stage, the developer should make the decision when to start the construction given that construction takes time τ to build and the maximum construction period is 4 years, meaning that the developer has *4-τ* years to wait for building without any additional cost (penalty). After that, the developer can defer completion of the construction for maximum 6 years with an increasing penalty each year that give six options to postpone the construction. The penalty for extension (κL) is proportional to the land premium *L* that has already been paid to the Government.

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[10] The details of derivation of this and the following equation about option thresholds are in the Appendix to Shen (2012). From Equation (4), we can find that if the rental yield equals zero, the American option collapses to a European option. But when the rental yield is positive, it has two opposing mechanisms through the denominator and numerator in the equation: on the one hand, the developer should exercise the option early to gain the rental income; on the other hand, because of "taking time to build", the developer is bound to lose the rental income of the construction period and should wait for more time to compensate for this loss.

[11] We do not consider the case where the Government denies the land conversion application. This scenario can be incorporated in the option pricing model by assuming a jump probability of the asset value to zero.

The developer has a total 10-τ years to wait but only 4-τ years without penalty. At each year after time t, the developer would make decisions between to build immediately and to defer one more year[12]. The value of construction opportunity at time t is given as:

max V_t, subject to ,

$$V(i,s) = \max\left[\frac{P(i,s)}{(1+\delta)^\tau} - I - \kappa L, e^{-r\sqrt{\Delta t}}(V(i+1,2s-1)\times q + V(i+1,2s)\times(1-q))\right] \quad (5)$$

$$i \leq 4-\tau, \kappa = 0;\ i > 4-\tau, \kappa > 0$$

$$i = t+1, \ldots, T-1, s = 1, \ldots, 2^{t-1}, T = 10-\tau$$

$$V(T,s) = \max\left[\frac{P(i,s)}{(1+\delta)^\tau} - I - \kappa L\right]$$

(6)

$$T = 10-\tau, s = 1, \ldots, 2^{T-1}$$

To secure the development opportunity, however, the developer has to apply for land use conversion and pay the conversion premium. It is the accepted negotiation expectation that the land premium is set as the NPV of the land (current value of underlying property minus construction cost and developer's profit). In the application stage, the developer owns an American call option to exchange the land premium with the value of the development opportunity. The expiration period for this call is the lease periods h_L for the agriculture land. The value of this compound American call is,

max C_t, subject to,

$$C(i,s) = \max\left[V(i,s) - (P(i,s) - I), e^{-r\sqrt{\Delta t}}(C(i+1,2s-1)\times q + C(i+1,2s)\times(1-q))\right] \quad (7)$$

$$i = t+1, \ldots, T-1, s = 1, \ldots, 2^{t-1}$$

$$C(T,s) = V(T,s) - (P(T,s) - I)$$

(8)

[12] By regulation, the penalty for an additional year delay is a portion of land price. In the following numerical examples this penalty is assumed to be associated with construction cost and it is scaled by deferment periods.

$T = h_L, s = 1, \dots, 2^{T-1}$

The compound option is valuable for two reasons. First, the developer is able to select the optimal timing to exercise the application before the agriculture land lease expires; second, the land premium is calculated with the conventional NPV rule typically without considering the flexibility value in the construction period. The initial stage to acquire agricultural land for the land bank involves the NPV decision to compare the compound option value of agricultural land and its acquisition cost. This stage may have occurred much earlier as the company which owns the land may have acquired it decades ago. In comparison with the land premium that may be payable upon conversion and construction cost, the acquisition cost for raw land is likely to be small, and the model thus focuses on the optimal timing for investing the land premium and the construction cost.

