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Topological Analysis of Longitudinal Networks

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Abstract

Longitudinal networks evolve over time through the addition or deletion of nodes and edges. A longitudinal network can be viewed as a single static network that aggregates all edges observed over some time period (i.e., structure of network is fixed), or as a series of static networks observed in different point of time over the entire network observation period (i.e., structure of network is changing over time). By following a topological approach (i.e., static topology and dynamic topology), this paper first proposes a framework to analyze longitudinal networks. In static topology, SNA methods are applied to the aggregated network of entire observation period. Smaller segments of network data (i.e., short-interval network) that are accumulated in less time compared to the entire network observation period are used in dynamic topology for analysis purpose. Based on this framework, this study then conducts a topological analysis of email communication networks of an organization during its different operational conditions to explore changes in the behavior of actor-level dynamics.

1. Introduction

Researchers have been exploring longitudinal networks to understand the micro mechanisms in the process of network formation and development over time. The study of longitudinal networks has therefore been the subject of intense research interest in recent years. Most studies in current literature give emphasis to the holistic view of network for studying evolutionary dynamics of networks [1-5] while underestimating the importance of the node-level view of networks. Based on a topological framework, this paper posits an actor-level approach about how to analyze longitudinal networks.

In the social science literature, the presence of methods for the analysis of longitudinal networks is noticed for quite some time. The dominant method for analyzing longitudinal networks includes Markov models and Multi-agent simulation models. Continuous time Markov chains for modelling longitudinal networks were proposed as early as 1977 by Holland and Leinhardt [6], which have been significantly improved later by many researchers [1, 4, 7-9]. For modelling longitudinal networks, exponential random graph and stochastic actor-oriented models are the two Markovian methods proposed by Robins et al. [8] and Snijders et al. [10]. In these two approaches of analysis, links are modeled as random variables that can be in different states (e.g., positive, negative, or neutral) at different time. The purpose is to examine which network effect fits the empirical data and better accounts for the observed structural changes. These two Markovian approaches to longitudinal network analysis are efficient to detect and describe network changes as well as to explain why these changes occur. However, they may have convergence issues in the presence of sufficiently large abrupt endogenous (i.e., structure based) and exogenous (i.e., attribute based) social changes. Moreover, these models have computational limitations for a very large social network data. In Multi-agent simulation model, members in a social network are often modeled and implemented as computer agents who have the abilities to behave and make decisions based on certain criteria. The collective behaviors' of all members in a network will determine how the network evolves from one structure to another. Evolutionary models often use multi-agent simulation. Carley et al. [11] use multi-agent technology to simulate the evolution of covert networks such as terrorist groups. Moreover, using a multi-agent system called DYNET, they perform a "what-if" analysis to anticipate how a network adapts to environmental changes such as the removal of a central member [11, 12]. A simulation model can be a powerful tool for predicting a network's future. However it often oversimplifies the behavior and decision-making of humans, and may not be able to model the complex reality of social networks. Unlike these two models (i.e., Markov models and Multi-agent simulation model) and other available statistical methods (e.g., panel regression), this paper proposes an innovative approach to capture dynamics of longitudinal networks.

Study of the analysis of longitudinal networks is very important for a number of reasons. First, longitudinal studies are unique in their ability to provide useful data about individual change over time [13]. Second, they can provide more efficient estimators than cross-sectional designs with the same number and pattern of observations [13]. Third, this type of study allows the flexibility to shift the focus of the study whilst data is being collected [14]. In

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longitudinal study, data is collected over time. Therefore, longitudinal analysis of previously collected data can be conducted while collecting the present data. Based on the output of this early analysis, the principal direction of the study could be changed, if required. Finally, longitudinal studies are able to separate aging effects (changes over time within individuals) from cohort effects (differences between subjects at baseline) [14].

Using the methods and measures of social network analysis (SNA), a social network can be analyzed by either static topology, or dynamic topology, or a combination of both [15, 16]. In static topology, SNA methods are applied to aggregated network data that have been accumulated over the entire period of observation [16, 17]. On the flip side, SNA methods are employed to the smaller intervals of data collection period in order to study how network interactions change over time in dynamic topology [17, 18]. For instance, a dynamic topology could be a daily or weekly or even monthly network analysis of a university email communication networks that evolve over five years, while static topology considers only one network - the single aggregated network of five years. Figure 1 illustrates a schematic difference between these two types of SNA topologies. According to this figure, for static network analysis purpose SNA methods are applied to the aggregated network (i.e., the upper shaded network inside the square) at the end of day three. In contrast, SNA methods are applied to each day network for research analysis purposes in dynamic topology (i.e., the three lower shaded networks inside squares). There is no aggregated network considered for SNA in this topology.

