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# Optimal Phasor Data Concentrator Installation for Traffic Reduction in Smart Grid Wide-Area Monitoring Systems 

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#### Abstract

As one of the core components in wide-area monitoring systems (WAMS), phasor measurement units (PMUs) acquire highly accurate and time-synchronized phasor data at high frequency for smart grid monitoring, protection, and control. Despite the advantages of PMUs, they do generate much data and create a heavy burden on the communication network. One way of alleviating such burden is to install phasor data concentrators (PDC) across the power system to concentrate data generated by the PMUs. Although PDCs are expensive as well, this may still be a much cheaper and more practical option than building a high bandwidth network for WAMS. Therefore, it is very important to solve the optimal PDC installation problem so as to achieve a desired level of traffic reduction. This paper is the first to address this problem and we give solutions for the IEEE 14-bus, 30 -bus, and $\mathbf{5 7}$-bus systems.


## I. Introduction

Synchronized by Global Positioning Systems (GPS), phasor measurement units (PMU) acquire highly accurate and timesynchronized phasor data at high frequency for the wide-area monitoring system (WAMS) [1] [2]. Significant improvements in the performance of power system monitoring, protection, and control have been demonstrated with PMUs, rendering PMUs among the key components of smart grid [3] [4].

As more PMUs are installed, the underlying communication network may face significant challenges due to the large volume of traffic it has to carry. The situation will get worse when future applications for smart grid are deployed and more traffic is generated. When the network gets overwhelmed by the heavy traffic, excessive communication delays are incurred, adversely affecting the performances of WAMS and many other applications. One possible solution is to invest in a network with much higher capacity, but this may be prohibitively expensive. Therefore, phasor data concentrator (PDC), a device that is capable of concentrating phasor data from multiple PMUs, has the potential to serve as a more affordable and practical alternative,.

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Fig. 1. A typical wide-area monitoring system (WAMS).

According to [5], [6], and [7], the major functions of a PDC include:

1) Data concentration. PMU data passing through a PDC can be compressed and concentrated so less traffic will be generated downstream.
2) Time alignment. Data received from multiple PMUs are aligned by a PDC according to their time stamps and combined into a single packet. Outdated PMU data are dropped so they do not unnecessarily occupy bandwidth downstream.
3) Data check. Corrupted or incorrect data can be detected and dropped by PDC so that such data do not have to be sent to the control center.
4) Local data archiving. A PDC usually comes with a local database that is capable of storing all PMU data it has processed over some time period.
In the traditional WAMS design, as shown in Fig. 1, only one PDC is needed in a power system at the control center to gather and process all PMU data. However, if some more

PDCs besides the one at the control center are deployed across the system, not only is the total amount of PMU traffic reduced, but the reliability of WAMS can also be enhanced thanks to the local database at each PDC.

Intuitively, if every PMU is installed together with a PDC, the minimum amount of PMU traffic will result. However, due to the relatively high cost of PDC, it may not be the desired solution for every power company. Therefore, it will be important for a power company to determine the least number of PDCs required such that the desired performance can be achieved. But no existing work has addressed this yet.

Therefore, the main objective of this paper is to propose an optimal PDC installation (OPI) model that computes the minimum number of PDCs needed so as to achieve a desired traffic reduction. The proposed problem is then solved on the IEEE 14-bus, 30 -bus, and 57 -bus systems. Although there are other advantages of having extra PDCs in a WAMS, due to the space limitations, we only consider traffic reduction in this paper. Reliability enhancement will be discussed in our future work.

The rest of the paper is organized as follows. After the introduction, we present our proposed OPI model in Section II. Numerical studies and results are given in Section III. The paper concludes in Section IV.

## II. Optimal PDC Installation Problem

In this section, we present the OPI problem that models how to optimally install additional PDCs in a power system as so to reduce the traffic in WAMS. Before discussing the OPI problem formulation in Section II-B, we will first introduce the traffic model in WAMS in Section II-A.

