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Author(s)	Du, B; Lam, J; Zou, Y; Shu, Z
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Stability and Stabilization for Markovian Jump Time-Delay Systems With Partially Unknown Transition Rates

Baozhu Du, James Lam, *Fellow, IEEE*, Yun Zou, and Zhan Shu, *Member, IEEE*

Abstract—This paper focuses on the stability analysis and controller synthesis of continuous-time Markovian jump time-delay systems with incomplete transition rate descriptions. A general stability criterion is formulated first for state- and input-delay Markovian jump time-delay systems with fully known transition rates. On the basis of the proposed condition, an equivalent condition is given under the assumption of partly known/unknown transition rates. A new design technique based on a projection inequality has been applied to design both state feedback and static output feedback controllers. All conditions can be readily verified by efficient algorithms. Finally, illustrative examples are provided to show the effectiveness of the proposed approach.

Index Terms—Time-delay system, Markovian jump linear system, stability, stabilization, state/output feedback.

I. INTRODUCTION

INITIALLY introduced by Krasovskii and Lidskii in 1961 [1], Markovian jump linear systems (MJLSs) combine a part of the state which takes values continuously and the other part of the state which takes values discretely. They represent a special class of hybrid systems with many operation modes and each mode corresponds to a deterministic dynamic system. The switching amongst the system modes is governed by a Markov process which takes values in a finite set. Markovian jump time-delay systems (MJTDSs) are often used to model dynamic systems whose structures are subject to abrupt changes and their extensive applications have been applied to many physical systems with time delay, such as communication systems, manufacturing systems, aircraft control, target tracking, robotics, solar receiver control, neural networks, and power systems (see [2]–[8] and references cited therein). When the estimation and control problems related to such systems are of major concern in control systems, the issues of robust stability and robust stabilization of MJTDSs

have attracted much attention (see [9]–[11]). In most studies of MJTDSs, uncertainties always affect the plant models in which the external disturbance is modeled as a nonlinear function [12], [13] or polytopic/norm-bounded uncertainties are introduced into system matrices with full knowledge of their bounds or structures [14]–[17]. All the transition rates in the corresponding Markov jumping process, as a crucial factor, are assumed to be completely accessible. In practice, incomplete transition probabilities of the corresponding Markov chain are often encountered especially if adequate samples of the transitions are costly or time-consuming to obtain. In such a case, delay-free MJLSs with uncertain transition probabilities have been studied in [18], [19], in which the robust methodologies were adopted to cope with some compact sets with polytopic-type or norm-bounded structure in the transition probabilities matrix. To relax the assumption that all the transition rates are known completely, a new concept with partially known transition probabilities was proposed [20], [21], [23]–[25] for continuous-time/discrete-time Markovian jump delay-free systems, in which the information of some elements in the stationary transition rate matrix is completely unknown.

Typical controller synthesis of MJTDSs is concerned with stochastic stabilization, whereas from the point of practical application the stabilization problem is more significant. Sun *et al.* developed sufficient conditions of the robust stochastic stabilization for uncertain MJLSs with input delay and designed memory controllers in [26]. For a class of uncertain MJLSs with (multiple) delays in both state and input signals, and delay-dependent robust stochastic stability analysis and controller synthesis were focused on in [28] under the assumption that the delays are constant and unknown but with known upper bounds. Unfortunately, these results are only applicable to systems whose mode transition rates are completely known when no information of the upper bound of the time delay is known in advance. Little work has been devoted to synthesis of MJLSs with state and input delays, especially the case of MJLSs with partially unknown transition rates. Delay-independent and mode-dependent stabilization and guaranteed cost control via the dynamic output feedback controllers have been designed [29]. Delay-dependent output feedback stabilization for MJTDSs was obtained in [9], and the proof of the main results contained an error which was indicated in [30]. Hence, it still remains a challenge to investigate the analysis and synthesis of MJTDSs with partially known transition rates by state or output feedback control.

This paper is concerned with the study of the analysis and synthesis problems of MJTDSs with incomplete description of

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B. Du and Y. Zou are with the School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China (e-mail: dubaozhu@gmail.com).

J. Lam is with the Department of Mechanical Engineering, University of Hong Kong, Hong Kong, China.

Z. Shu is with the Electro Mechanical Engineering Group, Faculty of Engineering and the Environment, University of Southampton, Southampton SO17 1BJ, UK.

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their transition rates, both of the state feedback and the output feedback. The rest of this paper is arranged as follows. A general stability criterion is formulated in Section 2 for state- and input-delay MJLSs with fully known transition rates. In Section 3, an equivalent form of the stability criterion is given in terms of LMIs under the assumption of partly known/unknown transition rates. In Section 4, the output feedback stabilizability analysis is developed. Illustrative examples are provided in Section 5 to show the effectiveness of the method.

Notation: The notations in this paper are standard. Throughout this paper, let \mathbb{Z}_+ be the set of natural numbers; \mathbb{R} be the set of real numbers; \mathbb{R}^n denotes the n -dimensional Euclidean space; $\mathbb{R}^{m \times n}$ is the set of $m \times n$ matrices for which all entries belong to \mathbb{R} . The standard vector norm in \mathbb{R}^n will be denoted by $\|\cdot\|$. The matrix norm is the operator norm induced by the standard vector norm and will be denoted by the same notation $\|\cdot\|$. $(\Omega, \mathcal{F}, \mathcal{P})$ is a complete probability space, where Ω is the space of elementary events, \mathcal{F} is a σ -field and \mathcal{P} is the probability measure, satisfying the conditions that it is right continuous and \mathcal{F} contains all \mathcal{P} -null sets. For real symmetric matrices X and Y , the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). 0 is a null matrix and I is the identity matrix with an appropriate dimension. The superscript “ T ” represents the transpose of the matrix and the asterisk “ $*$ ” in a matrix stands the term which is induced by symmetry. $\text{col}\{\cdot\}$ denotes a matrix column with blocks given by the matrices in $\{\cdot\}$. A block diagonal matrix with diagonal blocks A_1, A_2, \dots, A_r will be denoted by $\text{diag}\{A_1, A_2, \dots, A_r\}$. For any matrix $P \in \mathbb{R}^{n \times n}$, $\langle P \rangle$ denotes a diagonal block matrix in which the first matrix block is P and the remaining blocks are zeros, that is, $\langle P \rangle = \text{diag}\{P, 0, \dots, 0\}$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