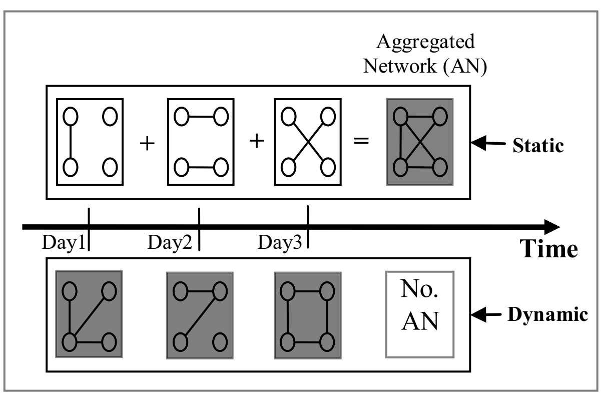

Figure 1: Illustration of static and dynamic topology of social network analysis (SNA)

This research aims to develop a framework, which is based on topological analysis, for analyzing longitudinal networks. It further intends to provide a relevant example which could provide the evidence about the applicability and usefulness of this proposed framework. The following two questions motivate this study:

(i) How the structural positions (which is measured using a common SNA measure) of different actors change over time in longitudinal networks?
(ii) What are the impacts of the changes in structural positions of actors in short-interval networks on their positions in the aggregated network?

The synopsis of this paper is as follows. Section 2 illustrates the research framework. A topological analysis of email communication network of a large organization is conducted, based on the proposed research framework, in section 3. This is followed by the discussion of this research in section 4. Finally, there is a conclusion and future research direction of this study in section 5.

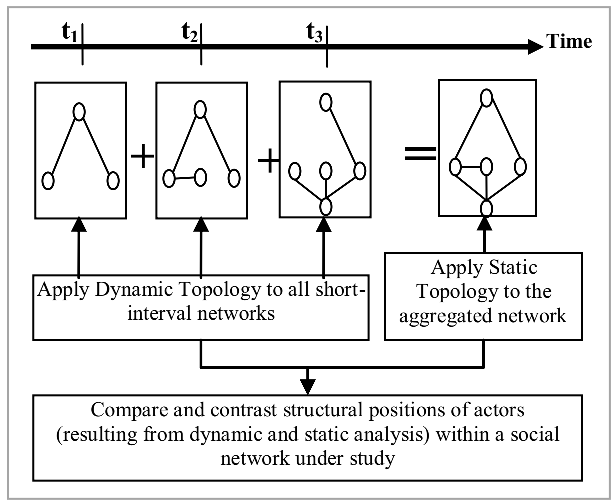

Figure 2: Research framework for topological analysis of longitudinal social networks

2. Research Framework

In analyzing longitudinal networks, the involvement of individual actor is observed and analyzed in (i) all networks that evolve in shorter period of time (i.e., short-interval networks); and (ii) in the single aggregated network which aggregates all edges observed in all networks of (i). To capture dynamics of networks that emerge in shorter period, dynamic topology needs to be followed. On the other hand, static topology has to be carried out for the single aggregated network. For this reason, to capture dynamics of longitudinal networks both static and dynamic topological analyses of networks need to be carried out.

This study develops, based on topological approach, a framework to capture dynamics of longitudinal networks. Figure 2 illustrates this framework. In this figure, dynamic topology is applied to short-interval networks observed at time t_1, t_2, and t_3 (where $t_3>t_2>t_1$) in order to capture network dynamics of individual actor. The network positions of all actors are quantified, by using a measure of social network analysis (say degree centrality), in these three short-interval networks for conducting a comparative analysis of the structural positions of actors over time. This comparison enables to explore how an individual actor change its structural behaviour over time compared to the other actors within the network. Static topology is then used for the aggregated network, which is the accumulation of the three short-interval networks. In this topological analysis, the network positions of all actors in the aggregated network are explored using the same social network measure used in dynamic topology (i.e., degree centrality). Finally, structural positions of all actors in short-interval networks (from dynamic topology) are compared and contrast with their respective positions in the aggregated network (from static topology). As this comparative analysis considers both the pattern of the changes of structural behavior of actors in short-interval networks and their structural positions in the aggregated network, it will eventually lead to better understanding about the dynamics of the longitudinal network under study.