## A. WAMS Traffic Model

When a PMU is installed on a bus, it measures the voltage phasors on this bus and calculates those on all its neighboring buses. In other words, a PMU installed on a bus with $k$ branches will transmit $(k+1)$ phasor data to the control center. According to the IEEE synchrophasor data standard [6] and [7], the amount of data that PMU $i$ generates per second, $D_{i}$, can be estimated by:

$$
\begin{equation*}
D_{i}=((k+1) \times p+\alpha) \times F_{s} \tag{1}
\end{equation*}
$$

where $p$ is the size of the data portion in a single phasor data frame, $\alpha$ is the size of the frame overhead generated by this PMU, and $F_{s}$ is the configured phasor data frame reporting rate. For instance, when $p=500$ Bytes and $\alpha=50$ Bytes, a PMU installed on a bus with 4 branches and configured to run at 120 frames per second, namely, $F_{s}=120 \mathrm{fps}$, will generate slightly over 300 kilo-Bytes per second traffic towards the control center.

Since the size of overhead is normally very small compared with the size of phasor data, $\alpha$ can be dropped from $D_{i}$, resulting in a linear function of $k$ :

$$
\begin{equation*}
D_{i}=(k+1) \times p \times F_{s} \tag{2}
\end{equation*}
$$



Fig. 2. A six-bus system with two PMUs installed.

In order to simplify our study, we assume that the communication links in the system are homogeneous. Then, the traffic in WAMS without PDC, $g$, measured in the unit of kilo-Byte-link per second, can be calculated as:

$$
\begin{equation*}
g=\sum_{i \in \mathbf{U}} D_{i} P_{i}^{c} \tag{3}
\end{equation*}
$$

where $\mathbf{U}$ is the set containing all buses where PMUs are installed, $D_{i}$ is the size of PMU data from Bus $i$, and $P_{i}^{c}$ is the shortest distance (measured by number of links) between Bus $i$ and the control center.

Let us consider an example system as shown in Fig. 2, where two PMUs are installed on Bus 1 and Bus 4, respectively, and the control center is located on Bus 3. To deliver the PMU data from Bus 1 to the control center, $5 p F_{s}$ kilo-Bytes per second traffic has to be carried over the link between Bus 1 and Bus 2 and that between Bus 2 and Bus 3. Similarly, to deliver the PMU data from Bus $4,3 p F_{s}$ kilo-Bytes per second traffic is carried over the link between Bus 3 and Bus 4. Therefore, the total WAMS traffic in this system, $g$, equals $13 p F_{s}$ kilo-Byte-link per second.

## B. OPI Problem Formulation

The objective of OPI is to minimize the number of PDCs installed in a given power system such that the total WAMS traffic is reduced from $g$ to a desired amount $\zeta \times g$. Let us describe the traffic reduction capability of a PDC by a function, $h()$. In other words, a PDC changes $D_{i}$ PMU data to $h\left(D_{i}\right)$, $0<h\left(D_{i}\right) \leq D_{i}$.

Then the OPI problem for an $N$-bus system is formulated as:

$$
\begin{align*}
\min . & \|\mathbf{C}\| \\
\text { s.t. } & f(\mathbf{C}) \leq \zeta \times g \tag{4}
\end{align*}
$$

where $\mathbf{C}$ is the set containing all buses where PDCs are to be installed plus the bus where the control center is located. $f(\mathbf{C})$ is the minimum WAMS traffic given the PDC installation set C. $f(\mathbf{C})$ is calculated as follows:

$$
\begin{equation*}
f(\mathbf{C})=\sum_{i \in \mathbf{U}} \min _{j \in \mathbf{C}}\left\{D_{i} P_{i, j}+h\left(D_{i}\right) P_{j}^{c}\right\} \tag{5}
\end{equation*}
$$

where $P_{i, j}$ is the shortest distance between Bus $i$ and Bus $j$. We define $P_{i, j}=0$ if $i=j$, forcing $D_{i} P_{i, j}$ in (5) to zero when a PDC is installed on the same bus as a PMU.

