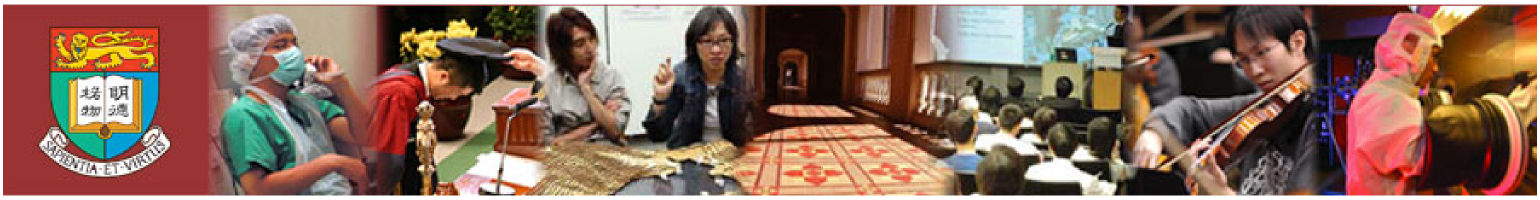

Title	Reduced-order dissipative filtering for discrete-time singular systems
Author(s)	Feng, Z; Lam, J
Citation	The 22nd IEEE International Symposium on Industrial Electronics (ISIE 2013), Taipei, Taiwan, 28-31 May 2013. In Conference Proceedings, 2013, p. 1-6
Issued Date	2013
URL	http://hdl.handle.net/10722/190024
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Reduced-order Dissipative Filtering for Discrete-time Singular Systems

Zhiguang Feng James Lam

***Abstract*—This paper is concerned with the reduced-order dissipative filtering problem of discrete-time singular systems. By considering an equivalent representation of the solution set, a necessary and sufficient dissipativity condition of singular systems is proposed in terms of strict LMI. By using the system augmentation approach, a reduced-order filter is designed such that the filtering error system is admissible and strictly (Q, S, R)-dissipative. A numerical example is presented to demonstrate the usefulness of the derived theoretical results.**

***Index Terms*—Augmentation Approach; Dissipative Filter; Reduced-order Filtering; Singular Systems.**

I. Introduction

Singular systems, also called descriptor systems, differential-algebraic systems, can often better describe the behavior of physical systems and have extensive application in many practical areas, such as chemical processes [8], economic systems [11], and circuit systems [15]. Due to the theoretical importance and practical application of singular systems, many researchers have been devoting their attention on the singular system and a lot of results have been obtained, for examples stability and stabilization [25]; H_∞ performance analysis and control [1], [10]; passivity and passification [28], [29]; dissipativity analysis and dissipative control [5], [22]; model reduction [20], [24].

Dissipativity property provides a uniform framework considering the gain and phase information simultaneously, which have played an important role in control theory and applications [21]. Many basic tools such as the passivity theorem, bounded real lemma, Kalman-Yakubovic-Popov (KYP) lemma and circle criterion are generalized in the dissipativity theory which has attracted considerable attention [2], [4], [19], [22]. For continuous-time singular systems, without constraining the choice of the system realization, a necessary and sufficient dissipativity condition is established and a state feedback control problem is solved in [13]. The corresponding output feedback controller design method is provided in [14]. However, the results in [13] and [14] are described in non-strict LMIs which lead to some computation difficulties. The new KYP lemma for the dissipativity of singular system is characterized in terms of strict LMI in [3]. For discrete-time singular systems, only one necessary and sufficient dissipativity condition is proposed in [2] which involves the condition in terms of non-strict LMIs. This motivates us to establish a necessary and sufficient condition with a strict LMI.

On the other hand, the filtering problem for dynamic systems has received great attention due to its practical significance [17], [18]. Some results about filtering problem for standard state space system have been extended to singular systems. The full-order and reduced-order H_∞ filtering problems of singular systems are investigated in [23] and [27], respectively. By using a similar method, the reduced-order energy-to-peak filtering problem for continuous-time and discrete-time singular systems is tackled in [30] and [31], respectively. It should be pointed out that the conditions obtained in [23], [27], [30] and [31] involve a rank constraint and non-strict inequality constraints which give rise to computation complexity. A reduced-order l_2-l_∞ filter design method for discrete-time singular systems is given in terms of strict LMI in [12]. For time-delay singular systems, full-order and reduced-order H_∞ filters are designed in [6] and [7], respectively. To the best knowledge of the authors, no reduced-order dissipative filter design method for discrete-time singular systems has been reported.

In this paper, by giving an equivalent representation of the solution set, a necessary and sufficient dissipativity condition is proposed in terms of strict LMI. Then a reduced-order filter design method is given based on the augmentation system approach such that the filtering error system is admissible and strictly (Q, S, R)-dissipative. A numerical example is given to illustrate the effectiveness of the obtained results.

The rest of this paper is briefly outlined as follows. In Section II, the reduced-order dissipative filtering problem is formulated. A necessary and sufficient dissipativity condition of singular system is given and the reduced-order filter is designed in Section III. Illustrative examples are provided in Section IV to show the effectiveness of our results. We conclude the paper in Section V.