II. PRELIMINARIES

Consider the following MJLS with a constant delay in the state and the input, which is defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\Sigma : \begin{cases} \dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t-d) + B(r(t))u(t) \\ \quad + B_d(r(t))u(t-d) \\ y(t) = C(r(t))x(t) \\ x(t) = \varphi(t), \quad u(t) = \phi(t), \quad r(0) = r_0, \quad t \in [-\bar{d}, 0] \end{cases}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input and $y(t) \in \mathbb{R}^p$ is the measurement output. $\{r_t\}$ is a homogeneous finite state Markov process with right continuous trajectories, which takes values in a finite state space $S = \{1, 2, \dots, N\}$ ($N \in \mathbb{Z}_+$) with generator $\Pi = \{\pi_{ij}\}$ ($i, j \in S$). The mode transition probabilities are given as

$$\Pr\{r_{t+\Delta t} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t) & i \neq j \\ 1 + \pi_{ii}\Delta t + o(\Delta t) & i = j \end{cases} \quad (1)$$

where $\Delta t > 0$, and $\lim_{\Delta t \rightarrow 0} (o(\Delta t)/(\Delta t)) = 0$. Here, $\pi_{ij} \geq 0$ ($i, j \in S, i \neq j$) denote the switching rate from mode i to mode j , and $\pi_{ii} = -\sum_{j=1, j \neq i}^N \pi_{ij}$ for all $i \in S$. For each $r_t = i \in S$, the system matrices of the i th mode are denoted by $(A_i, A_{di}, B_i, B_{di}, C_i)$ which are known real constant matrices

representing the nominal system. The constant delay d satisfies $0 < d \leq \bar{d}$ where \bar{d} is an upper bound of d .

Furthermore, when the transition rates in Π of the Markov process are considered to be partially accessible, that is, some elements in Π are unknown, for notational clarity, we denote the set $S = S_K^i \cup \bar{S}_K^i$ ($\forall i \in S$) with

$$S_K^i \triangleq \{j : \pi_{ij} \text{ is known}\}, \\ \bar{S}_K^i \triangleq \{j : \pi_{ij} \text{ is unknown}\},$$

and

$$\pi_K^i \triangleq \sum_{j \in S_K^i} \pi_{ij}, \quad \mathbf{P}^i \triangleq \sum_{j=1}^N \pi_{ij} P_j, \quad \mathbf{P}_K^i \triangleq \sum_{j \in S_K^i} \pi_{ij} P_j$$

throughout the paper. Some definitions and useful lemmas on time-delay system properties are provided for subsequent technical development of the paper.

Definition 1 (Orthogonal Complement Matrix): For a full row (column) rank matrix F , F^\perp (possibly non-unique) is called the right (left) orthogonal complement of F , if the matrix F^\perp is of maximum column (row) rank and satisfies $F F^\perp = 0$ and $F^\perp F^\perp > 0$ ($F^\perp F = 0$ and $F^\perp F^\perp > 0$).

Definition 2 (Stochastic Stability): [31] MJTDS Σ with input $u(t) = 0$ is said to be stochastically stable if, for the finite $\varphi(t) \in \mathbb{R}^n$ defined on $[-\bar{d}, 0]$ and all initial mode $r_0 \in S$, the following is satisfied

$$\lim_{t \rightarrow \infty} E \left\{ \int_0^t x^T(s, \varphi, r_0) x(s, \varphi, r_0) ds \right\} < \infty$$

where $x(t, \varphi, r_0)$ denotes the solution to the considered system at time t under the initial conditions $\varphi(t)$ and r_0 .

Lemma 1 (Finsler's Lemma): [32], [33] Consider real matrices $\Omega \in \mathbb{R}^{n \times n}$ and $F \in \mathbb{R}^{m \times n}$ such that $\Omega = \Omega^T$ and F has full row rank. Then the following statements are equivalent:

- 1) There exists a nonzero vector $\xi(t) \in \mathbb{R}^n$ such that $\xi^T(t)\Omega\xi(t) < 0$ and $F\xi(t) = 0$;
- 2) There exists a scalar $\mu \in \mathbb{R}$ such that $\Omega + \mu F^T F < 0$;
- 3) The following condition holds: $F^\perp \Omega F^\perp < 0$.

Lemma 2: [34], [33] Consider matrices W, U, V, S_1, S_2 , and suppose that $S_1 > 0, S_2 > 0$, and that U and V have full column and full row rank, respectively. Then there exists a matrix G satisfying the following matrix inequality:

$$(W + UGV)S_1(W + UGV)^T - S_2 < 0$$

if and only if the following two conditions hold:

$$U^\perp (S_2 - W S_1 W^T) U^{\perp T} > 0, \\ V^T (S_1^{-1} - W^T S_2^{-1} W) V^{T \perp} > 0.$$

In previous analysis of MJTDSs, it is often assumed that all the transition rates of the jumping process $\{r_t\}$ are completely accessible (that is, $\bar{S}_K^i = \emptyset, S_K^i = S$). Moreover, the transition rates with polytopic-type or norm-bounded uncertainties require the knowledge of bounds or structure of uncertainties, which can still be viewed as accessible transition rate knowledge. Therefore, our assumption on the transition rate matrix Π is more natural for MJTDSs.

A sufficient condition for the stability analysis of MJTDSs with complete description of transition rates is proposed first

before presenting the main results as follows, which will be used in the sequel.

Theorem 1: MJTDS Σ with $u(t) = 0$ and fully known transition rates is stochastically stable for any delay $0 < d \leq \bar{d}$ if there exist matrices $P_i > 0$, $Q > 0$, $R > 0$ such that the following LMIs hold for $i = 1, 2, \dots, N$,

$$\Upsilon^i = F_i^{\perp T} [\Phi^i + \langle \mathbf{P}^i \rangle] F_i^{\perp} < 0$$

where F_i^{\perp} is the orthogonal complement of the matrix

$$F_i = \begin{bmatrix} A_i & A_{di} & 0 & -I \\ I & -I & -I & 0 \end{bmatrix}$$

and

$$\Phi^i = \begin{bmatrix} Q & * & * & * \\ 0 & -Q & * & * \\ 0 & 0 & -\bar{d}^{-1}R & * \\ P_i & 0 & 0 & \bar{d}R \end{bmatrix}.$$