Since the traffic reduction capability of PDC is limited, no feasible solution can be obtained unless $\zeta$ satisfies:

$$
\begin{equation*}
1 \geq \zeta \geq \frac{f(\mathbf{U})}{g} \tag{6}
\end{equation*}
$$

This lower bound of acceptable $\zeta$ can be easily proved by contradiction.

Proof: Let $T_{i, j}=D_{i} P_{i, j}+h\left(D_{i}\right) P_{j}^{c}$, which represents the traffic needed for delivering $D_{i}$ PMU data from PMU $i$ to the control center. When a PDC is installed on this PMU bus $i, T_{i, i}=h\left(D_{i}\right) P_{i}^{c}$. Now assume this PDC is installed on bus $j^{\prime}, j^{\prime} \neq i$, and $T_{i, j^{\prime}}=D_{i} P_{i, j^{\prime}}+h\left(D_{i}\right) P_{j^{\prime}}^{c}$.

Since $h\left(D_{i}\right) \leq D_{i}$ and $P_{i}^{c} \leq P_{i, j^{\prime}}+P_{j^{\prime}}^{c}$ for any $j^{\prime}, T_{i, i} \leq$ $h\left(D_{i}\right) P_{i, j^{\prime}}+h\left(D_{i}\right) P_{j^{\prime}}^{c} \leq T_{i, j^{\prime}}$. Therefore, when $\|\mathbf{C}\|=\|\mathbf{U}\|$, $\sum T_{i, i} \leq \sum T_{i, j^{\prime}}$. And hence, for any $\mathbf{C}, f(\mathbf{U}) \leq f(\mathbf{C})$.

To understand Equations (4) and (5), let us consider the 6bus system shown in Fig. 2. Without any PDC, the total traffic equals $11 p F_{s}$ kilo-Byte-link per second. Assuming $h\left(D_{i}\right)=$ $\frac{D_{i}}{2}$, when one PDC is allowed, one can immediately see that this PDC should be placed either on Bus 1 or on Bus 4. When installed on Bus 1, $7 p F_{s}$ kilo-Byte-link per second traffic will result, and a PDC on Bus 4 will generate $9.5 p F_{s}$. Therefore, $f(\mathbf{C})=\frac{7}{11} g$ when $\|\mathbf{C}\|=1$. When two PDCs are allowed, installing them on Bus 1 and Bus 4, respectively, will result in $5.5 p F_{s}$ kilo-Byte link per second traffic. In other words, $f(\mathbf{C})=\frac{1}{2} g$ when $\|\mathbf{C}\|=2$, and further increasing the number of PDCs will not help. In this example, as can be concluded, if a power system manager desires $\frac{7}{11} \leq \zeta<1$, having one PDC installed on Bus 1 will be suggested; or if he/she desires $\frac{1}{2} \leq \zeta<\frac{7}{11}$, he needs to have two PDCs installed on Bus 1 and Bus 4, respectively.

## C. Complexity of OPI Problem

Consider a power system of $N$ buses (nodes) and $E$ branches (edges) with $P=\|\mathbf{U}\|$ PMUs installed $(P<N)$. Since $1 \leq\|\mathbf{C}\| \leq P$, the search space includes $P$ different $\|\mathbf{C}\|$ values. For any given $\|\mathbf{C}\|$, installing these PDCs onto $N$ possible buses will introduce $(N N$ any PDC installation scheme, (5) requires $2 N$ multiplications and $N$ additions. Besides, all values of $P_{i, j}$ and $P_{i}^{c}$ can be pre-computed, with complexity $N \times O\left(E^{2}\right)$.