Notation: The notation used throughout the paper is standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $P > 0$ (≥ 0) means that P is real symmetric and positive definite (semi-definite); I and 0 refer to the identity matrix and zero matrix with compatible dimensions; $\star$ stands for the symmetric terms in a symmetric matrix and $\mathrm{sym}(A)$ is defined as $A + A^T$; l_2 refers to the space of square summable infinite vector sequences. $\bullet$ represents matrices that are not relevant with our discussion; $\|\cdot\|$ refers to the Euclidean vector norm; for two vectors u, v, $\langle u, v\rangle_\tau$ is defined as $\sum_{k=0}^{\tau} u^T(k)v(k)$. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

This work was supported in part by GRF 7140/11E and SZSTI Basic Research Scheme JCYJ20120831142942514.

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II. System Description and Preliminaries

Consider a discrete-time singular system

$$\begin{cases} Ex(k+1) = Ax(k) + B_w w(k), \ x_0 = x(0) \\ z(k) = Cx(k) + D_w w(k) \\ y(k) = C_y x(k) + D_y w(k) \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $w(k) \in \mathbb{R}^l$ represents a disturbance which belongs to l_2; $z(k) \in \mathbb{R}^q$ is the controlled output; $y(k) \in \mathbb{R}^g$ is the measurement output; matrices E, A, B_w, C, D_w, C_y and D_y are constant matrices with appropriate dimensions and $\mathrm{rank}(E) = r \le n$. In order to estimate controlled output $z(k)$, the following reduced-order filtering is constructed:

$$\begin{cases} \hat{x}(k+1) = A_f \hat{x}(k) + B_f y(k), \ \hat{x}(0) = 0 \\ \hat{z}(k) = C_f \hat{x}(k) + D_f y(k) \end{cases} \quad (2)$$

where $\hat{x}(k) \in \mathbb{R}^m$ $(0 < m \le n)$ is the state vector of the filter; $\hat{z}(k) \in \mathbb{R}^q$ is the estimation of $z(k)$; matrices A_f, B_f, C_f and D_f are filter parameters to be determined.

Denote $\breve{x}(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T$ and the estimation error $\breve{z}(k) = z(k) - \hat{z}(k)$, then the filtering error singular system derived from the singular system in (1) and the filter in (2) is

$$\begin{cases} \breve{E}\breve{x}(k+1) = \breve{A}\breve{x}(k) + \breve{B}_w w(k) \\ \breve{z}(k) = \breve{C}\breve{x}(k) + \breve{D}_w w(k) \end{cases} \quad (3)$$

where

$$\breve{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \ \breve{A} = \begin{bmatrix} A & 0 \\ B_f C_y & A_f \end{bmatrix}, \ \breve{B}_w = \begin{bmatrix} B_w \\ B_f D_y \end{bmatrix}$$
$$\breve{C} = \begin{bmatrix} C - D_f C_y & -C_f \end{bmatrix}, \ \breve{D}_w = D_w - D_f D_y$$

Our aim is to design a filter in (2) such that the filtering error system in (3) is admissible and strictly (Q, S, R)-dissipative.

By using simple algebraic manipulations, the following equations hold:

$$\breve{A} = \tilde{A} + HK\tilde{C}_y, \ \breve{B}_w = \tilde{B} + HK\tilde{D}_y$$
$$\breve{C} = \tilde{C} + JK\tilde{C}_y, \ \breve{D}_w = D_w + JK\tilde{D}_y \quad (4)$$

where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \ \tilde{B} = \begin{bmatrix} B_w \\ 0 \end{bmatrix}, \ \tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$
$$\tilde{C}_y = \begin{bmatrix} 0 & I \\ C_y & 0 \end{bmatrix}, \ \tilde{D}_y = \begin{bmatrix} 0 \\ D_y \end{bmatrix}$$
$$H = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \ J = \begin{bmatrix} 0 & -I \end{bmatrix}, \ K = \begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix}$$

Then the system in (3) can be rewritten as:

$$\begin{cases} \breve{E}\breve{x}(k+1) = \tilde{A}\breve{x}(k) + H\breve{u}(k) + \tilde{B}w(k) \\ \breve{z}(k) = \tilde{C}\breve{x}(k) + J\breve{u}(k) + D_w w(k) \\ \breve{y}(k) = \tilde{C}_y \breve{x}(k) + \tilde{D}_y w(k) \end{cases} \quad (5)$$

with $\breve{u}(k) = K\breve{y}(k)$. Therefore, the filter design problem of system (1) is equivalent to design a matrix K for the system in (5) such that the filtering error system in (3) is admissible and strictly (Q, S, R)-dissipative.

Before moving on, we give some definitions and lemmas which will be used in deriving the main results.

