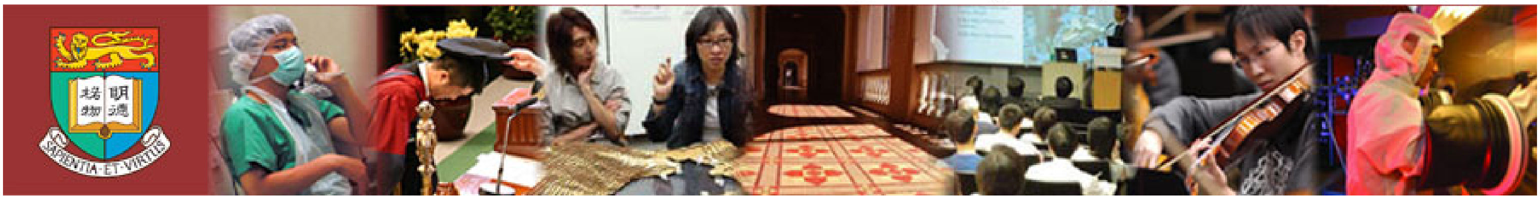

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An Incentive Scheme for Non-Cooperative Social Networks under the Independent Cascade Model

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***Abstract*—In this paper we analyze influence maximization for noncooperative social networks under the Independent Cascade Model. We propose a model of noncooperative nodes and prove some interesting properties of this model. Based on this, we further develop a game-theoretic model to characterize the behavior of noncooperative nodes, and design a Vickrey-Clarke-Groves-like scheme to incentivise cooperation. An advertiser can resolve the negative effect of noncooperation with our proposed solution. Evaluation on large social networks demonstrates the importance of cooperation and the effectiveness of our proposed incentive scheme in maximizing influence. We also discuss the budget allocation between seed nodes activation and incentives to non-seed nodes.**

***Keywords*—influence maximization; cooperative; social network**

I. Introduction

Identifying pilot users is critical for social-network advertisers to spread product adoption influence in the potential customer base, and thus to maximize client revenues. The famous influence maximization problem [1] is to select "optimal" initial seed nodes. Nevertheless, non-pilot/intermediate user may also influence the spreading of product adoption. Generally, pilot users are more likely to help forward the product information to their social neighbors because they feel "special" as early adopters and have a sense of duty for accepting certain incentives (e.g., free sample or discount) from advertisers [2]. However, other users may not be willing to pass on the influence (or are noncooperative) since the action incurs cost (e.g., time, credibility, privacy, etc.). Therefore, we believe it is important to investigate influence maximization in noncooperative social networks.

In this paper, we study the influence maximization problem in a social network in which nodes are noncooperative in propagating the influence. Firstly we generalize the standard Independent Cascade Model (ICM) to take node noncooperation into consideration, and prove some nice properties of the corresponding model. Then we design a Vickrey-Clarke-Groves-like (VCG-like) incentive mechanism to stimulate user cooperation. We use data from a large academic collaboration network to evaluate our strategy. Results validate the importance of cooperation and the effectiveness of our proposed incentive scheme in maximizing influence. We also discuss the budget allocation between seed nodes activation and providing incentives to non-seed nodes.

We proceed as follows. Section II describes related work, Section III, model and property, Section IV, incentive scheme, Section V, evaluation, and Section VI, conclusion.

II. Related Work

Prior work on solving influence maximization problem focuses on formulating influence propagation models [3] and related algorithmic optimization problems [4]. [1] show the NP-hardness of the corresponding optimization problem and provide a greedy seed node selection heuristic that can achieve near-optimal performance. [5] applies game theory to study the phenomena of innovation spreading. However, these work do not account for the heterogeneity of cooperativeness between the seed and ordinary nodes during the influence propagation process. In the field of computer communications, the problem of noncooperative routing and load balancing have been studied (see, e.g. [6] and [7], respectively) as examples of the impact of noncooperation on networked systems. In this paper we address the noncooperation problem in the online social advertising scenario. A natural approach to overcome node noncooperation is to provide incentives. Designing incentive mechanism has long been a hot topic in networking research. For example, [8] studies incentive issues in participatory sensing applications and design a Reverse Auction based Dynamic Price (RADP) mechanism to stimulate user participation. The VCG auction scheme has also been applied to design incentive schemes. [9] implements a variation of the VCG scheme in a mobile ad hoc network consisting of selfish nodes so that all nodes will report the true information.