Therefore, with all constants dropped, the complexity of the OPI problem can be estimated as follows:

$$
\begin{align*}
& P \times\binom{ N}{\|\mathbf{C}\|} \times N+N \times O\left(E^{2}\right) \\
= & O(N \times P \times N!) \tag{7}
\end{align*}
$$

Due to the space limitations, we will leave the proof of NP-completeness of this problem to our future work.


Fig. 3. IEEE 14-bus system [8].

## III. Numerical Study

In this section, we will solve the OPI problem on the IEEE 14-bus, 30-bus, and 57-bus systems. The minimum number of PMUs are placed such that the systems are fully "observable" [9]. The $h\left(D_{i}\right)$ we will use take this linear form for simplicity:

$$
\begin{equation*}
h\left(D_{i}\right)=0.56 \times D_{i}, \tag{8}
\end{equation*}
$$

which is an estimation provided by a PMU manufacturer [10].
Let $\eta_{\|\mathbf{C}\|}$, calculated as

$$
\begin{equation*}
\eta_{\|\mathbf{C}\|}=\frac{f(\|\mathbf{C}\|)}{g} \tag{9}
\end{equation*}
$$

denote the percentage of WAMS traffic reduction with $\|\mathbf{C}\|$ PDC. For a given $\zeta$, if $\exists i$ such that $\eta_{i} \leq \zeta<\eta_{i-1}$, then the solution to (4) is $\|\mathbf{C}\|=i$.

Since OPI is NP-complete, we can only obtain the optimal solution by exhaustive enumeration. Given the number of PDCs allowed, we obtain all possible PDC installation strategies and compare the resulting $\eta$ values. The computation time needed increases exponentially with the problem size, but we managed to get optimal solutions for systems with up to 57 buses.

## A. IEEE 14-bus System

As shown in Fig. 3, four PMUs have been installed on Bus 2, Bus 6, Bus 7, and Bus 9, respectively. With the control center located on Bus 8, these four PMUs will generate $49 p F_{s}$ kilo-Byte-link per second traffic, if no PDCs are installed across the system.

If one PDC is allowed to be installed, placing this PDC on Bus 4 will reduce the WAMS traffic to $40.2 p F_{s}$, which is around $18 \%$ reduction from the no-PDC case. When one more PDC is allowed, placing the two PDCs on Bus 2 and Bus 6, respectively, will reduce the traffic to $33.6 p K_{s}$, which is only

TABLE I
Results of OPI for IEEE 14-BUS SYSTEM

| $g=49.0$ |  |  |
| :--- | :--- | :--- |
| PMU Locations: 2679 |  |  |
| $\\|\mathbf{C}\\|=1$ | $f(\\|\mathbf{C}\\|)=40.2$ | $\eta_{1}=0.820$ |
|  | PDC Locations | 4 |
| $\\|\mathbf{C}\\|=2$ | $f(\\|\mathbf{C}\\|)=33.6$ | $\eta_{2}=0.686$ |
|  | PDC Locations | 26 |
| $\\|\mathbf{C}\\|=3$ | $f(\\|\mathbf{C}\\|)=29.2$ | $\eta_{2}=0.596$ |
|  | PDC Locations | 269 |
| $\\|\mathbf{C}\\|=4$ | $f(\\|\mathbf{C}\\|)=27.4$ | $\eta_{2}=0.560$ |
|  | PDC Locations | 2679 |



Fig. 4. $\quad \eta$ with respect to number of PDCs in IEEE 14-bus system [8].
$68.6 \%$ of what was required with no PDC. The traffic hits the minimum value of $27.4 p F_{s}$, namely, $56 \%$ of the no-PDC case, when the number of PDCs reaches four, each installed on the four PMU buses, respectively. The detailed PDC installation locations and resulting traffic can be found in Table I.

With $\eta_{1}, \eta_{2}, \eta_{3}$, and $\eta_{4}$ computed and shown in Fig. 4, problem (4) can be easily solved for any user input $\zeta$. For instance, when a power system manager desires to reduce the WAMS traffic by $40 \%$, namely, $\zeta=0.6$, we can see from Fig. 4 that three PDCs are needed to achieve the target. From Table I, we can obtain the solution as $\mathbf{C}=\{2,6,9\}$.