This topological framework could potentially be used to capture different features of actor-oriented dynamic behaviour (e.g., activity of actor, actor popularity, level of involvement, capacity to control the flow of information, and embeddedness of actors in a network) of longitudinal networks. This will create an innovative way to analyze networks that evolve over time. For example, an organizational communication network could be explored to understand staff collaboration dynamics both in the short-interval networks and in the aggregated network during different phases of the operational running of that organization. To give another example about the applicability of this framework, the role of individual actor could be identified in the evolution of any longitudinal network.

3. Example: Application of the Proposed Research Framework

The proposed research framework, as illustrated in Figure 2, is exercised to capture the dynamics of a longitudinal email communication network of a large organization during its different operational conditions (i.e., '*crisis*' and '*normal*' period). In doing so, this study also designs and conducts four *Comparative Analysis techniques* (CATs) to compare and contrast network level involvements of actors between short-interval networks and the aggregated network.

3.1. Data Source and Social Network Measure for Communication Network

The email communication dataset from Enron, named as Enron email corpus, has been analyzed for comparative analysis purpose in this study. This corpus was released by Federal Energy Regulatory Commission (FERC) in May, 2002. Shetty and Adibi [19] from University of Southern California created a MySQL database of this corpus. They also cleaned the database by removing a large number of duplicate emails, computer generated folders, junk data, invalid email addresses etc. The resulting dataset contains 252,759 messages from 20,294 distinctive users. In the area of organizational science and social networking research, the Enron corpus is of great value because it allows the academic to conduct research on real-life organization over a number of years.

The principal focus of this example is to explore organizational email communication networks, using the proposed research framework, to find out characteristics of actor dynamics associated with the '*normal*' and '*crisis*' period. It is well documented in the literature that a drastic form of critical loss, which was being started to flourish in the beginning of the third quarter of 2001, occurs in Enron during the final quarter of 2001 [20]. Therefore, this study chooses segments of communication networks for the '*normal*' and '*crisis*' periods: (i) July-December, 2000 for the '*normal*' email communication dataset; and (ii) July-December, 2001 for the '*crisis*' email communication dataset. For conducting static and dynamic topological analysis, it is required to consider short-interval networks and an aggregated network for both the '*normal*' and '*crisis*' period. For short-interval networks, weekly email communication networks are chosen because in a period of seven days there found a standard number communication exchanges among staff in the Enron email corpus.

The context of this example is email communication network. One of the important and primary uses of graph theory and network analysis is the identification of the most important actor(s) within a social network. Various researchers use different terms like 'importance' or 'prominence' to describe this important network measure. A prominent actor occupies a distinctive location in the network that may lead to high visibility relative to other actors. Prominence is frequently attributed to actors who have many ties in the network because such a position is associated with high visibility and ability to influence a large number of people [21]. Knoke and Burt [22] also considered an actor to be prominent if the ties of that actor make it particularly visible compared to other actors within the network. Hence, the concept of degree centrality has been used to describe the prominence of an actor in our email communication networks. In particular, to capture longitudinal dynamics of network associated with the '*normal'* and '*crisis'* periods, this study uses out-degree centrality measure, which is defined by the following equation for an actor:

$$d(\mathrm{n_i}) = \sum_j X_{\mathrm{ij}}$$

Where, $d(n_i)$ is the out-degree centrality for the node or actor i and X refers to the adjacency matrix for network data. Also, relationships are defined as communication linkages among actors. Number of emails sent by the employees to others within their respective communication networks is used to quantify the out-degree centrality for all actors.

There are other approaches used by researchers for the measure of network prominence. Eigenvector centrality is one of them. This approach represents an actor's connectedness to highly connected peers, and takes all direct and indirect network paths from the focal actor into account [23]. Stefanone et al. [24] consider actors' social network density, brokerage, and reach in order to explore their local of control and pursuit of social capital in a distributed distance learning environment of engineering students.