Proof: Denote $\Phi^i(\bar{d}) \triangleq \Phi^i$ since it is a function of \bar{d} . Note that $\{(x(t), r_t), t \geq 0\}$ is not a Markov process in the underlying system Σ . To cast the model involved into the framework of the Markov processes, we define a new process $\{(x_t, r_t), t \geq 0\}$ by

$$x_t(s) = x(t+s), \quad t - \bar{d} \leq s \leq t$$

Then, similar to [29], we can verify that $\{(x_t, r_t), t \geq 0\}$ is a Markov process with an initial state $(\varphi(\cdot), r_0)$. Now, define a stochastic Lyapunov-Krasovskii functional for system Σ as

$$\begin{aligned} V(x_t, r_t) &= x_t^T(0)P_i x_t(0) + \int_{-d}^0 x_t^T(s)Qx_t(s) ds \\ &\quad + \int_{-d}^0 \int_{\theta}^0 \dot{x}_t^T(s)R\dot{x}_t(s) ds d\theta \end{aligned}$$

where $P_i = P(r(t))$ for $r_t = i$. Let \mathcal{L} be the weak infinitesimal generator [35] of the random process $\{x_t, r_t\}$. Then, by some algebraic manipulations for each $r_t = i$ ($i \in S$), it can be verified that

$$\begin{aligned} \mathcal{L}[V(x_t, r_t)] &= 2x_t^T(0)P_i \dot{x}_t(0) + \sum_{j=1}^N \pi_{ij} x_t^T(0)P_j x_t(0) \\ &\quad + x_t^T(0)Qx_t(0) - x_t^T(-d)Qx_t(-d) \\ &\quad + d\dot{x}_t^T(0)R\dot{x}_t(0) - \int_{-d}^0 \dot{x}_t^T(s)R\dot{x}_t(s) ds \\ &\leq 2x_t^T(0)P_i \dot{x}_t(0) + \sum_{j=1}^N \pi_{ij} x_t^T(0)P_j x_t(0) \\ &\quad + x_t^T(0)Qx_t(0) - x_t^T(-d)Qx_t(-d) \\ &\quad - d^{-1} \left(\int_{-d}^0 \dot{x}_t(s) ds \right)^T R \left(\int_{-d}^0 \dot{x}_t(s) ds \right) \\ &\quad + d\dot{x}_t^T(0)R\dot{x}_t(0) \\ &= \eta^T(t) [\Phi^i(d) + \langle \mathbf{P}^i \rangle] \eta(t), \end{aligned}$$

where

$$\eta(t) = \text{col} \left\{ x_t(0), x_t(-d), \int_{-d}^0 \dot{x}_t(s) ds, \dot{x}_t(0) \right\}.$$

Thus, $\mathcal{L}V(x_t, r_t) < 0$ if

$$\eta^T(t) [\Phi^i(d) + \langle \mathbf{P}^i \rangle] \eta(t) < 0. \quad (2)$$

By the Newton-Leibniz formula, we have

$$\begin{aligned} -\dot{x}_t(0) + A_i x_t(0) + A_{di} x_t(-d) &= 0, \\ x_t(0) - x_t(-d) - \int_{-d}^0 \dot{x}_t(s) ds &= 0, \end{aligned}$$

or equivalently,

$$F_i \eta(t) = \begin{bmatrix} A_i & A_{di} & 0 & -I \\ I & -I & -I & 0 \end{bmatrix} \eta(t) = 0.$$

The above equality and (2) hold together, by Finsler's lemma, if and only if

$$F_i^{\perp T} [\Phi^i(d) + \langle \mathbf{P}^i \rangle] F_i^{\perp} < 0,$$

which follows from $\Upsilon^i < 0$ and

$$\begin{aligned} &F_i^{\perp T} [\Phi^i(d) + \langle \mathbf{P}^i \rangle] F_i^{\perp} \\ &= F_i^{\perp T} [\Phi^i(\bar{d}) \\ &\quad - \text{diag}\{0, 0, (d^{-1} - \bar{d}^{-1})R, (\bar{d} - d)R\} + \langle \mathbf{P}^i \rangle] F_i^{\perp} \\ &= F_i^{\perp T} [\Phi^i(\bar{d}) + \langle \mathbf{P}^i \rangle] F_i^{\perp} \\ &= -F_i^{\perp T} \text{diag}\{0, 0, (d^{-1} - \bar{d}^{-1})R, (\bar{d} - d)R\} F_i^{\perp} \\ &< 0. \end{aligned}$$

Now, we are in a position to deal with the stochastic stability of Σ . In view of $\mathcal{L}V(x_t, r_t) < 0$, we conclude that there exists a scalar $\gamma_1 > 0$ such that

$$\mathcal{L}V(x_t, r_t) \leq \eta^T(t) [\Phi^i(\bar{d}) + \langle \mathbf{P}^i \rangle] \eta(t) \leq -\gamma_1 \|x(t)\|^2 < 0$$

for all $x(t) \neq 0$. Therefore, it is readily seen from Dynkin's formula that

$$E\{V(x_t, r_t)\} - E\{V(x_{\bar{d}}, r_{\bar{d}})\} \leq -\gamma_1 E \left\{ \int_{\bar{d}}^t x^T(s)x(s) ds \right\}$$

which can be used to deduce that

$$E \left\{ \int_{\bar{d}}^t x^T(s)x(s) ds \right\} \leq \gamma_1^{-1} E\{V(x_{\bar{d}}, r_{\bar{d}})\}.$$

Then, using a similar method given in [31], we have that there exists a scalar $\gamma_2 > 0$ such that, for any $t \geq 0$,

$$\sup_{-\bar{d} \leq t \leq \bar{d}} \|x(t)\|^2 \leq 4e^{4\gamma_2 \bar{d}} \sup_{-\bar{d} \leq t \leq 0} \|\varphi(t)\|^2.$$

Based on the stochastic Lyapunov-Krasovskii functional $V(x_t, r_t)$, it can be verified that there exists a scalar $\gamma_3 > 0$ such that

$$\begin{aligned} V(x_{\bar{d}}, r_{\bar{d}}) &\leq \gamma_3 \sup_{-\bar{d} \leq t \leq \bar{d}} \|x(t)\|^2 \\ &\leq 4\gamma_3 e^{4\gamma_2 \bar{d}} \sup_{-\bar{d} \leq t \leq 0} \|\varphi(t)\|^2. \end{aligned}$$

Hence,

$$\begin{aligned} &E \left\{ \int_0^t x^T(s)x(s) ds \right\} \\ &= E \left\{ \int_0^{\bar{d}} x^T(s)x(s) ds + \int_{\bar{d}}^t x^T(s)x(s) ds \right\} \\ &\leq 4(\bar{d} + \gamma_3 \gamma_1^{-1}) e^{4\gamma_2 \bar{d}} E \left\{ \sup_{-\bar{d} \leq t \leq 0} \|\varphi(t)\|^2 \right\}. \end{aligned}$$

Taking limit as $t \rightarrow \infty$, it is clear that

$$\lim_{t \rightarrow \infty} E \left\{ \int_0^t x^T(s, \varphi, r_0) x(s, \varphi, r_0) ds \right\} < \infty$$

which shows that MJTDS Σ is stochastically stable. This completes the proof. \square

For stability analysis, Theorem 1 provides a simple representation of stability analysis for MJLSs with a constant delay using the Newton-Leibniz formula and the Finsler Lemma.