## B. IEEE 30-bus System

According to [9], ten PMUs are placed on the IEEE 30bus system. With the control center located on Bus 8, the ten PMUs will inject $116 p F_{s}$ traffic into the network if no PDCs exist in the network. With one PDC added, the traffic will be reduced by $15.6 \%$ to $98.0 p F_{s}$, and two PDC will bring the traffic down by $22.4 \%$ to $90.0 p F_{s}$. At the extreme case, when ten PDCs are placed together with the ten PMU, the total traffic within the network becomes $65.0 p F_{s}$, only $56 \%$ of the original traffic. The solutions to all cases are shown in Table II.

By constructing the $\eta$ graph as shown in Fig. 5, it is very easy to find the solutions for any $\zeta$. For instance, when $\zeta=0.6$ is desired, we have $\eta_{8}<0.6<\eta_{7}$, indicating eight PDCs to be installed. Looking up Table II, we obtain the solution to be: $\mathcal{C}=\{2,6,10,12,19,23,25,29\}$.

TABLE II
Results of OPI for IEEE 30-bus system

| $g=116.0$ |  |  |
| :---: | :---: | :---: |
| PMU Locations: 2469101219232529 |  |  |
| $\\|\mathbf{C}\\|=1$ | $f(\|\|\mathbf{C}\|\|)=98.0$ | $\eta_{1}=0.844$ |
|  | PDC Locations | 6 |
| $\\|\mathbf{C}\\|=2$ | $f(\\|\mathbf{C}\\|)=90.0$ | $\eta_{2}=0.776$ |
|  | PDC Locations | 612 |
| $\\|\mathbf{C}\\|=3$ | $f(\\|\mathbf{C}\\|)=83.9$ | $\eta_{2}=0.723$ |
|  | PDC Locations | 61227 |
| $\\|\mathbf{C}\\|=4$ | $f(\\| \mathbf{C}\| \|)=79.5$ | $\eta_{2}=0.685$ |
|  | PDC Locations | 6101227 |
| $\\|\mathbf{C}\\|=5$ | $f(\\| \mathbf{C}\| \|)=76.4$ | $\eta_{2}=0.659$ |
|  | PDC Locations | 610122529 |
| $\\|\mathbf{C}\\|=6$ | $f(\\|\mathbf{C}\\|)=73.8$ | $\eta_{2}=0.636$ |
|  | PDC Locations | 61012192529 |
| $\\|\mathbf{C}\\|=7$ | $f(\|\|\mathbf{C}\|\|)=71.1$ | $\eta_{2}=0.613$ |
|  | PDC Locations | 6101219232529 |
| $\\|\mathbf{C}\\|=8$ | $f(\\| \mathbf{C}\| \|)=68.9$ | $\eta_{2}=0.594$ |
|  | PDC Locations | 26101219232529 |
| $\\|\mathbf{C}\\|=9$ | $f(\|\|\mathbf{C}\|\|)=66.72$ | $\eta_{2}=0.575$ |
|  | PDC Locations | 246101219232529 |
| $\\|\mathbf{C}\\|=10$ | $f(\\|\mathbf{C}\\|)=65.0$ | $\eta_{2}=0.560$ |
|  | PDC Locations | 2469101219232529 |

IEEE 30-bus System


Fig. 5. $\quad \eta$ with respect to number of PDCs in IEEE 30-bus system [8].

## C. IEEE 57-bus System

For the IEEE 57-bus system, 17 PMUs are placed. These 17 PMUs generate $273.0 p K_{s}$ traffic for WAMS, which can be reduced by $12.9 \%$ with the help of one PDC installed on Bus 38, or by $17.7 \%$ after installing two PDCs on Bus 24 and Bus 38, respectively. As shown in Table III, the traffic reaches the minimum value when 17 PDCs are installed, one on each PMU.