3.2. Design of Comparative Analysis Technique (CAT) to Follow Research Framework

Before describing the CATs that are followed to exercise the proposed research framework, this section first defines the following network terms that are used in this research example:

(a) Weekly Network: The network which was evolved, by means of email communications, amongst employees in each week.

(b) Aggregated Network: This is the aggregation of weekly networks. For the both 'normal' and 'crisis' period, it consists of 26 weekly networks.

(c) Top-rank list: List of actors who show most out-degree centralities in the weekly networks or in the aggregated network. To compare actor-level network dynamics for the year 2000 and 2001, this study considers top-ranked list of size 10. From the statistical distribution of emails sent by all actors in our research dataset, it is observed that top-10 actors sent significant number of emails in each weekly network, which has guided the choice of 10 for the size of top-ranked list.

(d) Centrality Overlap: In the process of comparing two top-rank lists of weekly networks, an actor is said to be overlapped if it is found in both top-rank lists. When two weekly top-rank lists are compared in terms of overlapping, centrality overlap simply counts the number of actors that are located in both top-rank lists. For any two weekly top-rank lists (say L_a and L_b) of size n, it can be defined by the following equation:

$$\text{Centrality Overlap} = \sum_{i=1}^{n}\sum_{j=1}^{n} x$$

$$Where, x = \begin{cases} 1 \;\; if \;\; L_a^i = L_b^j \\ 0 \;\; otherwise \end{cases}$$

And, L_a and L_b are the top-rank lists for week a and b ($a \neq b$)

The following four CATs have been conducted to carry out topological analysis for the email communication networks by following the proposed research framework.

3.2.1. Rank of Actors. First, for both the years of 2000 and 2001 the number of times the top-10 actors of the corresponding aggregated networks have been found in 26 weekly networks is compared in Table 1. During the *'normal'* period, top-10 actors of the aggregated network are found more times in the top-10 lists of 26 weekly networks compared to the *'crisis'* period.

Table 1: Number of times top-10 actors of the aggregated network are found in all weekly networks for the years of 2000 and 2001

ID of top-10 actors of the single aggregated network		Number of times top-10 actor of aggregated network are found in all weekly networks	
Y: 2000	*Y: 2001*	*Y: 2000*	*Y: 2001*
1093	253	25	19
253	347	18	17
5335	256	24	9
288	1654	24	16
3113	43960	20	16
703	280	19	11
2530	293	18	9
642	14557	15	7
4134	22786	11	8
6815	1637	11	5

Then, presence or absence of the top-10 actors (that are found most times in 26 weekly networks) in the corresponding aggregated networks are compared for both the years of 2000 and 2001. Top-10 actors of 26 weekly networks are also found in the aggregated network for the year 2000; however, for the year 2001 only 8 actors have position in the aggregated network as illustrated in Table 2.

Table 2: Illustration of presence or absence of top-10 actors, which are found most time in all weekly networks, in the aggregated networks for both *'normal'* and *'crisis'* period.

ID of top-10 actors that are found most times in all weekly networks		Number of times they are found in all weekly networks		Do they have position in the single aggregated network	
Y:2000	***Y:2001***	***Y:2000***	***Y:2001***	***Y:2000***	***Y:2001***
1093	253	25	19	Yes	Yes
288	347	24	17	Yes	Yes
5335	1654	24	16	Yes	Yes
3113	43960	20	16	Yes	Yes
703	280	19	11	Yes	Yes
253	293	18	9	Yes	Yes
2530	256	18	9	Yes	Yes
642	22786	15	8	Yes	Yes
6815	6815	11	8	Yes	No
4134	1078	11	8	Yes	No

Finally, the size of top-rank list is varied to explore positions of top-ranked actors (in terms of their presence or absence in aggregated network, and number of time they are found in 26 weekly networks) of weekly networks for both the years of 2000 and 2001. The result is shown in Table 3. With varying the top-ranked list size, it is observed that during the *'normal'* period top-ranked actors of 26 weekly networks are also found most time in the aggregated network compared to the *'crisis'* period (e.g., 10 versus 8 for the top-ranked list size of 10). With the increase of top-rank list size, this difference remains almost consistent. For the top-rank list size of 50, for example, this difference is 46 versus 42. That means, during the *'normal'* period top-ranked actors of all weekly networks are found most times in the aggregated network regardless of the size of the top-ranked list size. Top-ranked actors show higher tendency to have positions in the aggregated network during the '*normal*' period compared to the '*crisis*' period. Ranges of the number of times top-ranked actors (of weekly networks) found in 26 weekly networks are also higher for the *'normal'* period compared to the *'crisis'* period (e.g., 11-25 versus 8-19 for top-ranked list of 10). This difference remains almost unchanged with the change of top-ranked list size. For instance, for the top-ranked lists size of 50 this difference is 8-27 versus 9-24.