III. DELAY-DEPENDENT STABILITY ANALYSIS

Now, we are in a position to give the stability analysis in the case of partly unknown transition rates by applying the approach proposed in [36] for delay-free MJLSs.

Theorem 2: MJTDS Σ with $u(t) = 0$ and partially unknown transition rates is stochastically stable for any delay $0 < d \leq \bar{d}$ if there exist matrices $P_i > 0$, $Q > 0$, $R > 0$ such that the following LMIs hold for $i = 1, 2, \dots, N$,

$$\begin{aligned} \Upsilon_1^i &\triangleq F_i^{\perp T} [\Phi^i + \langle \mathbf{P}_1^i \rangle] F_i^{\perp} < 0, \quad \forall j \in \bar{S}_K^i, \text{ if } i \in S_K^i, \\ \Upsilon_2^i &\triangleq F_i^{\perp T} [\Phi^i + \langle \mathbf{P}_2^i \rangle] F_i^{\perp} < 0, \quad \forall j \in \bar{S}_K^i, \text{ if } i \in \bar{S}_K^i, \end{aligned}$$

where F_i^{\perp} and Φ^i are defined in Υ^i in Theorem 1 and

$$\begin{aligned} \mathbf{P}_1^i &= \mathbf{P}_K^i - \pi_K^i P_j \\ \mathbf{P}_2^i &= \mathbf{P}_K^i - \pi_K^i P_j + \pi_l (P_i - P_j) \end{aligned}$$

with π_l is a given lower bound of the unknown diagonal elements.

Proof: Two cases $i \in S_K^i$ and $i \in \bar{S}_K^i$ will be separately discussed for the stability analysis of system (II).

Case I: $i \in S_K^i$.

It is easy to get that $\pi_K^i \leq 0$ for $\sum_{j=1}^N \pi_{ij} = 0$. We only consider $\pi_K^i < 0$ here since $\pi_K^i = 0$ means that all the unknown elements in the i th row are zeros (for $\pi_{ij} \geq 0, \forall j \in \bar{S}_K^i$).

For all $j \in \bar{S}_K^i$, the element π_{ij} satisfies $\sum_{j \in \bar{S}_K^i} (\pi_{ij}) / (-\pi_K^i) = 1$ and $0 \leq (\pi_{ij}) / (-\pi_K^i) \leq 1$ (for $\pi_{ij} \geq 0$), then we get

$$\begin{aligned} \mathbf{P}^i &= \mathbf{P}_K^i + \sum_{j \in \bar{S}_K} \pi_{ij} P_j \\ &= \mathbf{P}_K^i - \pi_K^i \sum_{j \in \bar{S}_K} \frac{\pi_{ij}}{-\pi_K^i} P_j \\ &= \sum_{j \in \bar{S}_K} \frac{\pi_{ij}}{-\pi_K^i} \mathbf{P}_1^i \end{aligned}$$

and

$$\begin{aligned} \Upsilon^i &= F_i^{\perp T} [\Phi^i + \langle \mathbf{P}^i \rangle] F_i^{\perp} \\ &= \sum_{j \in \bar{S}_K} \frac{\pi_{ij}}{-\pi_K^i} F_i^{\perp T} [\Phi^i + \langle \mathbf{P}_1^i \rangle] F_i^{\perp} \\ &= \sum_{j \in \bar{S}_K} \frac{\pi_{ij}}{-\pi_K^i} \Upsilon_1^i. \end{aligned}$$

Therefore, $\Upsilon^i < 0$ is equivalent to $\Upsilon_1^i < 0$ for all $j \in \bar{S}_K^i$.

Case II: $i \in \bar{S}_K^i$.

Since π_{ii} is unknown, we get the relationships that $\pi_K^i \geq 0$ and $-\pi_{ii} \geq \pi_K^i$. Also, we only consider $-\pi_{ii} > \pi_K^i$ here since if $-\pi_{ii} = \pi_K^i$, then all the unknown elements in the i th row are zeros (for $\pi_{ij} \geq 0, \forall j \in \bar{S}_K^i, j \neq i$).

Likewise, since $\sum_{j \in \bar{S}_K, j \neq i} (\pi_{ij}) / (-\pi_{ii} - \pi_K^i) = 1$ and $0 \leq (\pi_{ij}) / (-\pi_{ii} - \pi_K^i) \leq 1$, we know that

$$\begin{aligned} \mathbf{P}^i &= \mathbf{P}_K^i + \pi_{ii} P_i + \sum_{j \in \bar{S}_K, j \neq i} \pi_{ij} P_j \\ &= \mathbf{P}_K^i + \pi_{ii} P_i + (-\pi_{ii} - \pi_K^i) \sum_{j \in \bar{S}_K, j \neq i} \frac{\pi_{ij}}{-\pi_{ii} - \pi_K^i} P_j \\ &= \sum_{j \in \bar{S}_K, j \neq i} \frac{\pi_{ij}}{-\pi_{ii} - \pi_K^i} [\mathbf{P}_K^i + \pi_{ii} (P_i - P_j) - \pi_K^i P_j]. \end{aligned}$$

Denoting $\mathbf{M}^i(\pi_{ii}) \triangleq \mathbf{P}_K^i + \pi_{ii} (P_i - P_j) - \pi_K^i P_j$, then

$$\begin{aligned} \Upsilon^i &= F_i^{\perp T} [\Phi^i + \langle \mathbf{P}^i \rangle] F_i^{\perp} \\ &= \sum_{j \in \bar{S}_K, j \neq i} \frac{\pi_{ij}}{-\pi_{ii} - \pi_K^i} F_i^{\perp T} [\Phi^i + \langle \mathbf{M}^i(\pi_{ii}) \rangle] F_i^{\perp} \end{aligned}$$

which means that $\Upsilon^i < 0$ is equivalent to $F_i^{\perp T} [\Phi^i + \langle \mathbf{M}^i(\pi_{ii}) \rangle] F_i^{\perp} < 0$ for all $j \in \bar{S}_K^i$ and $j \neq i$.