Following the same approach, for any given $\zeta$, we look up the corresponding $\|\mathbf{C}\|$ from Fig. 6, and then obtain the exact solution from Table III. For instance, when $\zeta=0.6$, since $\eta_{12}<0.6<\eta_{11}, 12$ PDC are needed. Looking up Table III, we obtain the solution as $\mathbf{C}=$ $\{1,4,9,20,24,29,32,36,38,41,46,51\}$.

## D. Discussions

As can be observed from Fig. 4, Fig. 5, and Fig. 6, $\eta$ is a decreasing function of $\|\mathbf{C}\|$ with positive second-order

TABLE III
Results of OPI for IEEE 57-Bus system

| $g=273.0$ |  |  |
| :---: | :---: | :---: |
| PMU Locations: 1492024252829323638414446515457 |  |  |
| $\\|\mathbf{C}\\|=1$ | $f(\|\|\mathbf{C}\|\|)=237.8$ | $\eta_{1}=0.871$ |
|  | PDC Locations | 38 |
| $\\|\mathbf{C}\\|=2$ | $f(\|\|\mathbf{C}\|\|)=224.6$ | $\eta_{2}=0.823$ |
|  | PDC Locations | 2438 |
| $\\|\mathbf{C}\\|=3$ | $f(\\|\mathbf{C}\\|)=211.8$ | $\eta_{2}=0.776$ |
|  | PDC Locations | 92438 |
| $\\|\mathbf{C}\\|=4$ | $f(\|\|\mathbf{C}\|\|)=203.0$ | $\eta_{2}=0.744$ |
|  | PDC Locations | 9243238 |
| $\\|\mathbf{C}\\|=5$ | $f(\\|\mathbf{C}\\|)=196.0$ | $\eta_{2}=0.718$ |
|  | PDC Locations | 924323841 |
| $\\|\mathbf{C}\\|=6$ | $f(\\|\mathbf{C}\\|)=188.9$ | $\eta_{2}=0.692$ |
|  | PDC Locations | 4924323841 |
| $\\|\mathbf{C}\\|=7$ | $f(\|\|\mathbf{C}\|\|)=182.4$ | $\eta_{2}=0.668$ |
|  | PDC Locations | 14924323841 |
| $\\| \mathbf{C}\| \|=8$ | $f(\\|\mathbf{C}\\|)=176.2$ | $\eta_{2}=0.645$ |
|  | PDC Locations | 1492429323841 |
| $\\|\mathbf{C}\\|=9$ | $f(\\|\mathbf{C}\\|)=172.24$ | $\eta_{2}=0.631$ |
|  | PDC Locations | 149242932384146 |
| $\\|\mathbf{C} \mid\\|=10$ | $f(\|\|\mathbf{C}\|\|)=168.3$ | $\eta_{2}=0.616$ |
|  | PDC Locations | 149202429323841 46 |
| $\\|\mathbf{C} \mid\\|=11$ | $f(\|\|\mathbf{C}\|\|)=164.8$ | $\eta_{2}=0.603$ |
|  | PDC Locations | $\begin{aligned} & 149202429323638 \\ & 4146 \end{aligned}$ |
| $\\|\mathbf{C}\\|=12$ | $f(\|\|\mathbf{C}\|\|)=162.1$ | $\eta_{2}=0.594$ |
|  | PDC Locations | $\begin{aligned} & 149202429323638 \\ & 414651 \end{aligned}$ |
| $\\|\mathbf{C}\\|=13$ | $f(\|\|\mathbf{C}\|\|)=159.5$ | $\eta_{2}=0.584$ |
|  | PDC Locations | $\begin{aligned} & 149202429323638 \\ & 41465154 \end{aligned}$ |
| $\\|\mathrm{C} \mid\\|=14$ | $f(\|\|\mathbf{C}\|\|)=156.8$ | $\eta_{2}=0.575$ |
|  | PDC Locations | $\begin{aligned} & 149202429323638 \\ & 4146515457 \end{aligned}$ |
| $\\|\mathbf{C}\\|=15$ | $f(\|\|\mathbf{C}\|\|)=155.5$ | $\eta_{2}=0.570$ |
|  | PDC Locations | $\begin{aligned} & 149202428293236 \\ & 384146515457 \end{aligned}$ |
| $\\|\mathbf{C}\\|=16$ | $f(\|\|\mathbf{C}\|\|)=154.2$ | $\eta_{2}=0.565$ |
|  | PDC Locations | $\begin{array}{llllllll} 149 & 20 & 24 & 25 & 28 & 29 & 32 \\ 36 & 38 & 41 & 46 & 51 & 54 & 57 \end{array}$ |
| $\\|\mathbf{C}\\|=17$ | $f(\|\|\mathbf{C}\|\|)=152.9$ | $\eta_{2}=0.560$ |
|  | PDC Locations | $\begin{array}{lllllll} 149 & 20 & 24 & 25 & 28 & 29 & 32 \\ 36 & 38 & 41 & 44 & 46 & 51 & 54 \\ 57 \end{array}$ |