Definition 1. [26]

1) *The singular system in (1) is said to be regular if* $\det(sE - A)$ *is not identically zero.*
2) *The singular system in (1) is said to be causal if* $\deg\{\det(sE - A)\} = \mathrm{rank}\ (E)$.
3) *The singular system in (1) is said to be stable if the moduli of the roots of* $\det(zE - A) = 0$ *are less than 1.*
4) *The singular system in (1) is said to be admissible if it is regular, causal, and stable.*

Definition 2. [2] *The system in (1) is said to be strictly* (Q, S, R)*-dissipative if there exists a scalar* $\alpha > 0$ *and under zeros initial state* $x_0 = 0$*, the following inequality holds:*

$$G(z, w, \tau) = \langle z, Qz \rangle_\tau + 2\langle z, Sw \rangle_\tau + \langle w, Rw \rangle_\tau$$
$$\ge \alpha \langle w, w \rangle_\tau, \forall \tau \ge 0 \quad (6)$$

As in [2], $Q \le 0$ is assumed. Consequently, there exists a matrix $Q_-^{\frac{1}{2}} \ge 0$ satisfying $-Q = (Q_-^{\frac{1}{2}})^2$.

Lemma 1. [2] *Let the matrices Q, S and R be given with Q and R symmetric. Then the system in (1) is admissible (when $w(k) = 0$) and strictly (Q, S, R)-dissipative, if and only if there exists a symmetric and invertible matrix X such that*

$$E^T X E \ge 0 \quad (7)$$

$$\begin{bmatrix} A^T X A - E^T X E & A^T X B_w - C^T S & C^T Q_-^{\frac{1}{2}} \\ \star & Z & D_w^T Q_-^{\frac{1}{2}} \\ \star & \star & -I \end{bmatrix} < 0 \quad (8)$$

where $Z = B_w^T X B_w - D_w^T S - S^T D_w - R$.

Lemma 2. *The following two sets are equivalent:*

$$\mathcal{X}_1 = \{X \in \mathbb{R}^{n \times n} : E^T X E \ge 0, \mathrm{rank}(E^T X E) = r, X = X^T\}$$
$$\mathcal{X}_2 = \{X = P - E_0^T U E_0 : P > 0, E_0 E = 0, E_0 E_0^T > 0, E_0 \in \mathbb{R}^{(n-r) \times n}, U = U^T\}$$

Proof. Sufficiency: When $X \in \mathcal{X}_2$, we have $E^T X E = E^T P E \ge 0$ and $\mathrm{rank}(E^T P E) = r$ which implies $X \in \mathcal{X}_1$.

Necessity: Without loss of generality, we set $E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ and $X = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_3 \end{bmatrix}$, where $X_1 = X_1^T \in \mathbb{R}^{r \times r}$ and $X_3 = X_3^T \in \mathbb{R}^{(n-r) \times (n-r)}$. Then we have $E_0 = \begin{bmatrix} 0 & I \end{bmatrix}$ and it yields from $E^T X E \ge 0$ that $X_1 \ge 0$. Combining with $\mathrm{rank}(E^T X E) = \mathrm{rank}(X_1) = r$, we have $X_1 > 0$. By constructing $P = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_2^T X_1^{-1} X_2 + \epsilon I \end{bmatrix} > 0$ with $\epsilon > 0$ and $U = U^T = X_2^T X_1^{-1} X_2 + \epsilon I - X_3$, we have $X = P - E_0^T U E_0$. □

III. Reduced-order dissipative filtering

A. Dissipativity analysis

In this subsection, a new necessary and sufficient dissipaitivity condition of discrete-time singular systems is given in terms of strict LMI based on Lemma 2.

Theorem 1. *Let the matrices Q, S and R be given with Q and R symmetric. The following statements are equivalent:*

(i) *System (1) is admissible and strictly (Q,S,R)-dissipative.*

(ii) *There exist matrices $P>0$, and $U=U^T$ such that the following LMI holds:*

$$\begin{bmatrix} -E^TPE+A^TVA & A^TVB_w-C^TS & C^TQ_-^{\frac{1}{2}} \\ \star & \Gamma & D_w^TQ_-^{\frac{1}{2}} \\ \star & \star & -I \end{bmatrix} < 0 \tag{9}$$

where $V=P-E_0^TUE_0$, and $\Gamma=B_w^TVB_w-D_w^TS-S^TD_w-R$.

(iii) *There exist matrices $P>0$, $U=U^T$, $\mathcal{F}$ and $\mathcal{G}$ such that the following LMI holds:*

$$\begin{bmatrix} -\mathcal{E}^T\mathcal{P}\mathcal{E}+\mathrm{sym}(\mathcal{L}^T\mathcal{S}+\mathcal{F}\mathcal{A}) & -\mathcal{F}+\mathcal{A}^T\mathcal{G}^T \\ \star & \mathcal{V}-\mathcal{G}^T-\mathcal{G} \end{bmatrix} < 0 \tag{10}$$

where

$$\mathcal{E}=\begin{bmatrix} E & 0 \\ 0 & I\end{bmatrix},\ \mathcal{P}=\begin{bmatrix} P & 0 \\ 0 & R\end{bmatrix},\ \mathcal{L}=\begin{bmatrix} C & D_w\end{bmatrix}$$

$$\mathcal{A}=\begin{bmatrix} A & B_w \\ Q_-^{\frac{1}{2}}C & Q_-^{\frac{1}{2}}D_w\end{bmatrix},\ \mathcal{V}=\begin{bmatrix} V & 0 \\ 0 & I\end{bmatrix},\ \mathcal{S}=\begin{bmatrix} 0 & -S\end{bmatrix}$$