Since π_{ii} is unknown and π_l is a lower bound of $\pi_{ii}, \forall i \in S_K^i$, that is, $\pi_l \leq \pi_{ii} < -\pi_K^i$. There exists a sufficiently small scalar $\epsilon > 0$, π_{ii} may take any value between $[\pi_l, -\pi_K^i - \epsilon]$, and then π_{ii} can be further written as a convex combination

$$\pi_{ii}(\beta) = -\beta \pi_K^i - \beta \epsilon + (1 - \beta) \pi_l$$

where β takes value arbitrarily in $[0, 1]$. Thus, $F_i^{\perp T} [\Phi^i + \langle \mathbf{M}^i(\pi_{ii}) \rangle] F_i^{\perp} < 0$ holds if and only if the following two boundary conditions hold simultaneously,

$$F_i^{\perp T} [\Phi^i + \langle \mathbf{M}^i(\pi_{ii}(0)) \rangle] F_i^{\perp} < 0, \quad (3)$$

$$F_i^{\perp T} [\Phi^i + \langle \mathbf{M}^i(\pi_{ii}(1)) \rangle] F_i^{\perp} < 0, \quad (4)$$

where

$$\begin{aligned} \mathbf{M}^i(\pi_{ii}(0)) &= \mathbf{P}_K^i + \pi_l (P_i - P_j) - \pi_K^i P_j = \mathbf{P}_2^i, \\ \mathbf{M}^i(\pi_{ii}(1)) &= \mathbf{P}_K^i - (\epsilon + \pi_K^i) (P_i - P_j) - \pi_K^i P_j \\ &= \mathbf{P}_K^i - \pi_K^i P_i - \epsilon (P_i - P_j). \end{aligned}$$

Due to $\epsilon > 0$ can be arbitrarily small, (4) holds if and only if

$$F_i^{\perp T} [\Phi^i + \langle \mathbf{P}_K^i - \pi_K^i P_i \rangle] F_i^{\perp} < 0$$

which indeed is a particular case of inequality (3) when $j = i$, $\forall j \in \bar{S}_K^i$. Hence,

$$F_i^{\perp T} [\Phi^i + \langle \mathbf{M}^i(\pi_{ii}) \rangle] F_i^{\perp} < 0$$

is equivalent to (3) holds, which is precisely $\Upsilon_2^i < 0$.

Therefore, in the case of partially known transition rates, one can readily conclude that system Σ is stochastically stable if $\Upsilon_1^i < 0$ and $\Upsilon_2^i < 0$ for $i \in S_K^i$ and $i \in \bar{S}_K^i$ ($i = 1, 2, \dots, N$), respectively. This completes the proof. \square

Remark 1: Using the properties that the sum of each row is zero in the transition rate matrix, together with convex combination technique, a sufficient condition for the stability analysis for MJTDSs with incomplete transition rates is derived in Theorem 2. The knowledge of the transition rate matrix Π has been fully used. It should be noted that if there exists $i \in S_K^i$, all the elements in the i th row of Π are fully known, that is, $\bar{S}_K^i = \emptyset$, the corresponding inequalities for i th row to checking the stability property go back to the form of $F_i^{\perp T} [\Phi^i + \langle \mathbf{P}^i \rangle] F_i^{\perp} < 0$.

Remark 2: For the problem of stability analysis of MJTDSs with partial information on transition rates, a free-connection weighting matrix method has been proposed recently [37] to compute an upper bound of the allowable time delay by introducing some equalities involving a lot of slack matrices, some of which have been proved redundant in [38] on improving the margin of the time delay. Specifically, the form of the stability criterion given in [37] may not be amenable to designing a stabilizing controller due to the presence of cross-product terms of the slack matrices and the controller gains.

IV. DELAY-DEPENDENT STABILIZATION

Let us consider the stabilization problem for system Σ in the presence of partially unknown transition rates via a mode-dependent static output feedback controller of the following form:

$$C_O : u(t) = K_i^O y(t).$$

The closed-loop system is given by

$$\Sigma^O : \dot{x}(t) = A_i^O x(t) + A_{di}^O x(t-d)$$

where $A_i^O = A_i + B_i K_i^O C_i$ and $A_{di}^O = A_{di} + B_{di} K_i^O C_i$.

If A_i and A_{di} are replaced with A_i^O and A_{di}^O , respectively, in the inequalities $\Upsilon_j^i < 0$ ($j = 1, 2$) in Theorem 2, the non-linear coupling terms of the controller gains and the Lyapunov matrices will appear due to the two orthogonal complements. Hence, much difficulty exists to design a controller by some existing stabilizing approaches [14], [16], [26], [28], [30], [36]. The following theorem will present a sufficient criterion to overcome such difficulties by a new technique.

Theorem 3 (Output Feedback): Closed-loop system Σ^O with partially known transition rates is stochastically stable for any delay $0 < d \leq \bar{d}$, if there exist matrices $0 < \hat{P}_i \in \mathbb{R}^{n \times n}$, $0 < \hat{Q} \in \mathbb{R}^{m \times m}$, $0 < \hat{R} \in \mathbb{R}^{n \times n}$, nonsingular matrices $X_i \in \mathbb{R}^{2n \times 2n}$, and matrices $K_i^O \in \mathbb{R}^{m \times p}$ such that

$$\begin{aligned} \Upsilon_{O1}^i &\triangleq \begin{bmatrix} \hat{\Phi}^i + \langle \hat{\mathbf{P}}_l^i \rangle - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} < 0, \\ &\quad \forall j \in \bar{S}_K^i, \text{ if } i \in S_K^i, \\ \Upsilon_{O2}^i &\triangleq \Upsilon_{S1}^i + \langle \pi_i(\hat{P}_i - \hat{P}_j) \rangle < 0, \\ &\quad \forall j \in \bar{S}_K^i, \text{ if } i \in \bar{S}_K^i, \end{aligned}$$

hold for $i = 1, 2, \dots, N$, where

$$\begin{aligned} F_i &= \begin{bmatrix} A_i + B_i K_i^O C_i & A_{di} + B_{di} K_i^O C_i & 0 & -I \\ I & -I & -I & 0 \end{bmatrix}, \\ \hat{I}_1 &= \text{diag}\{I, 0, 0, I\}, \\ \hat{I}_2 &= \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \end{aligned}$$

and $\hat{\Phi}^i, \hat{\mathbf{P}}_l^i$ are Φ^i, \mathbf{P}_l^i , respectively, defined in Theorems 1 and 2 with P_i, Q, R replaced with $\hat{P}_i, \hat{Q}, \hat{R}$.