Fig. 6. $\quad \eta$ with respect to number of PDCs in IEEE 57-bus system [8].


Fig. 7. IEEE 30-bus system [8].
derivatives. This means that a power company will benefit the most from the first few PDCs installed, and afterwards, the extra benefits obtained by installing more PDCs keep decreasing until a PDC is installed on every PMU bus.

Besides, when comparing the results shown in Table I, Table II, and Table III, we observe that, in most cases, an optimal PDC installation location is where a PMU is installed. However, exceptions do occur. For the IEEE 30-bus system, when $\|\mathbf{C}\|=3$ and $\|\mathbf{C}\|=4$, one of the optimal PDC locations is Bus 27, a non-PMU bus. To better see this, let us refer to the system graph shown in Fig. 7, where three PDCs have been installed on Bus 6, Bus 12, and Bus 27, respectively. Our numerical results indicate that the PDC at Bus 27 will only handle PMU data sent from Bus 25 and Bus 29. The traffic generated by these two PMUs are $(4+3+0.56 \times 2 \times(4+3)) p F_{s}=14.84 p F_{s}$ kilo-Byte-link per second. However, if we move this PDC to Bus 25 or to Bus 29 , the resulting traffic generated by these two PMU will become $15.72 p F_{s}$ and $17.04 p F_{s}$, respectively. Interestingly, Bus 27 remains an optimal PDC location until $\|\mathbf{C}\| \geq 5$, when both Bus 25 and Bus 29 have a PDC installed with them.

Last but not least, as shown in Section II-C, the complexity of the OPI model is $N$ !. This makes it almost impossible to solve in large systems. In fact, the exhaustive enumeration approach we used will work for power systems with up to 57 buses. For bigger systems, the computation time becomes excessive. In the future, we will look into meta-heuristic algorithms, such as chemical reaction optimization [11], to solve problems of larger sizes.

## IV. CONCLUSION

As phasor measurement units generate increasing volume of data for wide-area monitoring systems, the underlying communication network may get overwhelmed, causing severe and unacceptable communication delays for smart grid applications. This potential problem can be overcome by building new
high-bandwidth communication infrastructure, but a cheaper and more practical alternative is to use PDCs to reduce the traffic generated by PMUs. Based on this, we propose an optimal PDC installation (OPI) problem that determines the minimum number of PDCs required for a desired reduction in WAMS traffic. In this paper, the problem is solved on the IEEE 14-bus, 30-bus, and 57-bus systems using an exhaustive search algorithm. However, we note that the problem, with some reasonable simplifications, can be formulated into an integer linear programming (ILP) problem. We will present our ILP formulation and solutions in our future work. We hope this paper will serve as a foundation for further research in this important area.

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