III. System Model

We first formulate the influence maximization problem and introduce the noncooperative ICM. Following our prior work [10] which proved the submodularity of the noncooperative Linear Threshold Model (LTM), we further present several useful properties of the noncooperative ICM.

A. Problem formulation

We consider an online social network (OSN) as a directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$, where $\mathcal{V}$ is the set of nodes (OSN users) and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges (social ties) in the network. We also denote by $\mathcal{N}_u \subseteq \mathcal{V}$ the set of neighbors of node u. Each node in the system can either be active or inactive. As more neighbors of an inactive node become active, it is more likely

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to switch to being active. A node cannot return to the inactive state once it becomes active. All nodes are inactive at the beginning of the influence propagation process and marketing practitioners initially activate K nodes to seed the information cascade in the social network. The process ends when no more nodes can be activated. The influence maximization problem is defined as follows: Determine the K-node seed set to achieve the maximal expected active nodes at the end of the process.

B. Diffusion models

The Independent Cascade Model (ICM) [11] is a popular diffusion model for the propagation effect. In ICM, a node i activated at time t has a probability $p_{i,j}$ to successfully activate its inactive neighbor j at time $t+1$. Node i does not have any further opportunities to activate j again whether it succeeds or not.

C. Noncooperative influence maximization under ICM

Traditional ICM implicitly assumes that nodes in the system will not reserve their influence capacities during the propagation process. To account for non-cooperativeness the standard ICM is generalized such that Node j is activated by Node i with probability $\alpha_{i,j} \cdot p_{i,j}$, where $\alpha_{i,j} \in [0, 1]$ is the cooperativeness level of Node i on its neighbor Node j. We assume that node cooperativeness levels are static during the entire diffusion process.

D. Properties of the noncooperative ICM

We now discuss some nice properties of the noncooperative ICM. First we define a set function $\sigma(\cdot)$ to be submodular if $\sigma(S \cup \{v\}) - \sigma(S) \geq \sigma(T \cup \{v\}) - \sigma(T)$ for all $v \in \mathcal{V} \setminus T$ and $S \subseteq T$, i.e., $\sigma(\cdot)$ satisfies a "diminishing returns" requirement: the marginal gain from adding a node to a set T is at most the same as the marginal gain from adding the same node to a subset of T. In addition, we say that $\sigma(\cdot)$ is monotone if $\sigma(T) \geq \sigma(S)$ for all $S \subseteq T$, that is, $\sigma(\cdot)$ will at least stay the same after adding elements to the original set. We also define a greedy algorithm as follows: starting from an empty set, the algorithm iteratively selects a seed which achieves the highest incremental change of $\sigma(\cdot)$. [12] proves that a non-negative, monotone submodular objective function can be approximated to within a factor of $(1-1/e)$ (around 63%, here e is the base of the natural logarithm) using the greedy algorithm.

Theorem 1. *[12] The greedy algorithm is a $(1-1/e)$ approximation for a non-negative, monotone submodular objective function.*

[1] further proves that the greedy algorithm can also achieve $(1 - 1/e)$ approximation for the influence maximization problem by proving that the final influence function $\sigma(\cdot)$, which is the expected number of the final active nodes in the network at the end of the diffusion process, is submodular. Based on [1], we prove that the influence function under the proposed noncooperative ICM also satisfies the requirement of submodularity, so that a greedy algorithm can also achieve the same $(1 - 1/e)$ performance guarantee.

Lemma 1. *[1] The influence function $\sigma(\cdot)$ is submodular for an arbitrary instance of the ICM.*

Theorem 2. *The influence function $\sigma(\cdot)$ of noncooperative ICM is submodular.*

Proof: Since the cooperativeness parameters $\alpha_{i,j}$ are static, the noncooperative ICM is equivalent to a standard ICM in which $p'_{i,j} = \alpha_{i,j} \cdot p_{i,j}$. Thus, according to Lemma 1, the influence function of noncooperative ICM is also submodular. ■

Proving that the influence function under noncooperative ICM also satisfies the requirement of submodularity not only shows that the model has a performance guarantee, but also implies that the incentive needed for the advertising campaign should show similar property, since the amount of incentive needed is closely related to the seed-node set size. It is also intuitively satisfying that incentive as a function of seed-node set size would show a "diminishing returns" property. The detailed study of the relationship between the amount of incentive and seed-node set size in noncooperative influence maximization problem will be our future work.