Proof: Closed-loop system Σ^O is stochastically stable, by Theorem 2, if there exist matrices $P_i > 0, Q > 0$ and $R > 0$ such that, for $i = 1, 2, \dots, N$,

$$\begin{aligned} F_i^{\perp T} [\Phi^i + \langle \mathbf{P}_1^i \rangle] F_i^{\perp} < 0, \quad \forall j \in \bar{S}_K^i, \text{ if } i \in S_K^i, \\ F_i^{\perp T} [\Phi^i + \langle \mathbf{P}_2^i \rangle] F_i^{\perp} < 0, \quad \forall j \in \bar{S}_K^i, \text{ if } i \in \bar{S}_K^i, \end{aligned}$$

F_i is full row rank and its right orthogonal complement satisfies $F_i F_i^{\perp} = 0$ and $F_i^{\perp T} F_i^{\perp} > 0$. It follows that

$$\begin{aligned} F_i^{\perp T} F_i^T &= (F_i^{\perp T} F_i^{\perp})^T = 0, \\ F_i^{\perp T} F_i^{\perp} &= (F_i^{\perp T} F_i^{\perp})^T > 0, \end{aligned}$$

which results that $F_i^{\perp T}$ is the left orthogonal complement of F_i^T , that is, $F_i^T F_i^{\perp} = F_i^{\perp T} F_i^T$. The inequalities for the stability of the closed-loop system $F_i^{\perp T} (\Phi^i + \langle \mathbf{P}_l^i \rangle) F_i^{\perp} < 0, l = 1, 2$, are equivalent to

$$F_i^T F_i^{\perp T} (\Phi^i + \langle \mathbf{P}_l^i \rangle) F_i^{\perp T} F_i^T < 0, \quad l = 1, 2. \quad (5)$$

On the other hand, based on Lemma 2, set $S_{1i} = \delta_i I \in \mathbb{R}^{4n \times 4n}$ with sufficiently large scalars $\delta_i > 0$, and $S_{2i} = S_{1i} - (\Phi^i + \langle \mathbf{P}_l^i \rangle) > 0, (l = 1, 2)$. We have that the inequalities in (5) hold if there exist full row rank matrices $G_i = [G_{1i} \ 0 \ G_{2i}] \in \mathbb{R}^{2n \times 4n}$ with $G_{1i}, G_{2i} \in \mathbb{R}^{2n \times n}$ such that

$$\begin{aligned} \delta_i (I + F_i^T G_i) (I + F_i^T G_i)^T \\ + (\Phi^i + \langle \mathbf{P}_l^i \rangle) - \delta_i I < 0, \quad l = 1, 2. \quad (6) \end{aligned}$$

Hence, system Σ^O is stochastically stable via an output feedback controller C_O if (6) holds. Next, we will derive an equivalent condition of (6).

Denote

$$T_i = \begin{bmatrix} G_{1i}^T & 0 \\ 0 & W_i^T \\ G_{2i}^T & 0 \end{bmatrix} \in \mathbb{R}^{4n \times 4n}$$

where $W_i \in \mathbb{R}^{2n \times 2n}$ is any nonsingular matrices. By Schur complement equivalence, (6) is equivalent to

$$\begin{bmatrix} \Phi^i + \langle \mathbf{P}_l^i \rangle - \delta_i I & * \\ I + G_i^T F_i & -\delta_i^{-1} I \end{bmatrix} < 0.$$

Pre- and post multiplying the above inequality with $\text{diag}\{I, T_i^T\}$ and $\text{diag}\{I, T_i\}$, respectively, it turns to be

$$\begin{aligned} &\begin{bmatrix} \Phi^i + \langle \mathbf{P}_l^i \rangle - \delta_i I & * \\ T_i^T + T_i^T G_i^T F_i & -\delta_i^{-1} T_i^T T_i \end{bmatrix} \\ &= \begin{bmatrix} \Phi^i + \langle \mathbf{P}_l^i \rangle - \delta_i I & * & * \\ G_i + G_i G_i^T F_i & -\delta_i^{-1} G_i G_i^T & * \\ [0 \ W_i \ 0] & 0 & -\delta_i^{-1} W_i W_i^T \end{bmatrix} \\ &< 0. \end{aligned}$$

Since the matrix G_i is full row rank, pre- and post multiplying the above inequality with $\text{diag}\{I, (G_i G_i^T)^{-1}, I\}$ and $\text{diag}\{I, (G_i G_i^T)^{-1}, I\}$, respectively, it is equivalent to

$$\begin{bmatrix} \Phi^i + \langle \mathbf{P}_l^i \rangle - \delta_i I & * & * \\ (G_i G_i^T)^{-1} G_i + F_i & -\delta_i^{-1} (G_i G_i^T)^{-1} & * \\ [0 \ W_i \ 0] & 0 & -\delta_i^{-1} W_i W_i^T \end{bmatrix} < 0 \quad (7)$$

which, by Schur complement equivalence, is also equivalent to

$$\begin{bmatrix} \Phi^i + \langle \mathbf{P}_l^i \rangle - \delta_i \hat{I}_1 & * \\ (G_i G_i^T)^{-1} G_i + F_i & -\delta_i^{-1} (G_i G_i^T)^{-1} \end{bmatrix} < 0. \quad (8)$$

Due to the particular structure of G_i , it is easy to know that $(G_i G_i^T)^{-1} G_i$ is full row rank and has a similar form to G_i ,

which can be denoted by $X_i \hat{I}_2$ where X_i is a nonsingular matrix. Then,

$$\begin{aligned} (G_i G_i^T)^{-1} &= (G_i G_i^T)^{-1} G_i G_i^T (G_i G_i^T)^{-1} \\ &= X_i \hat{I}_2 \hat{I}_2^T X_i^T \\ &= X_i X_i^T. \end{aligned}$$

Hence, (8) is equivalent to

$$\delta_i \begin{bmatrix} \delta_i^{-1} \Phi^i + \delta_i^{-1} \langle \mathbf{P}_i^i \rangle - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} < 0.$$

By setting $\hat{P}_i = \delta_i^{-1} P_i$, $\hat{Q} = \delta_i^{-1} Q$, $\hat{R} = \delta_i^{-1} R$, the inequality (7) holds if $\Upsilon_{Ol}^i < 0$ ($l = 1, 2$) respectively for two cases. This completes the proof. \square

Remark 3: One feature of the stability characterizations given in Theorems 2 and 3 is that there is no product term involving the Lyapunov matrices P_i , Q , R and the system matrices, which is suitable for determining the robust stability of the system with polytopic-type uncertainties satisfying the following real convex polytopic constraint:

$$\begin{aligned} (A_i, A_{di}, B_i, B_{di}, C_i) &\in \Omega \\ \Omega &\triangleq \left\{ (A_i(\alpha), A_{di}(\alpha), B_i(\alpha), B_{di}(\alpha), C_i(\alpha)) \right. \\ &= \left. \sum_{j=1}^p \alpha_j (A_{ij}, A_{dij}, B_{ij}, B_{dij}, C_{ij}) \right\} \\ &\times \left. \sum_{j=1}^p \alpha_j = 1, \alpha_j \geq 0 \right\} \end{aligned}$$

where A_{ij} , A_{dij} , B_{ij} , B_{dij} and C_{ij} ($j = 1, \dots, p$) are constant matrices with appropriate dimensions and α_j ($j = 1, \dots, p$) are time-invariant uncertainties. A similar treatment could be taken as that in [39], [40].