Proof. (i) $\Longleftrightarrow$ (ii): The equivalence between item (i) and item (ii) are obtained by using Lemma 1 and Lemma 2.
(iii) $\Rightarrow$ (ii): The following LMI can be derived by pre-multiplying and post-multiplying (10) with $\begin{bmatrix} I & \mathcal{A}^T\end{bmatrix}$ and $\begin{bmatrix} I & \mathcal{A}^T\end{bmatrix}^T$:

$$\begin{aligned}\bar{V} =& \begin{bmatrix} -E^TPE+A^TVA & A^TVB_w-C^TS \\ \star & B_w^TVB_w-D_w^TS-S^TD_w-R\end{bmatrix} \\ &-\begin{bmatrix} C^TQC & C^TQD_w \\ \star & D_w^TQD_w\end{bmatrix} \\ &< 0\end{aligned}$$

which is equivalent to (9) by utilizing Schur complement equivalence.
(ii) $\Rightarrow$ (iii): By employing Schur complement equivalence, condition (9) is equivalent to

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \star & \Gamma_{22}\end{bmatrix} = -\mathcal{E}^T\mathcal{P}\mathcal{E}+\mathrm{sym}(\mathcal{L}^T\mathcal{S})+\mathcal{A}^T\mathcal{V}\mathcal{A}<0$$

where

$$\begin{aligned}\Gamma_{11} &= -E^TPE+A^TVA-C^TQC \\ \Gamma_{12} &= A^TVB_w-C^TS-C^TQD_w \\ \Gamma_{22} &= B_w^TVB_w-D_w^TS-S^TD_w-R-D_w^TQD_w\end{aligned}$$

and $\mathcal{P}$, $\mathcal{A}$, $\mathcal{V}$ are defined in (10). On the other hand, there always exist a matrix $\mathcal{G}$ such that $\mathcal{V}-\mathcal{G}^T-\mathcal{G}<0$ and

$$\begin{bmatrix} -\mathcal{E}^T\mathcal{P}\mathcal{E}+\mathrm{sym}(\mathcal{L}^T\mathcal{S})+\mathcal{A}^T\mathcal{V}\mathcal{A} & 0 \\ 0 & \mathcal{V}-\mathcal{G}^T-\mathcal{G}\end{bmatrix} < 0 \tag{11}$$

By pre-multiplying and post-multiplying (11) by $\begin{bmatrix} I & -\mathcal{A}^T \\ 0 & I\end{bmatrix}$ and $\begin{bmatrix} I & -\mathcal{A}^T \\ 0 & I\end{bmatrix}^T$, it yields that

$$\begin{bmatrix} \Gamma_2 & \mathcal{A}^T(-\mathcal{V}+\mathcal{G}+\mathcal{G}^T) \\ \star & \mathcal{V}-\mathcal{G}-\mathcal{G}^T\end{bmatrix} < 0 \tag{12}$$

with $\Gamma_2=-\mathcal{E}^T\mathcal{P}\mathcal{E}+\mathrm{sym}(\mathcal{L}^T\mathcal{S})+\mathcal{A}^T\mathrm{sym}(\mathcal{V}-\mathcal{G})\mathcal{A}$. By setting $\mathcal{F}=\mathcal{A}^T(\mathcal{V}-\mathcal{G})$, we get inequality (10). □

Remark 1. *The advantage of Item 3 of Theorem 1 lies in separating the Lyapunov matrix P and the system matrices A and C which is very useful for the controller design problem and utilized widely [16]. However, if we use Item 3 of Theorem 1 to design the matrix K with input matrices B and D, the terms $F_1Q_-^{\frac{1}{2}}(C+DKC_y)$ and $F_1(A+BKC_y)$ will appear with $\mathcal{F}=\begin{bmatrix} F_1 & F_2 \\ F_3 & F_4\end{bmatrix}$, which makes the condition in (10) difficult to solve. Therefore, the separation of the controller K and system matrices D and B will be helpful for solving the filtering design problem.*

B. Filter design

In this section, we will firstly give a necessary and sufficient condition of designing the matrix K for system (5). Based on this result, a tractable filtering design method is proposed.