Note that we have to use Monte-Carlo simulations to estimate $\sigma(\cdot)$ because there is no explicit formula for the influence function. This means that we can obtain a $(1 - 1/e - \epsilon)$ approximation with small ϵ if we run a large number of simulations.

IV. The VCG-like incentive scheme

We introduce an incentive mechanism to solve the node noncooperation problem in this section. We first introduce a game-theoretic framework to model node noncooperation in influence propagation. Next, we describe the incentive method and derive some nice properties of the mechanism, namely, individual-rationality (IR) and incentive-compatible (IC). Then we compare the proposed scheme to a fixed price incentive mechanism to show some of its other advantages. Finally, we discuss implementation details of the proposed mechanism.

A. A VCG-like incentive mechanism to solve the noncooperation problem

We define $C(i)$ as the cost of individual node i during the influence diffusion process. The utility of an individual node without payment should be

$$\begin{aligned} U_i &= -C(i) \\ &= -D \cdot \sum_{j\, neighbor\, of\, i} \alpha_{i,j} \cdot p_{i,j} \end{aligned} \tag{1}$$

In (1) we model $C(i)$ as the sum of the influence probabilities Node i imposes on all its neighbors mainly to reflect the fact that the more a single node can impact its friends, the more reward it will ask for from the initiator of the viral marketing campaign, because "influence" here is considered a scarce commodity. Also an influential node (e.g., a celebrity) in the social network may have already expended a large amount of resources (e.g., time, money, privacy, etc.) in order to cultivate its impact. $D \geq 0$ is the cost-of-influence parameter, which converts the amount of influence an individual node exerts into cost. It is assumed to be constant over the whole network and known to the operator in our model.

The action of Node i is denoted as $\alpha_i = (\alpha_{i,1}, \alpha_{i,2}, \ldots, \alpha_{i,|\mathcal{N}_i|})$, in which $0 \leq \alpha_{i,j} \leq 1$, $j = 1, 2, \ldots, |\mathcal{N}_i|$. We also assume that all nodes determine their actions (i.e., cooperativeness levels) at the beginning of the game simultaneously.

Theorem 3. *Without payment, the strategy $\hat{\alpha}_i$ which constitutes the Nash equilibrium should be $\hat{\alpha}_{i,j} = 0$, j neighbor of i.*

Proof: Suppose that Node i chooses an action α_i different from $\hat{\alpha}_i$, with $\alpha_i = (\alpha_{i,1}, \alpha_{i,2}, \ldots, \alpha_{i,|\mathcal{N}_i|})$, *s.t.* $\exists \alpha_{i,j} \neq 0$. From the utility function (1) we can see that Node i can obtain a better payoff by setting $\alpha_{i,j} = 0$. Thus α_i is a strictly dominated action and cannot be used in any Nash equilibrium. So the strategy which constitutes the Nash equilibrium should be in the form $\hat{\alpha}_{i,j} = 0$, j neighbor of i. ■

Define VCG-like payment to Node i as

$$\begin{aligned} M_i &= q \cdot (\sigma(A) - \sigma_{-i}(A)) + C(i) \\ &= q \cdot (\sigma(A) - \sigma_{-i}(A)) + D \cdot \sum_{j\, neighbor\, of\, i} \alpha_{i,j} \cdot p_{i,j} \end{aligned} \tag{2}$$

where $\sigma(A) - \sigma_{-i}(A)$ is the difference of the expected final active node set size when Node i exists, and the expected size if Node i does not exist. $q \geq 0$ is the amount of reward the initiator is willing to pay for a successful activation. In other words, (2) means that besides compensating the individual cost $C(i)$, the initiator will additionally pay Node i for its contribution during the influence diffusion stage.