In the special case where state-feedback is applicable (that is, $C_i = I$), we have a state feedback controller as follows:

$$C_S : u(t) = K_i^S x(t).$$

The closed-loop system becomes

$$\Sigma^S : \dot{x}(t) = A_i^S x(t) + A_{di}^S x(t-d)$$

where $A_i^S = A_i + B_i K_i^S$, $A_{di}^S = A_{di} + B_{di} K_i^S$.

Theorem 4 (State Feedback): Closed-loop system Σ^S with partially known transition rates is stochastically stable for any delay $0 < d \leq \bar{d}$, if there exist matrices $0 < \hat{P}_i \in \mathbb{R}^{n \times n}$, $0 < \hat{Q} \in \mathbb{R}^{n \times n}$, $0 < \hat{R} \in \mathbb{R}^{n \times n}$, nonsingular matrices $X_i \in \mathbb{R}^{2n \times 2n}$ and matrices $K_i^S \in \mathbb{R}^{m \times n}$ such that, for $i = 1, 2, \dots, N$,

$$\begin{aligned} \Upsilon_{O1}^i &< 0, \quad \forall j \in \bar{S}_K^i, \text{ if } i \in S_K^i, \\ \Upsilon_{O2}^i &< 0, \quad \forall j \in \bar{S}_K^i, \text{ if } i \in \bar{S}_K^i, \end{aligned}$$

hold where $\Upsilon_{Ol}^i < 0$ ($l = 1, 2$) are defined in Theorem 3 in which

$$F_i = \begin{bmatrix} A_i + B_i K_i^S & A_{di} + B_{di} K_i^S & 0 & -I \\ I & -I & -I & 0 \end{bmatrix}.$$

When all the elements in the transition rate matrix Π are unknown except for a lower bound π_l of π_{ii} ($\forall i \in S$), the considered MJTDSs are subject to a special case of a switching model under arbitrary switching (see [41] for the time-varying delay case). Meanwhile, a natural conclusion can be directly drawn from Theorems 4 and 3 to this case.

Corollary 1: For any delay $0 < d \leq \bar{d}$, MJTDS Σ with fully unknown transition rates is stochastically stable via state or output feedback control, if there exist matrices $0 < \hat{P}_i \in \mathbb{R}^{n \times n}$, $0 < \hat{Q} \in \mathbb{R}^{n \times n}$, $0 < \hat{R} \in \mathbb{R}^{n \times n}$, nonsingular matrices $X_i \in \mathbb{R}^{2n \times 2n}$ and K_i such that

$$\begin{bmatrix} \hat{\Phi}^i + \langle \pi_l(\hat{P}_i - \hat{P}_j) \rangle - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} < 0$$

hold for $i = 1, 2, \dots, N$, where $\hat{\Phi}^i$, \hat{I}_1 , \hat{I}_2 and π_l are defined in Theorem 3 and,

$$F_i = \begin{bmatrix} A_i + B_i K_i & A_{di} + B_{di} K_i & 0 & -I \\ I & -I & -I & 0 \end{bmatrix}$$

with $K_i = K_i^S \in \mathbb{R}^{m \times n}$ in the state feedback case and $K_i = K_i^O \in \mathbb{R}^{m \times p}$ in the static output feedback case, respectively.

V. CONTROLLER COMPUTATION

Observing the stabilization conditions in Theorems 3 and 4 and Corollary 1, there are nonlinear terms $X_i X_i^T$. This gives rise to the non-convexity of the synthesis problems. The controller design problem amounts to searching feasible solutions of the quadratic matrix inequalities, such that the stabilizability of system (II) can be guaranteed via the state feedback controllers or the static output feedback controllers. We now adopt an effective algorithm proposed in [42], [43] to compute the controller gains. The nonconvex problem in Theorem 3 for the static output feedback control is chosen as an example to illustrate the details of the algorithm. A similar algorithm can be constructed to deal with the nonconvex problems in other cases in Theorem 4 and Corollary 1.

To accommodate the term $X_i X_i^T$, we introduce additional design variables $Y_i \in \mathbb{R}^{2n \times 2n}$. Since

$$(X_i - Y_i)(X_i - Y_i)^T \geq 0$$

for any matrices X_i and Y_i , it can be obtained that

$$-X_i X_i^T \leq -X_i Y_i^T - Y_i X_i^T + Y_i Y_i^T \quad (9)$$

with equalities hold when $X_i = Y_i$. By substituting the relationship in (9) into $\Upsilon_{Ol}^i < 0$ ($l = 1, 2$), sufficient conditions for the existence of the static output feedback controller gains K_i^O are given in the following theorem.

Theorem 5: The stabilizability conditions in Theorem 3

$$\begin{aligned} \Upsilon_{O1}^i &= \begin{bmatrix} \hat{\Phi}^i + \langle \hat{\mathbf{P}}_i^i \rangle - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} < 0, \\ &\quad \forall j \in \bar{S}_K^i, \text{ if } i \in S_K^i, \\ \Upsilon_{O2}^i &= \Upsilon_{S1}^i + \langle \pi_l(\hat{P}_i - \hat{P}_j) \rangle < 0, \\ &\quad \forall j \in \bar{S}_K^i, \text{ if } i \in \bar{S}_K^i \end{aligned}$$

hold if and only if there exist matrices $0 < \hat{P}_i \in \mathbb{R}^{n \times n}$, $0 < \hat{Q} \in \mathbb{R}^{n \times n}$, $0 < \hat{R} \in \mathbb{R}^{n \times n}$, nonsingular matrices $X_i \in \mathbb{R}^{2n \times 2n}$, $Y_i \in \mathbb{R}^{2n \times 2n}$ and matrices $K_i^O \in \mathbb{R}^{m \times p}$ such that

$$\bar{\Upsilon}_{O1}^i \triangleq \begin{bmatrix} \hat{\Phi}_1^i + \langle \hat{P}_i \rangle - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i Y_i^T - Y_i X_i^T + Y_i Y_i^T \end{bmatrix} < 0, \\ \forall j \in \bar{S}_K^i, \text{ if } i \in S_K^i, \\ \bar{\Upsilon}_{O2}^i \triangleq \bar{\Upsilon}_{O1}^i + \langle \pi_l(\hat{P}_i - \hat{P}_j) \rangle < 0, \\ \forall j \in \bar{S}_K^i, \text{ if } i \in \bar{S}_K^i$$

hold for all $i = 1, 2, \dots, N$, in which, $\hat{\Phi}_1^i$, \hat{P}_i^i , F_i , \hat{P}_i , \hat{Q} , \hat{R} , and \hat{I}_1 , \hat{I}_2 are defined in Theorem 3.