The Land Development Model for land bought at Public Auction

The third model combines aspects of the first and second cases. If the developer buys the land from public auction organized by the Government, the land development is similarly constrained with respect to limited time as the second case. The land premium is given at time period *t* as,

max V_t, subject to,

$V(i,s) = \max\left[\frac{P(i,s)}{(1+\delta)^{\tau}} - I - \kappa L, e^{-r\sqrt{\Delta t}}(V(i+1,2s-1) \times q + V(i+1,2s) \times (1-q))\right]$
(9)

$i \leq 4-\tau, \kappa = 0;\ i > 4-\tau, \kappa > 0$

$i = t+1, \dots, T-1, s = 1, \dots, 2^{t-1}, T = 10-\tau$

$V(T,s) = \max\left[\frac{P(i,s)}{(1+\delta)^{\tau}} - I - \kappa L\right]$
(10)

$T = 10-\tau, s = 1, \dots, 2^{T-1}$

Similar to Equation (4) land from market, the early exercise threshold for auction land with penalty is[13]:

$$P_{(T-n)} \geq \frac{e^{r}-e^{-(n-1)r}+e^{r}\kappa_{T-n}-e^{-(n-1)r}\kappa_{T}}{e^{r}-e^{r-n\delta}} e^{\delta\tau} I$$
(11)

These land development models differ from the classic models in real option theory, and incorporate real flexibility generated from the institutional environment within which developers actually operate rather than theoretical flexibility that may not exist in practice. We argue that actual flexibility in development as considered here would greatly influence the investment decisions and thus real option values. The firms that hold different options behave differently, as these options may have various exercise boundaries and timing that are particular to the firm – in a sense, comparable to proprietary options.

3.2 General Real Estate Development Model with Firm Factors

In this section we incorporate financing constraints, opportunity cost, and direct interactions between firm properties and real options into the real estate development model. The effects of direct interactions function to influence the property value positively or construction cost negatively. The financial constraint has an impact on financial cost in addition to construction cost, while the opportunity cost of capital shared in the firm may influence the payoff of an early exercise decision. For simplicity, we develop a general real option model with these firm characteristics for the base case, the land acquired from the market, which is most common in the real option literature. The model is scalable, however[14].

The value of the development opportunity with above firm factors can be priced at time *t* as,

max V_t, subject to,

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[13] From Equation (11) we see that the penalty, in addition to rental yield, influences the investment threshold.

[14] Originally we set up separate models for direct interaction, financing cost and opportunity cost. We thank the referees for suggesting one general model for these firm factors. In the next section, we study the effects of firm factors on both Market Land and Auction Land.

$$V(i,s) = \max\,[\left(\frac{\alpha P(i,s)}{(1+\delta)^{\tau}} - (1-\beta)(1+\gamma)I\right) e^{(k-r)\sqrt{\Delta t}}, e^{-r\sqrt{\Delta t}}(V(i+1,2s-1) \times q + V(i+1,2s) \times (1-q))]$$

(12)

$i = t+1, \dots, T-1, s = 1, \dots, 2^{t-1}$

$$V(T,s) = \max\,[\frac{\alpha P(i,s)}{(1+\delta)^{\tau}} - (1-\beta)(1+\gamma)I]$$

(13)

$T = h - \tau, s = 1, \dots, 2^{T-1}$

In the above equation (12) and (13), the additional parameters are defined as follows: parameter α and β measure the direct interactions with respect to property value and construction cost; parameter γ is additional financing cost to the construction cost as the firm has to rely on external capital like debt or equity; parameter k is the risk-adjusted rate of return for capital in the firm (the opportunity cost of re-allocating the capital to other projects as in Han and Park, 2008).

The investment threshold for the option model with direct interactions and financing cost can be derived as,

$$P_{(T-n)} \geq \frac{(1-\beta)(1+\gamma)(e^{r} - e^{-(n-1)r})}{\alpha(e^{r} - e^{r-n\delta})} e^{\delta\tau} I$$

(14)

And the investment threshold with opportunity cost only is,

$$P_{(T-n)} \geq \frac{e^{k} - e^{-(n-1)r}}{e^{k} - e^{r-n\delta}} e^{\delta\tau} I$$

(15)

The means and sources of these four firm factors can be summarized in Table 1:

[Insert Table 1]

In practice, a firm can select different parameters in the general real estate development model in accordance with its circumstances. For instance, if the firm has existing properties around the proposed project which would generate positive synergy, it can set the parameter α to be a number that is larger than 1 with its own estimation of the magnitude of synergy[15]; the remaining parameters can be set equal to 0 or r[16]. In the next section, we solve the binomial models and use numerical cases to illustrate how these additional factors affect investment threshold and option value.