Our proposed VCG-like incentive scheme is different from the standard scheme in the following two aspects. First, in traditional mechanism design theory, the goal of VCG auction is to encourage each selfish agent in the game to disclose its private information ("types") to the auctioneer [13]. For example in [14], under the VCG payment scheme, each node may choose to report its true forwarding cost so that the least cost path can be found correctly. But the objective of our proposed incentive mechanism is to ensure that each selfish node is cooperative in the sense that they will exert all its influence capacity (i.e. $\alpha_{i,j} = 1$, $j = 1, 2, \ldots, |\mathcal{N}_i|$ for an arbitrary Node i). Second, in standard VCG, the cost the operator has to compensate is the reported value claimed by the selfish node, while in (2) the cost $C(i)$ is the actual cost Node i has exerted taking its cooperativeness level into consideration.

Although there are some differences, the proposed VCG-like payment (2) is similar to the standard VCG payment formula in structure in that they both consist of two parts: premium, and some kind of "cost" (reported value or true value). More importantly, the proposed VCG-like scheme shares some nice properties of the standard scheme, and such properties will be discussed next.

First we introduce Lemma 2 which is useful in showing that the VCG-like incentive scheme will encourage nodes in the network to be cooperative. In order to prove the lemma, an equivalent view of the ICM proposed in [1] needs to be described first.

The probability $p_{i,j}$ in ICM represents the likelihood Node i will activate Node j when Node i becomes active while at the same time Node j is inactive. The outcome of this random event can be viewed as the flipping of a coin of bias $p_{i,j}$. In fact we can flip the coin corresponding to each of the edges at the beginning of the cascading process and the result will only be revealed when Node i is active while its neighbor Node j is inactive. This change is equivalent to the original cascading process. After all the coins have been flipped in advance, we declare edges in G for which the coin flip result in heads as live and the remaining edges as blocked. In this graph, it is clear that a node will be active at the end of the cascading process if it is on a path consisting of only live edges from the target set A. Further we can see that the number of nodes that are active at the end of the cascading process will be the number of the nodes that are on paths consisting of only live edges from the target set A. This equivalent view also shows that the final activated set size under ICM is an order-independent outcome, that is, if a node has several newly activated neighbors, the order of their activating attempts will not affect the final result. For a detailed discussion on the equivalent view, readers are referred to [1].

Lemma 2. *The expression $\sigma(A) - \sigma_{-i}(A)$ is always non-negative, i.e., $\sigma(A) - \sigma_{-i}(A) \geq 0$.*

Proof: Based on the order-independent equivalent view of the ICM process [1], we can divide the diffusion process of one sample point X in a sample space S into two steps. The first step is to simulate the diffusion process in the whole graph, but assuming all the incoming edges of Node i to be "blocked" and Node i itself to be inactive. The active set size at the end of the first step is thus $\sigma_{X,-i}(A)$. In the second step, we keep the original states of incoming edges (blocked or live) of i, and activate Node i if it is in the original seed set A. The result at the end of the second step is thus $\sigma_X(A)$. If Node i is activated first at the beginning of step two (i.e. $i \in A$), then it is obvious that $\sigma_X(A) > \sigma_{X,-i}(A)$. Consider the case in which $i \notin A$, if there is a path from some node in A to i consisting entirely of live edges, then Node i will be active and in turn may possibly initiate a cascading process (i.e. $\sigma_X(A) > \sigma_{X,-i}(A)$), if not, Node i will end up inactive and the diffusion process ends (i.e. $\sigma_X(A) = \sigma_{X,-i}(A)$). In general, $\sigma_X(A) \geq \sigma_{X,-i}(A)$. Since

$$\sigma(A) = \sum_{X \in S} P[X] \sigma_X(A) \tag{3}$$

and

$$\sigma_{-i}(A) = \sum_{X \in S} P[X] \sigma_{X,-i}(A) \tag{4}$$

So $\sigma(A) - \sigma_{-i}(A) \geq 0$. ■

Lemma 3. *[15] Let $p' \in [0,1]^{|\mathcal{E}|}$ be the true influence* *probabilities on each edge. Given a target set A, then*

$$\sigma(A) = \sum_{i=1}^{n} u_i(p', A) + |A| \tag{5}$$

where $u_i(p', A)$ is the expected number of neighbors activated by Node i, given the target set.