Table 3: Illustration of the variability of frequency and range of the number of times position occupied by actors (that are found most times in all weekly networks) in the top-ranked lists with the change of top-rank list size

Top-rank list size	Number of actors (that are found most times in all weekly networks) have position in the aggregated network (considering same top-rank list size)		Range of the number of times top-ranked actors are found in all weekly networks (considering same top-rank list size)	
	Y: 2000	***Y: 2001***	***Y: 2000***	***Y: 2001***
10	10	8	11-25	8-19
20	19	16	11-26	9-21
30	27	25	11-27	10-23
40	38	33	9-27	9-24
50	46	42	8-27	9-24

3.2.2. Centrality Overlap. First, the number of centrality overlap among all weekly networks is tested for both the years of 2000 and 2001. As there are 26 weekly networks, there is a total of 325 ($^{26}C_2$) pair of weekly networks. There is an increased number of centrality overlap during the *'normal'* period compared to the *'crisis'* period as noticed in Table 4. The number of centrality overlaps increase with the increase in top-ranked list size for both the *'normal'* and *'crisis'* time.

Table 4: Varying centrality overlap (for the year 2000 and 2001) among the all weekly networks

Top-rank list size	Number of centrality overlap among weekly networks	
	Y: 2000	***Y: 2001***
5	910	444
10	1895	975
15	2859	1595
20	3897	2286
25	5127	3048
30	6308	3661
35	7157	4487
40	8198	5198
45	9101	5739
50	9482	6123

Then, the number of centrality overlap between each of weekly networks and the aggregated network is compared for both the years of 2000 and 2001. As there are 26 weekly networks and only one aggregated network for each year, there are only 26 pairs of networks are considered for the overlap statistics for both the *'normal'* and *'crisis'* period. For this reason, number of overlaps, as showed in Table 5, is significantly low compared to the statistics of the first 'centrality overlap' test (mentioned in Table 4). However, trend is similar to the first test – increased centrality overlaps in 2000 compared to 2001.

Table 5: Varying centrality overlap (for the year 2000 and 2001) between each of weekly networks and the aggregated network

Top-rank list size	Number of centrality overlap between each of weekly networks and the single aggregated network	
	Y: 2000	*Y: 2001*
5	92	57
10	185	117
15	268	184
20	369	252
25	481	332
30	575	397
35	674	467
40	781	548
45	860	619
50	927	696

3.2.3. Activity of Actors. The percentages of emails, that are sent by top-10 actors of each weekly network during the *'normal'* and *'crisis'* period, are compared. During the *'normal'* period top-10 actors of each weekly network sent more emails compared to the *'crisis'* period as depicted in Figure 3. Similar test is conducted to explore percentages of emails sent by top-10 actors of the aggregated network in each weekly network during the *'normal'* and *'crisis'* period. As illustrated in Figure 4, more emails were sent by top-10 actors of the aggregated network during the *'normal'* period compared to the *'crisis'* period.

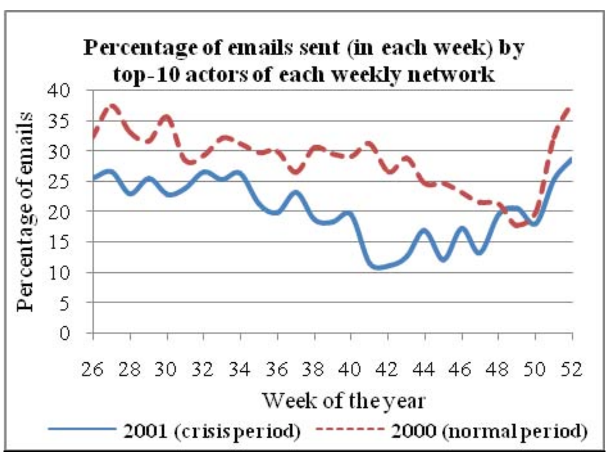

Figure 3: Percentage of emails sent by top-10 actors (in each week) of each weekly network.