Define $\bar{x}(k)=\begin{bmatrix} \breve{x}^T(k) & \breve{u}^T(k)\end{bmatrix}^T$ as a new state variable and the system in (5) is equivalent to the following augmentation one:

$$\begin{cases} \bar{E}\bar{x}(k+1) = \bar{A}\bar{x}(k)+\bar{B}_ww(k) \\ \breve{z}(k) = \bar{C}\bar{x}(k)+\bar{D}_ww(k)\end{cases} \tag{13}$$

where

$$\bar{E}=\begin{bmatrix} \breve{E} & 0 \\ 0 & 0\end{bmatrix},\ \bar{A}=\begin{bmatrix} \tilde{A} & H \\ K\tilde{C}_y & -I\end{bmatrix}$$

$$\bar{B}_w=\begin{bmatrix} \tilde{B} \\ K\tilde{D}_y\end{bmatrix},\ \bar{C}=\begin{bmatrix} \tilde{C} & J\end{bmatrix},\ \bar{D}_w=D_w$$

Before giving the main result, we first prove the equivalence of admissibility and dissipativity between the systems in (3) and (13). The following two equations are true:

$$\begin{aligned} z\bar{E}-\bar{A} &= \begin{bmatrix} zE-\tilde{A} & -H \\ -K\tilde{C}_y & I\end{bmatrix} \\ &= \begin{bmatrix} I & -H \\ 0 & I\end{bmatrix}\begin{bmatrix} z\breve{E}-\tilde{A}-HK\tilde{C}_y & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix} I & 0 \\ -K\tilde{C}_y & I\end{bmatrix}\end{aligned}$$

and

$$\begin{aligned} &\bar{C}(z\bar{E}-\bar{A})^{-1}\bar{B}_w+D_w \\ &= D_w+\begin{bmatrix} \tilde{C} & J\end{bmatrix}\begin{bmatrix} I & 0 \\ KC_y & I\end{bmatrix}\begin{bmatrix} (z\breve{E}-\tilde{A}-HK\tilde{C}_y)^{-1} & 0 \\ 0 & I\end{bmatrix} \\ &\quad\times\begin{bmatrix} I & H \\ 0 & I\end{bmatrix}\begin{bmatrix} \tilde{B}_w \\ K\tilde{D}_y\end{bmatrix} \\ &= D_w+JK\tilde{D}_y+(\tilde{C}+JK\tilde{C}_y)(z\breve{E}-\tilde{A}-HK\tilde{C}_y)^{-1} \\ &\quad\times(\tilde{B}_w+HK\tilde{D}_y) \\ &= \breve{C}(z\breve{E}-\breve{A})^{-1}\breve{B}_w+\breve{D}_w\end{aligned}$$

which derive that the determinants of $z\bar{E}-\bar{A}$ and $z\breve{E}-\breve{A}$ are the same, and the transfer functions of the systems in (3) and (13) are equal, respectively. By using the Definitions 1 and 2, the admissibility and dissipativity of system in (3) are equivalent to these in (13). By using the equivalence between items (i) and (iii) of Theorem 1, the following theorem proposes the dissipativity condition for system (13) by utilizing system augmentation.

Theorem 2. *The system in (13) is admissible and strictly (Q,S,R) dissipative if and only if there exist matrices $P>0$, $U=U^T$, $\mathcal{F}$ and $\mathcal{G}$ such that the following LMI holds:*

$$\begin{bmatrix} -\bar{\mathcal{E}}^T\mathcal{P}\bar{\mathcal{E}}+\mathrm{sym}(\bar{\mathcal{L}}^T\mathcal{S}+\mathcal{F}\bar{\mathcal{A}}) & -\mathcal{F}+\bar{\mathcal{A}}^T\mathcal{G}^T \\ \star & \bar{\mathcal{V}}-\mathcal{G}^T-\mathcal{G} \end{bmatrix}<0 \quad (14)$$

where

$$\bar{\mathcal{E}}=\begin{bmatrix}\bar{E} & 0\\ 0 & I\end{bmatrix},\ \mathcal{P}=\begin{bmatrix}P & 0\\ 0 & R\end{bmatrix},\ \bar{\mathcal{L}}=\begin{bmatrix}\bar{C} & D_w\end{bmatrix},\ \mathcal{S}=\begin{bmatrix}0 & -S\end{bmatrix}$$

$$\bar{\mathcal{A}}=\begin{bmatrix}\bar{A} & \bar{B}_w\\ Q_-^{\frac{1}{2}}\bar{C} & Q_-^{\frac{1}{2}}D_w\end{bmatrix},\ \bar{\mathcal{V}}=\begin{bmatrix}\bar{V} & 0\\ 0 & I\end{bmatrix},\ \bar{V}=P-\bar{E}_0^TU\bar{E}_0$$

Remark 2. *It can be seen that the inequality in (14) is in terms of bilinear matrix inequality (BMI) which can be solved by utilizing the existing numerical method [9]. Moreover, the H_∞ filtering problem and the passivity filtering problem also can be addressed by setting $-Q=I$, $S=0$, $R=\gamma^2$ and $-Q=0$, $S=I$, $R=0$ in (14), respectively.*

Based on Theorem 2, the result of reduced-order filtering design for system (1) in terms of a tractable LMI condition is presented in the following theorem.