Theorem 4. *The strategy $\hat{\alpha}_i$ which constitutes the Nash equilibrium should be $\hat{\alpha}_{i,j} = 1$, j neighbors of i, under the VCG-like payment scheme.*

Proof: Consider an arbitrary Node i, and fix the cooperativeness levels of the other nodes. If Node i is cooperative (i.e. $\alpha_{i,j} = 1$, j neighbors of i), with the VCG-like payment, the utility function of Node i now becomes

$$\begin{aligned} U_i &= M_i - C(i) \\ &= q \cdot (\sigma(A) - \sigma_{-i}(A)) \\ &+ D \cdot \sum_{j\, neighbor\, of\, i} p_{i,j} - D \cdot \sum_{j\, neighbor\, of\, i} p_{i,j} \\ &= q \cdot (\sigma(A) - \sigma_{-i}(A)) \end{aligned} \tag{6}$$

In Lemma 2 we have proved that $\sigma(A) - \sigma_{-i}(A) \geq 0$. Since $q \geq 0$, so $U_i \geq 0$.

Let $\alpha'_i = (\alpha'_{i,1}, \alpha'_{i,2}, \ldots, \alpha'_{i,|\mathcal{N}_i|})$, *s.t.* $\exists \alpha'_{i,j} < 1$ be the cooperativeness level of a noncooperative Node i, then the utility becomes

$$\begin{aligned} U'_i &= M'_i - C'(i) \\ &= q \cdot (\sigma'(A) - \sigma'_{-i}(A)) + D \cdot \sum_{j\, neighbor\, of\, i} \alpha'_{i,j} \cdot p_{i,j} \\ &- D \cdot \sum_{j\, neighbor\, of\, i} \alpha'_{i,j} \cdot p_{i,j} \\ &= q \cdot (\sigma'(A) - \sigma'_{-i}(A)) \\ &= q \cdot (\sigma'(A) - \sigma_{-i}(A)) \text{(Since the cooperativeness levels of other nodes are fixed)} \end{aligned} \tag{7}$$

Actually the true influence probability vector p' in Lemma 3 can be represented as (p'_i, p'_{-i}), where $p'_i = (p'_{i,1}, p'_{i,2}, \ldots, p'_{i,|\mathcal{N}_i|})$ is the true influence probability Node i has on its neighbors while p'_{-i} is the true influence probability vector on all other edges in the graph. According to Lemma 3, the final expected active node set size contains the expected number of neighbors activated by each node, given target set A. For each i, Node i can influence more neighbors when it is cooperative, i.e. $u_i((p_i, p'_{-i}), A) \geq u_i((p'_i, p'_{-i}), A)$. Thus $\sigma(A) \geq \sigma'(A)$ and $U_i \geq U'_i$. The cooperative strategy always maximizes the node utility. ∎

Theorem 4 implies that the VCG-like incentive scheme satisfies two important properties. The first property is individual-rationality (IR), that is, for each player, it is always better (i.e. achieving at least no less utility) to join the game than not participating. Combining Lemma 2 and Theorem 4 we can see that the individual utility of Node i is always nonnegative (0 is the utility when not participating in the game) under the proposed incentive scheme, so our scheme is IR. The other property is incentive-compatible (IC) — each player prefers to act in accordance with the objective of the mechanism. Theorem 4 has proved that the dominant strategy for a single node is to be cooperative to exert all its influence capacity under the VCG-like scheme, which is exactly the design objective of the proposed scheme, so the scheme is also IC. IR and IC are also two nice properties of the standard VCG auction [16].

B. Advantages of the VCG-like scheme over the fixed price incentive scheme

Another possible, also intuitive incentive scheme is as follows:

$$\begin{aligned} M_i &= \varepsilon + C(i) \\ &= \varepsilon + D \cdot \sum_{j\, neighbor\, of\, i} \alpha_{i,j} \cdot p_{i,j} \end{aligned} \tag{8}$$

where ε can be any arbitrary positive number. Under this scheme, besides compensating for the individual cost $C(i)$, the operator will also pay a fixed amount of incentive ε.