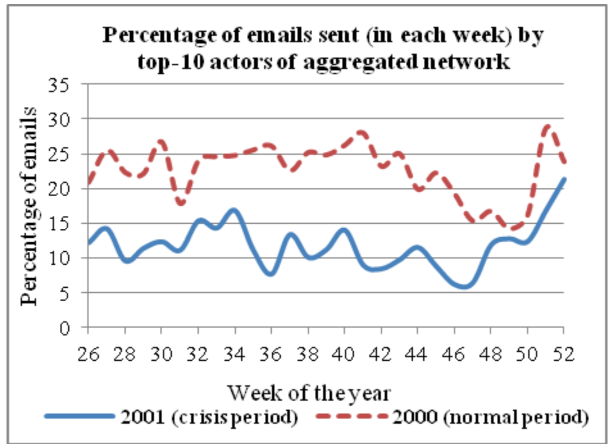

Figure 4: Percentage of emails sent by top-10 actors (in each week) of the aggregated network

3.2.4. Actor Participation. The number of actors participates (i.e., sent email to other) in the corresponding week of the year 2000 and 2001 is compared. Participation of actors is higher during the *'crisis'* period (i.e., 2001) compared to the *'normal'* period (i.e., 2000) as illustrated in Figure 5. There is a sharp increase in the participation statistics during the '*crisis*' period; however, it remains almost consistent during the '*normal*' period.

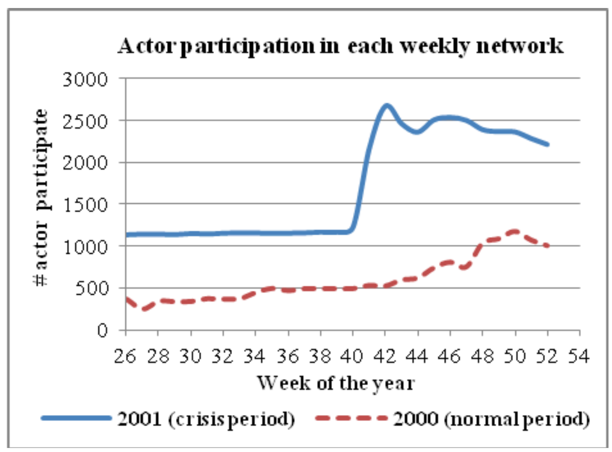

Figure 5: Actors' participation statistics in the weekly networks during the '*normal*' and '*crisis*' period.

3.3. Experimental Outcome from this Research Example

In this research example, the proposed topological framework is successfully exercised, by designing and conducting four CATs, to capture actor-oriented dynamics for a longitudinal communication network. From the first CAT (i.e., Rank of Actors) it is observed that positions of top-ranked actors, both in weekly networks and in the aggregated network, are more stable during the *'normal'* period: (i) top-10 actors of the aggregated network are found most times in weekly networks (Table 1); (ii) all of the top-10 actors (that are found most times in all weekly networks) are also found in the aggregated network (Table 2); and (iii) larger range of the number of times top-ranked actors are found in all weekly networks (Table 3). Less number of centrality overlaps (among all weekly networks, and between each of weekly networks and the aggregated network) are noticed during the *'crisis'* period from the second CAT (i.e., Centrality Overlap). From the third CAT (i.e., Activity of Actors), it is found that top-10 actors of both weekly networks and the aggregated network involved in more activity (i.e., sent more emails) during the *'normal'* period compared to the '*crisis*' period. More actors participate in the weekly networks during the '*crisis*' period as notice in the last CAT (i.e., Actor Participation).

From the outputs of four CATs, it is noticed that actors' were showing more stable communication behaviour during the '*normal*' period compared to the '*crisis*' period of Enron. Actors showed superior ranking in the top-ranked list, higher centrality overlap, more activity, and consisting participation statistics in the email networks during the '*normal*' period compared to the '*crisis*' period. This is because of Enron's organizational practice culture. Numerous post-mortem studies have commented on Enron's culture of individualized self-enrichment. Enron celebrated a culture of accelerated performance coupled with the pressure to innovate at speed [25, 26]. The highly individualistic and winner-takes-all culture was so powerful that it led some senior employees to blur the line between legal and illegal activity, through corrupt and unethical exploitation of accounting regulations. These senior people usually show prominent behaviour in the email communication network; however, when they realized the possible future economical downfall of Enron during mid 2001 they greatly reduced email communication [25]. Moreover, many senior employees quit Enron while new senior-level employees were hired during this time. These newly hired employees became prominent as they might have been involved with a particular task for a certain period of time during the '*crisis*' period [27]. Besides, as the organization was going through a period of crisis, some of the employees also left the organization due to job insecurity. It is plausible that these issues caused Enron's email network to show less stable communication behaviour during the '*crisis*' period compared to the '*normal*' period. Finally, for the analysis of longitudinal network the proposed topological framework of this study successfully captures these behavioral differences of Enron's communication network.