Theorem 3. *There exists a filter in (2) such that the filtering error system in (3) is admissible and strictly (Q,S,R)-dissipative if there exist matrices $P=\begin{bmatrix}P_{11} & P_{12}\\ \star & P_{22}\end{bmatrix}>0$, $U=U^T$, F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_3, G_{11}, G_{13}, G_{21}, G_{22} and G_3 such that the following LMI holds:*

$$\Theta=\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} & \Theta_{15} & -F_{21}+\tilde{C}^TQ_-^{\frac{1}{2}}G_3^T\\ \star & \Theta_{22} & \Theta_{23} & \Theta_{24} & \Theta_{25} & -F_{22}+J^TQ_-^{\frac{1}{2}}G_3^T\\ \star & \star & \Theta_{33} & \Theta_{34} & \Theta_{35} & -F_3+D_w^TQ_-^{\frac{1}{2}}G_3^T\\ \star & \star & \star & \Theta_{44} & \Theta_{45} & -G_{21}\\ \star & \star & \star & \star & \Theta_{55} & -G_{22}\\ \star & \star & \star & \star & \star & I-G_3-G_3^T \end{bmatrix}<0 \quad (15)$$

where

$$\begin{aligned}
\Theta_{11} &= -\breve{E}^TP_{11}\breve{E}+\mathrm{sym}(F_{11}\tilde{A}+LM\tilde{C}_y+F_{21}Q_-^{\frac{1}{2}}\tilde{C})\\
\Theta_{12} &= F_{11}H-LF_{12}+F_{21}Q_-^{\frac{1}{2}}J+\tilde{A}^TF_{13}^T+\tilde{C}_y^TM^T\\
&\quad +\tilde{C}^TQ_-^{\frac{1}{2}}F_{22}^T\\
\Theta_{13} &= -\tilde{C}^TS+F_{11}\tilde{B}+LM\tilde{D}_y+F_{21}Q_-^{\frac{1}{2}}D_w\\
&\quad +\tilde{C}^TQ_-^{\frac{1}{2}}F_3^T\\
\Theta_{14} &= -F_{11}+\tilde{A}^TG_{11}^T+\tilde{C}_y^TM^TL^T+\tilde{C}^TQ_-^{\frac{1}{2}}G_{21}^T\\
\Theta_{15} &= -LF_{12}+\tilde{A}^TG_{13}^T+\tilde{C}_y^TM^T+\tilde{C}^TQ_-^{\frac{1}{2}}G_{22}^T\\
\Theta_{22} &= \mathrm{sym}(F_{13}H-F_{12}+F_{22}Q_-^{\frac{1}{2}}J)\\
\Theta_{23} &= -J^TS+F_{13}\tilde{B}+M\tilde{D}_y+F_{22}Q_-^{\frac{1}{2}}D_w\\
&\quad +\tilde{D}^TQ_-^{\frac{1}{2}}F_3^T\\
\Theta_{24} &= -F_{13}+H^TG_{11}^T-F_{12}^TL^T+J^TQ_-^{\frac{1}{2}}G_{21}^T\\
\Theta_{25} &= -F_{12}+H^TG_{13}^T-F_{12}^T+J^TQ_-^{\frac{1}{2}}G_{22}^T\\
\Theta_{33} &= -R+\mathrm{sym}(F_3Q_-^{\frac{1}{2}}D_w-D_w^TS)\\
\Theta_{34} &= \tilde{B}^TG_{11}^T+\tilde{D}_y^TM^TL^T+D_w^TQ_-^{\frac{1}{2}}G_{21}^T\\
\Theta_{35} &= \tilde{B}^TG_{13}^T+\tilde{D}_y^TM^T+D_w^TQ_-^{\frac{1}{2}}G_{22}^T\\
\Theta_{44} &= P_{11}-\bar{E}_{01}^TU\bar{E}_{01}-G_{11}-G_{11}^T\\
\Theta_{45} &= P_{12}-\bar{E}_{01}^TU\bar{E}_{02}-LF_{12}-G_{13}^T\\
\Theta_{55} &= P_{22}-\bar{E}_{02}^TU\bar{E}_{02}-F_{12}-F_{12}^T
\end{aligned}$$

and $\bar{E}_0=\begin{bmatrix}\bar{E}_{01} & \bar{E}_{02}\end{bmatrix}$ with $\bar{E}_0\bar{E}=0$, $\bar{E}_0\bar{E}_0^T>0$, $\bar{E}_0\in\mathbb{R}^{(n+m+q-r)\times(n+2m+q)}$, $\bar{E}_{01}\in\mathbb{R}^{(n+m+q-r)\times(n+m)}$, $\bar{E}_{02}\in\mathbb{R}^{(n+m+q-r)\times(m+q)}$, $L^T=\begin{bmatrix}I_{m+q} & 0_{(m+q)\times(n-q)}\end{bmatrix}$. Then a desired filter can be obtained by $K=F_{12}^{-1}M=\left[\begin{array}{c|c}A_f & B_f\\ \hline C_f & D_f\end{array}\right]$.

