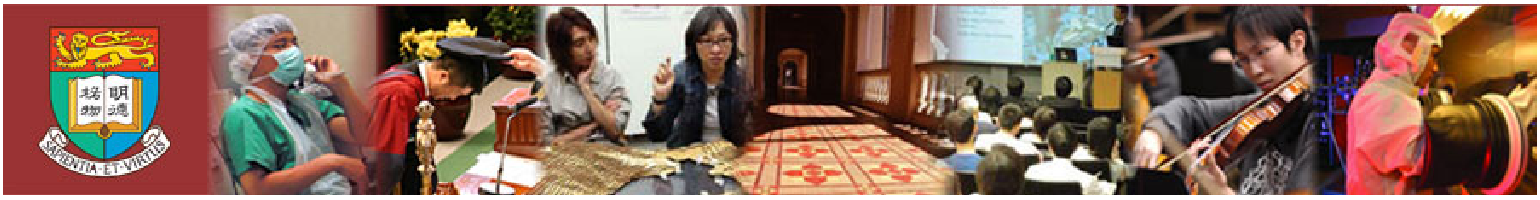

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Enhanced A-EFIE with Calderón Multiplicative Preconditioner

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***Abstract*—In this work, a Calderón multiplicative preconditioner (CMP) is proposed for the augmented electric field integral equation (A-EFIE) to improve the convergence. To avoid the imbalance between the vector potential and the scalar potential in the traditional EFIE, A-EFIE considers both the charge and the current as unknowns. After implementing the appropriate frequency scaling and the enforcement of charge neutrality, its formulation is also stable in the low-frequency regime and applicable for large-scale and complex problems. Instead of using other preconditioners, Calderón preconditioning converts the first kind integral equations into the second kind, thus improving the spectrum of the original A-EFIE system. The numerical results show that the resultant system with the combined methods is more stable at low frequencies and converges faster in the calculation of far-field scattering fields.**

I. Introduction

In the past decades, the traditional electric field integral equation (EFIE) has been widely used in the computation of field scattering, antenna design, and microwave circuit problems [1]. In order to solve multi-scale and complex problems, recent research efforts have been directed towards making the iterative system more stable in the low frequency regime or in the presence of dense meshes [2]-[14]. In the low-frequency regime, the size of the structure is much smaller than the wavelength. As the simulation frequency deceases, the EFIE tends to become increasingly ill-conditioned because of the imbalanced spectrum branches, which is known as the "low-frequency breakdown" problem.

The most of popular methods is based on the loop-tree or loop-star decomposition, which separates the electrostatic and magnetostatic physics [2]-[7]. However, it only rescales the spectrum branches of original EFIE, but does not modify the branches' shape. Hence, a remedy with a new set of loop hierarchical basis functions is introduced to properly regularize the solenoidal part of the EFIE and make the condition number grows only logarithmically with the number of unknowns [8]. In addition, Calderón-based preconditioners can also be used to improve the spectrum property and reduce the condition number by converting the matrix into the second kind integral equation [9]-[14].

On the other hand, the augmented electric field integral equation (A-EFIE) was proposed to include both the charge and the current as unknowns to avoid the imbalance between the vector potential and the scalar potential in the traditional EFIE, which was proved to be stable in the low frequency regime [13]-[14]. However, the shape of spectrum branches remains unchanged. Hence, in this work, the Calderón multiplicative preconditioner (CMP) is further proposed for the A-EFIE to alter the spectrum. Numerical results show that the convergence of the proposed formulation is the fastest in comparison with the traditional EFIE, A-EFIE, and CMP-based EFIE methods.

II. Augmented EFIE with Calderón Preconditioner

A. Formulation of A-EFIE

The A-EFIE system can be written in its matrix form as [15]-[16]

$$\begin{bmatrix} \overline{\mathbf{V}} & \overline{\mathbf{D}}^T \cdot \overline{\mathbf{P}} \\ \overline{\mathbf{D}} & k_0^2 \overline{\mathbf{I}} \end{bmatrix} \cdot \begin{bmatrix} ik_0 \mathbf{J} \\ c_0 \boldsymbol{\rho} \end{bmatrix} = \begin{bmatrix} \eta_0^{-1} \mathbf{b} \\ \mathbf{0} \end{bmatrix} \tag{1}$$

where the matrix $\overline{\mathbf{V}}$ represents the vector potential, the matrices $\overline{\mathbf{D}}$ and $\overline{\mathbf{P}}$ are the factorized components of the scalar potential matrix, the vector $\mathbf{J}$ and $\boldsymbol{\rho}$ represent the current and charge coefficients, and the vector $\mathbf{b}$ denotes the excitation.