It can be easily shown that under the fixed price incentive scheme, being cooperative (i.e. $\alpha_{i,j} = 1$, $j = 1, 2, \ldots, |\mathcal{N}_i|$ for an arbitrary Node i) is the weakly dominant strategy for a selfish node. In other words, the utility of an individual node is the same (i.e. ε) whether it is cooperative or not. However, Theorem 4 has already shown that under the VCG-like scheme, being cooperative is the strongly dominant strategy. That is to say, the individual utility is maximized if a selfish node chooses to exert all it influence capacity. From this aspect the VCG-like scheme is superior to the fixed price scheme.

Another drawback of the fixed price scheme is that every node can get the same premium ε regardless of its ability to impact others. That means the fixed price scheme is not "fair" in the sense that the specific contribution of an individual node during the influence diffusion process is ignored. Some "influential" nodes in the social network may thus find this property discouraging. In contrast, in the VCG-like scheme (2), $\sigma(A) - \sigma_{-i}(A)$, which is the difference of the final performance when Node i does not exist, exactly quantifies the contribution of Node i during the diffusion process. To conclude, being more "fair" is another advantage of the proposed VCG-like incentive scheme.

C. Some technical discussions on the VCG-like incentive scheme

1) How do we get the cooperativeness levels of nodes?

Suppose the social network marketer has already determined the influence probability on each edge in the network through various methods (e.g. machine learning techniques [17]). Since the marketer has to pay the real cost (i.e. $C(i)$ in (2)), it is vital for the proposed incentive scheme to get the cooperativeness levels (i.e. $\alpha_{i,j}$) of nodes correctly. In this model we assume that both nodes on the edge (u, v) have various information about the properties of the edge. There is a similar assumption in [15]. The difference is that in our proposed incentive scheme, it is obvious that the influencer (i.e. node u in edge (u, v)) has motivation to lie about its cooperativeness level in order to get a higher payment. So in the VCG-like incentive scheme, the influencee (i.e. node v in edge (u, v)) will report the influence probability $p'_{u,v}$ the influencer node u has exerted on it to the marketer, thus

$$\alpha_{u,v} = \frac{p'_{u,v}}{p_{u,v}} \tag{9}$$

2) How do we calculate the premium given to Node i (i.e. $\sigma(A) - \sigma_{-i}(A)$)?

Another possible concern on the VCG-like incentive scheme is the calculation of $\sigma(A) - \sigma_{-i}(A)$ via simulation. Since both terms of the premium require the expected final active node set size, the efficiency would be greatly improved if we can reduce the Monte-Carlo simulation times needed while preserving the accuracy of the result.

To solve this problem, from the proof of Lemma 2, we see that the expression $\sigma(A) - \sigma_{-i}(A)$ at one sample point X (i.e. $\sigma_X(A) - \sigma_{X,-i}(A)$) is exactly the marginal increase in the active set size at the end of the present step compared to the previous step. So for each simulation run we can store the value of $\sigma_X(A) - \sigma_{X,-i}(A)$ and the average number over all simulation runs is the result desired. By using this method we can avoid running two simulations separately and the efficiency is hence improved.

V. Evaluation

In this section, we study the effect of node noncooperation on the system performance in terms of the final active set size under ICM on real large academic collaboration networks.

A. Dataset and influence model

The dataset utilized for evaluation is Arxiv's co-authorship network under the General Relativity and Quantum Cosmology category [18]. The graph constructed contains 4158 nodes and 26850 edges. Each node is an author, and an edge between two authors i and j means that they have co-authored a paper. We consider the co-authoring relationships between two authors only once in case they have co-authored more than one paper. For ICM, we set the activation probability $p = 5\%$ and $p = 20\%$, respectively. We adopt the two-tiered, static node cooperativeness here. That is, we set $\alpha_{i,j} = 1$ if Node i belongs to the seed-node set and $\alpha_{i,j} = \alpha < 1$, otherwise.

B. Centrality measure utilized under noncooperative ICM

Since seed node selection will have no effect on the performance of the proposed incentive scheme, the selection strategy can be the neighborhood-removal heuristic proposed in [19], the greedy algorithm or the centrality-based (degree or betweenness) schemes [1]. For ease of simulation we choose the simple degree-based centrality metric, which measures the influence of a node in terms of its out degree.