Proof. Set

$$\begin{aligned}
F_1 &= \begin{bmatrix}F_{11} & LF_{12}\\ F_{13} & F_{12}\end{bmatrix},\ F_2=\begin{bmatrix}F_{21}\\ F_{22}\end{bmatrix}\\
G_1 &= \begin{bmatrix}G_{11} & LF_{12}\\ G_{13} & F_{12}\end{bmatrix},\ G_2=\begin{bmatrix}G_{21}\\ G_{22}\end{bmatrix}
\end{aligned}$$

and let the matrices $\mathcal{F}$ and $\mathcal{G}$ be the following forms:

$$\mathcal{F}=\begin{bmatrix}F_1 & F_2\\ 0 & F_3\end{bmatrix},\ \mathcal{G}=\begin{bmatrix}G_1 & G_2\\ 0 & G_3\end{bmatrix}$$

Then noting that $K=F_{12}^{-1}M$, we can obtain the inequality in (14) from the inequality in (15) by straightforward calculating. Therefore, the admissibility and dissipativity of system (13) which is equivalent to those of system (3) are proved. $\square$

Remark 3. *The non-singularity of the matrix F_{12} in Theorem 3 is satisfied without loss of generality. If it is not the case, then we can choose a sufficient small scalar θ such that $\bar{F}_{12}=F_{12}+\theta I$ satisfying the inequality in (15). Then the matrix K can be replaced with $\bar{F}_{12}^{-1}M$.*

Remark 4. *The reduced-order filtering problem is also investigated in [7] and [27], respectively. However, in order to obtain the desired filter parameters, the complex matrix structure is needed in them and the rank of the different of two decision variables should be less than the order of the filter in [27]. For our method, the filter parameters can be obtained directly by solving the LMI in (15) which avoids considering the rank constraint in [27] or constructing some complex matrices in [7] .*

IV. Illustrative example

In this section, an example is provided to illustrate the effectiveness of the proposed approach. Theorem 1 which provides a necessary and sufficient dissipativity condition will be used to check the applicability of the filter design methods.

In this example, a first-order filter in the form of (2) will be designed for the following discrete-time singular systems:

$$\begin{cases} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix} x(k+1) = \begin{bmatrix} -1 & 0.5 & 1 \\ -1 & -0.3 & 1 \\ 0.5 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} -0.1 \\ 0 \\ 0.1 \end{bmatrix} w(k) \\ z(k) = \begin{bmatrix} -3.2 & 0 & 3.2 \\ 3.2 & 0 & 1.6 \\ 0 & 0 & 3.2 \end{bmatrix} x(k) + \begin{bmatrix} -0.1 \\ 0.5 \\ 0.1 \end{bmatrix} w(k) \\ y(k) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix} w(k) \end{cases} \tag{16}$$

From the value of $\bar{E}$, we get

$$\bar{E}_{01} = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \bar{E}_{02} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By setting

$$Q = \begin{bmatrix} -0.6200 & 0.8000 & -0.1600 \\ 0.8000 & -1.2500 & 0.0500 \\ -0.1600 & 0.0500 & -1.0100 \end{bmatrix}$$

$$S = \begin{bmatrix} -0.1 \\ 0.5 \\ 0.2 \end{bmatrix}, \ R = 1.5$$

and solving the LMI in (15), the matrix K is obtained as follows:

$$K = \left[\begin{array}{c|ccc} 0.1256 & 1.3812 & -1.7594 & -0.7707 \\ \hline -0.0637 & 0.0899 & -0.5044 & -0.8806 \\ 0.1001 & 1.8047 & -1.3853 & -2.4015 \\ -1.0660 & -0.8425 & 1.1057 & 0.7424 \end{array}\right]$$

Then we get that the first-order filter is

$$\begin{cases} \hat{x}(k+1) = 0.1256\hat{x}(k) + \begin{bmatrix} 1.3812 & -1.7594 & -0.7707 \end{bmatrix} y(k) \\ \hat{x}(0) = 0 \\ \hat{z}(k) = \begin{bmatrix} -0.0637 \\ 0.1001 \\ -1.0660 \end{bmatrix} \hat{x}(k) \\ \quad + \begin{bmatrix} 0.0899 & -0.5044 & -0.8806 \\ 1.8047 & -1.3853 & -2.4015 \\ -0.8425 & 1.1057 & 0.7424 \end{bmatrix} y(k) \end{cases}$$

and the parameters of the filtering error system in (3) are given as follows:

$$\breve{E} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\breve{B}_w = \begin{bmatrix} -0.1000 \\ 0 \\ 0.1000 \\ 0.0610 \end{bmatrix}, \ \breve{D}_w = \begin{bmatrix} -0.0209 \\ 0.5597 \\ 0.1100 \end{bmatrix}$$

$$\breve{A} = \begin{bmatrix} -1.0000 & 0.5000 & 1.0000 & 0 \\ -1.0000 & -0.3000 & 1.0000 & 0 \\ 0.5000 & 0 & 1.0000 & 0 \\ -0.3782 & -0.3782 & -0.7707 & 0.1256 \end{bmatrix}$$

$$\breve{C} = \begin{bmatrix} -2.7855 & 0.4145 & 4.0806 & 0.0637 \\ 2.7806 & -0.4194 & 4.0015 & -0.1001 \\ -0.2632 & -0.2632 & 2.4576 & 1.0660 \end{bmatrix} \tag{17}$$

To check whether the obtained filtering error system is admissible and strictly (Q, S, R)-dissipative, Theorem 1 is utilized. By solving the LMI in (9), a feasible solution is found which shows the applicability and effectiveness of the method.