4. Numerical Solutions and Implications

We develop MATLAB programs to solve these real option models numerically. To increase accuracy, the step between two nodes in the binomial lattice is one month. To compare the effects of various flexibility, interactions and constraints, it is assumed that the underlying properties for each parcel of land in all previous cases have the same value, construction cost and construction period. The parameters for the different real estate development cases are shown in Table 2. We set up four cases for numerical analysis: Market Land with a 40 year lease period, Market Land with a 10 year lease period, Conversion Land and Auction Land.

[Insert Table 2]

4.1 Effects of Various Real Flexibilities

Through application of the binomial option models developed in Section 3.1, project values for the cases can be calculated as follows:

[Insert Table 3]

From Table 3, we observe that the flexibility of investment timing and the penalty influence the option values. Firstly, the term to maturity of land leases affect its value because of the timing flexibility. If the project can be deferred for more time, for instance, 40 years rather than 10

[15] As mentioned, direct and indirect interactions are an under-theorized and under-researched phenomenon in real options portfolio analysis, including real estate. Thus we are not able to specify which interactions may be relevant and what the appropriate parameters to measure these interaction effects are, thereby keeping our approach generic. It is likely that a corporation will be able to identify the interactions it may create between properties and so modify the model presented here for specific purposes.

[16] When all the parameters are set to "be equal to" in the conditions, the model defaults to the traditional binomial option pricing model. One implication from our model is that the firms would place different valuations on land given that their different expected interactions, financing cost and opportunity cost.

years, the option value increases to around 2.2332 (73.1128-70.8796). Secondly, the flexibility value in Conversion Land is significant[17], although in established practice the Government customarily ignores this flexibility in negotiations with the effect of handing it to developers costlessly. Thirdly, in the case of Auction Land we can see that even though flexibility is constrained, it is still valuable to defer construction in some situations. The option value is larger than the NPV value for the project if immediately developed. Fourth, from the Market Land and Auction Land cases we see that the development project with restriction and penalty has a lower value than the one without restriction (70.8796>69.8874). This difference is due to the penalty if the construction does not start according to regulation, which reduces waiting value. It is obvious that the flexibility and inflexibility embedded in the land lease terms and conditions has impact on its value.

Also, we see that the option value is significantly influenced by the rental yield and penalty rate. The trends of the changes in land values are presented in Table 4. We see that the rental yield and penalty have negative relationships with project value. From the trends, we find that when the rental yield is larger than 0.04, it is optimal to choose immediate development rather than deferment of projects (in Table 4, the option values are equal to the NPV with immediate development); and when the penalty is high enough (larger than 0.2), the developers would be better if they start the construction in two years before the required penalty payment. It is also shown that when the rental yield is small, the deferment is valuable to the developers even if there is a penalty for delay. In the row RY=0.01, the option values of Auction Land are all larger than the NPV of immediate development and the project value of waiting for two years to develop. The Government confers some flexibility in construction to developers and the penalty does not eliminate this flexibility value. For instance, in the case of RY=0.01 and Pen=0.2, the option value 71.5644 is larger than NPV of immediate development (68.0199) and the project value of waiting for two years to develop (71.5203), which means that the developer may wait for more than two years and pay the penalty if the property price is sufficiently high. Thus even if the land is bought at public auction and supposedly has rigid completion constraints, it still contains some flexibility value from waiting to develop.

[17] In the case of Conversion Land, the option value comes from two kinds of flexibility: conversion timing and the lower negotiated land premium based on NPV.

[Insert Table 4]

The exercise thresholds (*P/I*) by years for the market land with 10 years construction periods are presented in Table 5.

[Insert Table 5]

From Table 5, we can see that when the rental yield is close to zero (no dividend in the American option, rental yield at 0.001 in the table), the early exercise boundaries are so high (for instance, 72.3129 in Year 1) that early exercise is impossible. However, with increasing rental yields, the investment thresholds lower significantly. It is more likely that the option would be exercised before its expiration date. This result is consistent with the predictions of real option theory that foregone benefit is a good reason for early exercise of an American call option.