4. Discussion

Longitudinal networks evolve over time. Therefore, existing SNA measures (e.g., centrality [28]) and model (e.g., exponential random graph model [29] [30]), that are mostly applicable to static networks, cannot capture complete dynamics of longitudinal networks. This study proposed a novel framework for analyzing longitudinal networks. Node-level structural positions of actors in each of the short-interval networks are compared to their positions in the aggregated network. In other words, this framework is based on a cross comparison of the structural positions of actors in short-interval networks and in the aggregated network. This comparative approach to study networks ultimately facilitates the ease of capturing dynamics of individual actor in the longitudinal network.

The proposed research framework demonstrates the ability to identify and capture some special or irregular events that may happen to any longitudinal network. By continuously observing the node-level involvement of individual actor, this research framework can trace the point when a new actor joins the network, or the time an existing actor departs from the network, and the interval of time when an actor may be absent in the network or highly involved with other actors within the network. These kinds of instances could potentially be captured through the dynamic topology of the proposed research framework. The dynamic topology, which measures structural position of each actor, is applied to short-interval networks that constitute the aggregated network. Thus, a significant change to the structural measure for any actor indicates an alternation to the network participation (i.e., may leave the network, or join the network, or may hide from the network, or may join after hiding for some period) of that actor. Furthermore, it is also possible to associate these node-level changes to node attribute-level data. For instance, a new node may suddenly appear at a certain point in time displaying high levels of out-degree centrality in dynamic network topology and its appearance may be attribute to an organizational position (e.g., a vice-president has just been hired and started to direct and manage new and current projects).

To calculate the top-ranked list in the research example, this study considers out-degree centrality which represents activity of actors in a network. When examining particular measures of prominence, it is important to consider carefully whether a given measure is applicable to the network in question. In the context of organization, activities of actors reflect their level of involvement in the procedural processes of organizations. Other centrality measures (e.g., closeness centrality) and network measures (e.g., distance) could also be considered for measuring the top-ranked list. For instance, in-degree centrality, which indicates actor popularity [31], could potentially be used to calculate top-ranked list for political networks because level of the acceptability of actors is very important in this type of network. Similarly, in the context of advice seeking network, where a tie represents seeking advice or support, an actor with a high in-degree is especially prestigious. However, this measure is not suitable to measure prominence for our research context as in an organization many emails are sent as general notice, advice, and report. These types of emails will increase the tally of in-degree for all actors; however, they do not convincingly represent any prominent behaviour. On the other hand, betweenness centrality, which reflects actors' control of communication, might be the best choice to calculate the top-ranked list for a broker network. We noticed that in our dataset all the actors of the short-interval networks and the aggregated networks are not reachable and most actors belong to small clusters or cliques. Betweenness centrality therefore does not represent the prominence in our dataset.

Only four comparative analysis techniques (CATs) (i.e., Rank of Actor, Centrality Overlap, Activity of Actor, and Actor Participation) were designed and implemented for conducting topological analysis of email communication network in the research example. However, for topological analysis more CATs can be designed to compare actors' behaviour between short-interval networks and the aggregated network. For example, a correlation test could be conducted to investigate actors' level of involvement within the network (which could be captured by centrality measure) between any two consecutive short-interval networks, and between any short-interval network and the aggregated network. To give another example, the network size (i.e., number of actors participate in the network) for short-interval networks compared to the aggregated network could be analyzed to explore the statistics of entrance and departure of actors to the longitudinal network under study.