In order to test the admissibility and the dissipativity of system (3) with the parameters in (17) from simulation view, Fig. 1-Fig. 3 are depicted as follows. By giving the initial condition with $\breve{x}(0) = \begin{bmatrix} -1.6756 & -0.2870 & 0.9170 & 0 \end{bmatrix}^T$ and $w(k) = 0$, the state responses of system (3) is given in Fig.1 which illustrates the stability of the system. Combining the characteristic polynomial with $T(s) = \frac{-11750s^3+136008s^2-45229s+3768}{25000}$ which shows the regularity and casuality of the system, the admissibility of the system is obtained. To demonstrate the dissipativity of the system, we choose $w(k) = 0.1e^{-0.1k}sin(k)$ and zero initial conditions, the output signal $\breve{z}(k)$ and the performance signal $G(\breve{z}, w, \tau) = \langle z, Qz\rangle_\tau + 2\langle z, Sw\rangle_\tau + \langle w, Rw\rangle_\tau$ are proposed in Fig.2 and Fig. 3, respectively. From Fig. 3, we can see that $G(\breve{z}, w, \tau)$ is larger and equal than zero when $\tau \geq 0$. Then a sufficient small scalar $\alpha > 0$ can be always found such that the inequality in (6) holds, which shows the dissipativity of the system in (3).

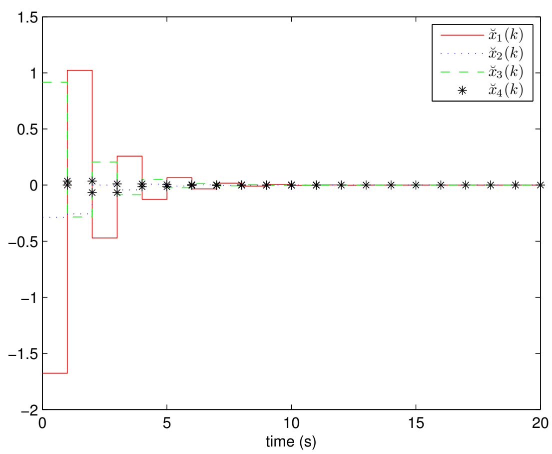

Fig. 1. State responses

V. Conclusion

The problem of reduced-order dissipative filtering of discrete-time singular systems by using an augmentation system approach has been investigated in this paper. A necessary and sufficient condition in terms of strict LMI has been proposed by considering an equivalent representation of the solution set. Augmentation system approach is utilized to solve the reduce-order dissipative filtering problem to guarantee the filtering error singular systems to be admissible and strictly (Q, S, R)-dissipative. The results presented in this paper are in terms of strict LMIs which make the conditions more tractable. Finally, a numerical example is given to demonstrate the effectiveness of our methods.

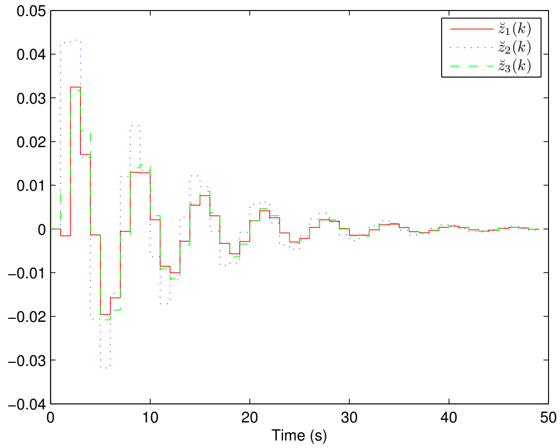

Fig. 2. Output signal $\breve{z}(k)$

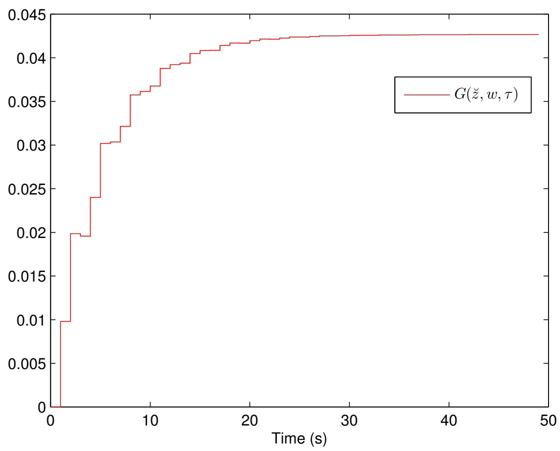

Fig. 3. Performance signal $G(\breve{z}, w, \tau) = \langle \breve{z}, Q\breve{z}\rangle_\tau + 2\langle \breve{z}, Sw\rangle_\tau + \langle w, Rw\rangle_\tau$

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