To see in which year the option would be exercised, we calculate the ratio between property value and construction cost in worsening scenarios. The worst scenario at year *t* is:

$$P_t = d^t P_0$$

The ratio for the development in the worst scenario is:

$$P_t/I = d^t P_0/I$$

Using the parameters (*d*, *P*, *I* and *t*) in the above numerical examples, the ratios of the worst scenario for each year are:

[Insert Table 6]

It should be noted that the exercise boundary for the worst scenario is only one situation (not all) where the option holder would decide to exercise its option early: if the state of property declines and the early exercise ratio in Table 5 is lower than the boundary for the worst scenario in Table 6, it is optimal to execute the project immediately. This is an approximation to determine whether to make early exercise decisions, because early exercise may also occur before the asset state reaches the worst scenario. Figure 1 shows the early exercise thresholds with various rental yields and also the boundary of the worst case scenario.

[Insert Figure 1]

Through the comparison of the ratios of early exercise thresholds and worst scenarios, we find that when the rental yield is larger than 0.03, the real estate project should be executed at year 1. The rental income is thus a determining factor for timing the execution of real estate projects, and for projects that have lower rental yield, waiting is still valuable. These results are consistent with the observations about option values in the previous section.

The penalty for the deferment also influences the development decision. Intuitively, the penalty would force the developer to immediately execute the project. The early exercise thresholds for the project with penalty are presented in Table 7.

[Insert Table 7]

From Table 7, we see that when the penalty is increasing, postponement beyond year two is more expensive and the investment threshold is higher. The penalty is thus an effective way to force the developer to start construction. If the penalty is small (0.02), the investment thresholds at the first years (from year 1 to year 4) are similar to that of case two without penalty (see Table 5). It indicates that waiting to invest is still valuable in the project with restrictions, but when the penalty is large, it is more likely that the developer would not defer after year two as the investment thresholds are substantially raised by the penalty. We present these results graphically in Figure 2.

[Insert Figure 2]

Overall, the results in this section confirm our predictions that the three categories of land have different values due to their individual institutional restrictions. In addition to the traditional rental income factor that influences early development, the penalty charged by the Government also forces developers to start construction earlier and results in lower land value, although it still leaves some room for "optimal" delay. This finding of the effect of restrictions on land value is consistent with studies of land use regulations which find that more restrictive regulations decrease land value (Ihlanfeldt, 2007, Ding, 2013).

4.2 Effects of Firm Factors on Option Value and Investment Timing

In this section we examine the effects of firm factors on option value and investment timing. The predictions of these effects are presented in Table 1. We examine the effects of these factors on Market Land (40 years), Market Land (10 years) and Auction Land. These firm factors also have an impact on Conversion Land, but we do not show the results of Conversion Land[18]. The results of option values with firm factors are presented in Table 8.

[Insert Table 8]

From Table 8, it is clear that firm factors change the project value in all three cases. In each case, the effects of four firm factors are consistent with the predictions in Table 1. Consider the Market Land case (40 years) as example. If in the model the parameter α=1.02 (2% increase in property value), the option value jumps to 74.7974 in comparison with the option value 73.1128 (1.6846 and 2.30% increase in land value) without any interactions. If the developer can save construction cost by 2%, the land value would increase by 0.2227 (0.3%). If the financing cost increases by 2%, the option value deceases by 0.2223, and if the risk-adjusted return of the firm's capital increases from 10% to 20%, the option value increases by 0.6116 through early development. The option value is most sensitive to the parameter α, which may indicate that developers should seek positive synergy if they wish to increase project profits. We can observe similar effects on the other two cases in Table 8. The firm factors influence the values of all categories of land. The implications from these results are that even for the same parcel of land, value may vary for different users because of the "proprietary value" attributable to firm factors and a firm's ability to exercise the opportunity.