B. Calderón Multiplicative Preconditioner (CMP)

The EFIE system with CMP in [9] can be written as

$$\overline{\mathbf{Z}}_{CWBC} \overline{\mathbf{G}}_m^{-1} \overline{\mathbf{Z}}_{RWG} \overline{\mathbf{I}} = \overline{\mathbf{V}}, \tag{2}$$

where the matrices $\overline{\mathbf{Z}}_{CWBC}$ and $\overline{\mathbf{Z}}_{RWG}$ are the inner and outer impedance matrices of squared EFIE system, discretized by Chen-Wilton-Buffa-Christiansen (CWBC) and Rao-Wilton-Glisson (RWG) functions, respectively. Hence, the first equation of A-EFIE system in (1) can be further regularized by multiplying itself as shown in (2), which becomes a well-conditioned second-kind Fredholm integral equation.

III. Numerical Results and Discussion

In this numerical example, an x-polarized plane wave impinges onto a PEC sphere from the direction. The sphere centers at the origin and has a radius of 1 m. We discretize the surface into 578 triangular patches with 867 inner edges as unknowns. From the results, we find that both the EFIE-based methods can converge at 10 MHz and 100 KHz, as shown in

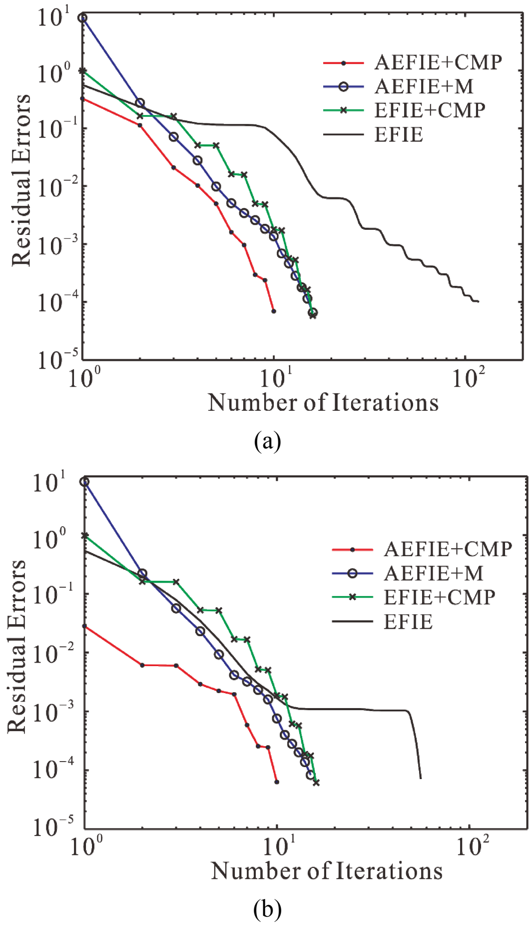

Fig. 1. Comparison of convergent history of the different iterative systems for the far-field scattering computation of a unit PEC sphere. (a) At 10 *MHz*. (b) At 100 *KHz*.

Fig. 1(a) and Fig. 1(b), respectively. It is interesting to notice that, the A-EFIE with constraint preconditioning (M) and CMP-EFIE methods have similar convergence speed, and both of them converge much faster than the traditional EFIE system. After applying CMP on the A-EFIE system, the convergence can be further improved with only 10 iterations for this example. It is found that both the preconditioned iterative systems converge with almost the same speeds at different frequencies, while the convergence of the traditional EFIE system is obviously different at two frequencies.

IV. Conclusion

In this paper, the Calderón projection method has been applied on the A-EFIE system. Numerical examples show that the convergence speed becomes much faster than the traditional EFIE system. Since the Calderón preconditioning converts the first kind integral equation of EFIE system into the second kind integral equation, the EFIE spectrum branches have been well bounded, leading to a well-conditioned interaction matrix. As a result, the combination of CMP and A-EFIE has the fastest convergent speed among different EFIE-based systems.

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