The methodological contribution of this paper is certainly noteworthy. By following static and dynamic topology, this study shows a framework about how to analyze longitudinal networks. Static topology can examine overall behaviour of actors (from the aggregated network) whereas dynamic topology is suitable for capturing temporal behaviour of actors over time. For capturing the overall dynamics of longitudinal social networks, it is therefore required to compare the temporal behaviour of actors, which can be captured using the dynamic topology, with the overall behaviour of actor as collected by the static topology. The four proposed CATs can do this comparison between static and dynamic topological analysis of social networks.

5. Conclusion and Future Research Direction

This study proposes a research framework based on network topology to capture the dynamics of longitudinal networks. Further, this paper shows an example where topological approach to analyze longitudinal network is employed to capture network dynamics of the email communication network of a large organization.

Recent studies on complexity research argue that dynamics of any complex network could be captured by having a holistic view of the complete network under study rather than investigating each possible link [32, 33]. This is especially true for very large complex networks. For these types of networks, traditional methods (e.g., exponential random graph models) have degeneracy, non-convergence, and computational issues. That means it is not mandatory to inspect each link in a network for understanding its evolutionary dynamics. Instead it is required to study network behavior and role of few dominant actors in a network to understand the complete evolutionary dynamics of that network. In this regard, the proposed topological framework of this study is very appropriate for capturing dynamics of longitudinal networks because this framework emphasizes on the actor-level involvement of prominent individual actor.

For small-sized networks, currently there are models (e.g., stochastic actor-based model and exponential random graph model) available in present literature to capture the dynamics of the overall network. For these types of networks, the computational complexity is not an issue due to their network size. Combined with these models, the proposed topological approach will bring new research challenges to develop new optimized algorithms for many current prediction problems of 'network science' research arena, such as 'link prediction', and 'community detection'. Not only that, as the proposed topological framework can explore structural dynamicity of actors in short-interval networks and can compare structural positions of actors in short-interval networks with their positions in the aggregated network, it has the potential to enhance our present knowledge about how to break, control, or destroy a network, which is a very important research topic in many contexts, such as, 'Drug Users' Network', 'Military Network in War Field', and 'Disease Spread Network'. For example, the identification of prominent actors (in terms of spreading capability of disease) in an over-time 'Disease Spread

Network' by applying the proposed research framework of this study would guide us to quarantine those prominent actors, which will eventually lead to better control of that 'Disease Spread Network'.

A more specific future research direction of this study is that the proposed framework for topological analysis of longitudinal network could be applied to measure the 'degree of dynamicity' showed by an individual actor in an evolving network. Dynamicity for an actor indicates the level of variances (in respect of a social network analysis measure) showed by that actor in different short-interval networks compared to its position in the aggregated network of a longitudinal network [34]. The following equation could be a possible way to measure 'degree of dynamicity' showed by an individual actor in a longitudinal network:

$$\text{DDA} = \frac{\sum_{i=1}^{m} | OV_{AN} - OV_{SIN}^{i} |}{m}$$

Where, DDA represents 'degree of dynamicity showed by individual actor', OV_{AN} indicates observed value (say degree) in the aggregated network for a particular actor, OV_{SIN}^{i} indicates observed value for the same SNA measure (i.e., degree) in the i^{th} short-interval network (SIN) for that actor, and *m* indicates the number of SINs considered in the analysis. By using the above equation for DDA, the 'degree of dynamicity' showed by the complete network could be captured for any given time-evolving network by the following equation:

$$\text{DDN} = \frac{\sum_{i=1}^{n} (DDA^{*} - DDA^{i})}{n}$$

Where, DDN represents 'degree of dynamicity' showed by the complete network, DDA^{*} is the highest observed 'degree of dynamicity' showed by individual actor in the network, DDA^{i} is the 'degree of dynamicity' for actor *i*, and *n* is the number of actors in the network. These two constructs (i.e., DDA and DDN) eventually enable us to compare the degree of dynamicity of two or more different longitudinal networks.

Two longitudinal networks are analyzed and compared using the proposed topological framework in this study. Actor-level statistics of email communication networks are compared for the '*normal*' and '*crisis*' period data. Apart from the four comparative analysis techniques (CATs) proposed in this paper, other statistical methods could be employed to explore actors' behaviour in longitudinal networks. For instance, control chart theory and methods could be incorporated to compare actor-level behaviour for a single dynamic network over time. This theory and its corresponding available methods can monitor a stochastic process over time, and rapidly detect statistically significant departures from typical behaviour.

6. References

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