We have shown that firm factors would change option values in Table 8; while they alter the values through varying investment timing. From Equation (14), we observe how interactions and financing cost affect investment thresholds: the interaction factors lower the investment thresholds that make early exercise more likely; and financing cost raises the investment threshold and depress option exercise. This result is consistent with the intuition that the developer would accelerate development if they observe potential positive synergies between

[18] Due to complexity we exclude numerical results for Conversion Land with firm factors for two reasons. First, the conversion period of agriculture land is long; and thus the developer may not anticipate the potential interactions and financial status over such a long time. Second, the option model of Conversion Land can be derived from two other scenarios as the restrictions on the development stage on Conversion Land are the same with Auction Land.

real estate projects. If there is additional financing cost, the property value should increase to compensate for this cost and the developer actually is forced to delay. In contrast, if the firm can invest its capital at its risk-adjusted return, it would execute the project early and capture higher returns from capital reallocation. We present the effects of firm factors on investment thresholds in Table 9. Panel A shows the investment thresholds of Market Land (10 years); and Panel B contains results for Auction Land.

[Insert Table 9]

Consistent with predictions in Table 1, positive interactions tend to lower investment thresholds both in Market Land and Auction Land; financing cost has opposing effects on investment thresholds; and risk-adjusted return substantially decreases the thresholds. Similar to the results in Han and Park (2008), we find that the risk-adjusted return is an important factor to decide whether to exercise the option early. For example, if the risk-adjusted return is 0.15 rather than the risk-free rate 0.1, the investment threshold in Year 1 turns to 3.1736 (rather than 4.0107) for both Market Land and Auction Land. In Year 3, the threshold lowers to 3.3032 rather than 4.5222 in Auction Land.

The results in Table 9 also indicate that the investment thresholds are higher for Auction Land than Market Land even under the consideration of firm factors. To compare the effects of firm factors on two categories of land, we present Figure 3 for Market Land and Auction Land with various factors.

[Insert Figure 3 here]

In sum, a firm's fundamentals influence real option values and exercise decisions by enhancing flexibility or imposing inflexibility as shown in the numerical results presented. The positive interactions enable developers to obtain more option value and induce earlier development, which is consistent with the real estate development models of Dong (2004) and Huang (2006). Financial constraints and financing cost would force the developer to wait longer and reduce the project value, similar to the conclusion of Boyle and Guthrie (2003). When the firm has several real options, it increases the flexibility value of early exercise and it earn dividend-like payments as well as use reclaimed capital to generate higher payoffs.

5. Conclusion

In this paper we developed practical and generalizable binomial real options valuation models for an actual real estate development environment, as may faced by a conventional real estate development firm with multiple projects (real options). Apart from the traditional real option theory, we consider in our models the institutional arrangements, direct interactions and financial constraints that influence flexibility in real estate development, all aspects of real estate developers' operating context that may be generalized across jurisdictions. It was argued that more practical factors should be incorporated into real option theory when it is applied in practice, and that real option theory should be applied at firm level to consider financial constraints in the firm and portfolio aspects among multiple assets. Numerical analyses based on the models confirm accepted industry folklore without much empirical or theoretical substance, and also shows that real flexibility in real estate projects and financial flexibility (inflexibility) influence project value and its exercise timing.

We draw at least four important implications from our results for real estate development firms. First, project value (land value) is influenced by the policies that restrict its embedded flexibility. The penalty for delaying effectively forces the developer to execute immediately its project, but it still leaves some valuable flexibility to the developer. This result is consistent with the empirical testing of timing option premium in auctioned land price in Hong Kong (Chiang et al., 2006). Second, the developer can gain extra benefits through the effects of direct interactions among its property portfolio by increasing property value or decreasing construction cost. These positive synergies lower the investment thresholds for the projects, which make early development more profitable. Third, financial constraints, which increase the cost of capital, decrease project value and enhance investment thresholds. Fourth, if the firm has many developable projects, it may exercise its projects early and free up the capital to invest in further projects. Overall, our models have shown that firm fundamentals can change project value, and that the firm can realize strategic advantages by capturing direct interactions, using lower cost of capital (financial flexibility). We have also shown that real option value is not only related to option characteristics revealed by financial option literature, but also influenced by the holder's circumstances and its ability to capture for the firm the flexibility embedded in the option. Also, importantly, to capture such flexibility a firm's fundamentals, like financial structure and asset portfolio, play a role in decisions to exercise options and the valuation of option-like projects. However, while the basic principle remains the same - more flexibility gives higher value to real options and the option holder - when it comes to execution in practice, the value of a real option is influenced by the structure and resources of the host firm.

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