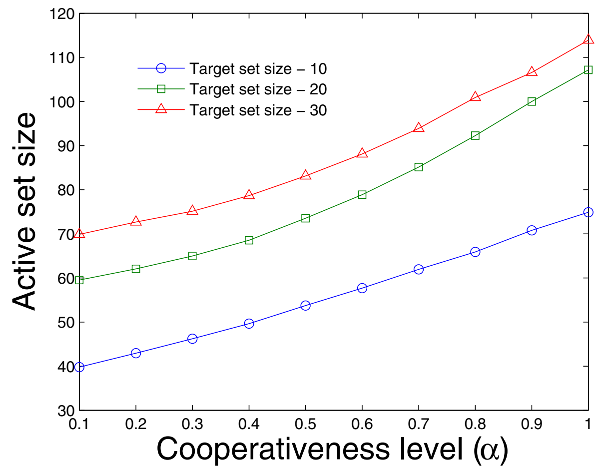

Fig. 1. Performance under ICM ($p = 5\%$) for different α and target set size

C. Result

Here we study the effect of node cooperativeness level on the performance of the degree-based seed node selection strategy. The results are shown in Figure 1 and Figure 2 under ICM with activation probability $p = 5\%$ and $p = 20\%$, respectively. The results are obtained as averages of 1000 simulation runs. The x-axis represents the cooperativeness level α and the y-axis represents the final active set size. From the figures we can see that the performance of the seed node selection scheme improves as α increases, and the performance of algorithms with a larger target set size is always superior. An intuitive implication for online social advertising practitioners is that in order to achieve an effective online viral marketing campaign, enough incentive should be offered to recruit enough initial product adopters to seed the influence cascade. In addition, incentives should be offered to other nodes in the system so that all nodes in the network would be cooperative to propagate the influence. We have proved in Theorem 4 that the Nash equilibrium for nodes under the proposed VCG-like incentive scheme is $\alpha = 1$, these simulation results also verify that the system performance (i.e. active set size) is optimal under the proposed incentive mechanism.

D. The budget allocation problem

Since the VCG-like incentive scheme requires a budget from the viral marketer, an interesting question arises: Since one only has limited budget, should one put all of the budget in selecting as many seed nodes as possible, or in making sure that all nodes in the network will be cooperative? There are some selection criteria suggested by the simulation results.

The final active set size as a function of the initial target set size under ICM is generally concave as shown in [19]. It is a natural result due to the submodularity of the influence function $\sigma(\cdot)$ ("diminishing return" property). We can also get the same conclusion from Figure 1 and Figure 2 because the

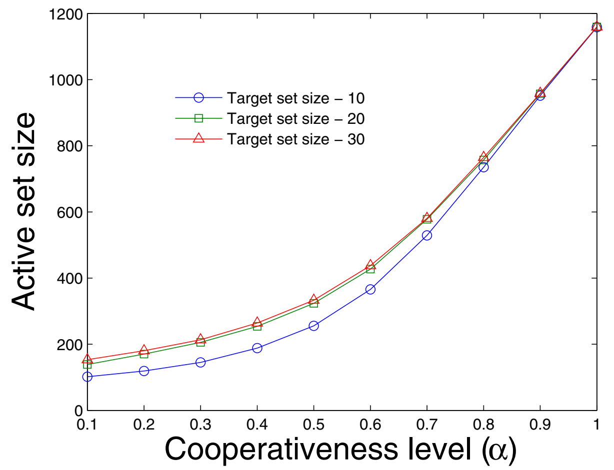

Fig. 2. Performance under ICM ($p = 20\%$) for different α and target set size

differences between the two simulation results under the same cooperativeness level α are getting smaller as the target set size increases. This phenomenon is most obvious for ICM when $p = 20\%$, in which case the performance of the influence maximization process under target set sizes of 20 and 30 are almost identical. It means that it may not be cost-effective for a viral marketer to use the budget entirely on the initial seed nodes. Figure 1 and Figure 2 also show that the final active set size as a function of the cooperativeness level α is convex, which means that the marginal increase in the performance of the influence maximization process is higher when nodes in the social network are more cooperative. Therefore, properly allocating the budget between initial seed node activation and incentives for non-seed nodes is an important problem, and we shall study this problem in our further research.

VI. Conclusion

In this paper, we investigate influence maximization in noncooperative social networks. We generalize ICM to take node noncooperation into consideration and provide a provable approximation guarantees for the noncooperative influence maximization problem. We also desgin a VCG-like incentive mechanism to solve the node noncooperation problem, showing it is IR and IC as well as having other nice properties. The evaluation based on noncooperative ICM shows the importance of cooperation and incentive in maximizing influence. In this study, we assume a two-tiered, static node cooperativeness in the system, i.e., seed nodes are cooperative to propagate the influence while ordinary nodes are only partly willing to do so. In the future, we plan to study the impact of noncooperation in other influence diffusion models, especially for those not satisfying the submodularity requirement. We also plan to study the proper allocation of the budget between initial seed node activation and incentives for non-seed nodes.

References

[1] D. Kempe, J. Kleinberg, and E. Tardos, "Maximizing the spread of influence through a social network," in *Proceedings of the ninth ACM SIGKDD*, 2003, pp. 137–146.

[2] J. E. Phelps, R. Lewis, L. Mobilio, D. Perry, and N. Raman, "Viral marketing or electronic word-of-mouth advertising: Examining consumer responses and motivations to pass along email," *Journal of Advertising Research*, vol. 44, no. 04, pp. 333–348, 2004.

[3] J. Leskovec, L. A. Adamic, and B. A. Huberman, "The dynamics of viral marketing," *ACM Trans. Web*, vol. 1, no. 1, 2007.

[4] P. Domingos and M. Richardson, "Mining the network value of customers," in *Proceedings of the seventh ACM SIGKDD*, 2001, pp. 57–66.

[5] H. Young, "The Diffusion of Innovations in Social Networks," *Economics Working Paper Archive*, 2000.

[6] T. Roughgarden and É. Tardos, "How bad is selfish routing?" *J. ACM*, vol. 49, no. 2, pp. 236–259, 2002.

[7] H. Kameda and O. Pourtallier, "Paradoxes in distributed decisions on optimal load balancing for networks of homogeneous computers," *J. ACM*, vol. 49, no. 3, pp. 407–433, Oct. 2002.

[8] J.-S. Lee and B. Hoh, "Sell your experiences: a market mechanism based incentive for participatory sensing," in *IEEE International Conference on Pervasive Computing and Communications*, 2010.

[9] L. Anderegg and S. Eidenbenz, "Ad hoc-VCG: a truthful and cost-efficient routing protocol for mobile ad hoc networks with selfish agents," in *Proceedings of the MobiCom*, 2003, pp. 245–259.

[10] Y. Yang, V. O. K. Li, and K. Xu, "Influence maximization in noncooperative social networks," in *Proceedings of the IEEE GLOBECOM*, 2012.

[11] J. Goldenberg, B. Libai, and E. Muller, "Using complex systems analysis to advance marketing theory development: Modeling heterogeneity effects on new product growth through stochastic cellular automata," *Academy of Marketing Science Review*, vol. 2001, no. 9, pp. 1–18, 2001.

[12] G. L. Nemhauser, L. A. Wolsey, and M. L. Fisher, "An analysis of approximations for maximizing submodular set functions," *Mathematical Programming*, vol. 14, pp. 265–294, 1978.

[13] Y. Narahari, D. Garg, R. Narayanam, and H. Prakash, *Game theoretic problems in network economics and mechanism design solutions*. Springer, 2009.

[14] H. Zhou, K.-C. Leung, and V. O. K. Li, "Auction-based schemes for multipath routing in selfish networks," in *Proceedings of the IEEE WCNC*, 2013.

[15] M. Mohite and Y. Narahari, "Incentive compatible influence maximization in social networks and application to viral marketing," in *The 10th International Conference on Autonomous Agents and Multiagent Systems*, 2011, pp. 1081–1082.

[16] V. Krishna, *Auction Theory*. Academic Press, 2002.

[17] A. Goyal, F. Bonchi, and L. V. Lakshmanan, "Learning influence probabilities in social networks," in *Proceedings of the third ACM international conference on Web search and data mining*, 2010, pp. 241–250.

[18] J. Leskovec, J. Kleinberg, and C. Faloutsos, "Graph evolution: Densification and shrinking diameters," *ACM Trans. Knowl. Discov. Data*, vol. 1, 2007.

[19] X. Fan and V. O. K. Li, "The probabilistic maximum coverage problem in social networks," in *Proceedings of the IEEE GLOBECOM*, 2011.