Proof. The sufficiency is obvious, and only the necessity proof needs to be carried out. Suppose that $\bar{\Upsilon}_{Ol}^i < 0$ ($l = 1, 2$) hold, there must exist scalars $v_i > 0$ such that $\bar{\Upsilon}_{Ol}^i + \text{diag}\{0, v_i I\} < 0$. Select a matrix $M_i \geq I$ and set $Y_i = X_i - v_i^{1/2} M_i^{-1/2}$, then we have

$$(X_i - Y_i)(X_i - Y_i)^T = v_i M_i^{-1} \leq v_i I.$$

Hence, $\bar{\Upsilon}_{Ol}^i < 0$ ($l = 1, 2$) hold. \square

The nonconvex problem in Theorem 5 points to an iterative approach to solve X_i and K_i^O , namely, if Y_i ($i \in S$) are fixed in advance, $\bar{\Upsilon}_{Ol}^i < 0$ ($l = 1, 2$) reduce to an LMI problem with respect to matrix variables \hat{P}_i , \hat{Q} , \hat{R} , X_i , K_i^O . The detailed process is shown in the following algorithm.

Algorithm ILMI

Step 1) Set $m = 0$, and select initial matrices $Y_i^{(m)}$ and a delay upper bound \bar{d} .

Step 2) Solve the following convex optimization problem with respect to $\hat{P}_i^{(m)} > 0$, $\hat{Q}^{(m)} > 0$, $\hat{R}^{(m)} > 0$, $X_i^{(m)} > 0$, $K_i^{O(m)}$ and $\varepsilon^{(m)}$,

$$\begin{aligned} & \text{minimize } \varepsilon^{(m)} \\ & \text{s.t. } \bar{\Upsilon}_{Sl}^i < \varepsilon^{(m)} I \quad (l = 1, 2) \end{aligned} \quad (10)$$

Denote $\varepsilon^{(m)*}$ as the minimized value of $\varepsilon^{(m)}$ satisfying (10). If $\varepsilon^{(m)*} \leq 0$, MJTDS Σ is stochastically stable via an output feedback controller C_O , STOP, else, go to Step 3.

Step 3) Solve the following convex optimization problem with respect to $\hat{P}_i^{(m)} > 0$, $\hat{Q}^{(m)} > 0$, $\hat{R}^{(m)} > 0$, $X_i^{(m)} > 0$, and $K_i^{O(m)}$,

$$\begin{aligned} & \text{minimize } \text{trace}(X_i^{(m)}) \\ & \text{s.t. } \bar{\Upsilon}_{Sl}^i < \varepsilon^{(m)*} I \quad (l = 1, 2) \end{aligned} \quad (11)$$

Denote $X_i^{(m)*}$ as the $X_i^{(m)}$ which minimizes $\text{trace}(X_i^{(m)})$.

Step 4) If $\|Y_i^{(m)} - X_i^{(m)*}\| < \delta$, a prescribed tolerance, go to Step 5, else, update $Y_i^{(m+1)}$ as

$$Y_i^{(m+1)} := X_i^{(m)*}$$

and set $m = m + 1$, then go to Step 2.

Step 5) MJTDS Σ may not be stabilizable via output feedback control. STOP.

Remark 4: For a given MJTDS Σ , it is stabilizable via static output feedback if inequalities $\bar{\Upsilon}_{Ol}^i < 0$ ($l = 1, 2$) have feasible solutions. In order to search for these feasible solutions, inequality (9) provides a crucial rule to update $X_i^{(m)}$ in Algorithm ILMI. The optimization in Step 3 guarantees that the sequence $\{\text{trace}(X_i^{(m)})\}$ is bounded and it is a monotonic decreasing sequence with $Y_i^{(m)} := X_i^{(m)*}$ for fixed $\varepsilon^{(m)*}$ when $m > h$, where h is a positive integer [44]. On the other hand, the sequence $\{\varepsilon^{(m)*}\}$ is guaranteed to be monotonically decreasing. To make this fact clear, let us denote $\bar{\Upsilon}(Y_i, X_i) \triangleq \bar{\Upsilon}_{Ol}^i$ ($l = 1, 2$). For a fixed $Y_i^{(m)}$, one has $\bar{\Upsilon}(Y_i^{(m)}, X_i^{(m)*}) < \varepsilon^{(m)*} I$ from Step 3. With an updated $Y_i^{(m+1)} = X_i^{(m)*}$ in Step 4, we get

$$\begin{aligned} \bar{\Upsilon}(Y_i^{(m+1)}, X_i^{(m)*}) &= \bar{\Upsilon}(X_i^{(m)*}, X_i^{(m)*}) \\ &\leq \bar{\Upsilon}(Y_i^{(m)}, X_i^{(m)*}) \\ &< \varepsilon^{(m)*} I \end{aligned}$$

which is derived from

$$\begin{aligned} & -X_i^{(m)*} Y_i^{(m+1)T} - Y_i^{(m+1)} X_i^{(m)*T} + Y_i^{(m+1)} Y_i^{(m+1)T} \\ &= -X_i^{(m)*} X_i^{(m)*T} \\ &\leq -X_i^{(m)*} Y_i^{(m)T} - Y_i^{(m)} X_i^{(m)*T} + Y_i^{(m)} Y_i^{(m)T} \end{aligned}$$

based on the relationship shown in (9). Therefore, there exists an $X_i^{(m+1)}$ such that $\bar{\Upsilon}(Y_i^{(m+1)}, X_i^{(m+1)}) < \varepsilon^{(m)*} I$ is feasible with the fixed $Y_i^{(m+1)}$. Meanwhile, in the optimization problem (10) in Step 2,

$$\varepsilon^{(m+1)*} = \min \left\{ \varepsilon^{(m+1)} \mid \bar{\Upsilon}(Y_i^{(m+1)}, X_i^{(m+1)}) < \varepsilon^{(m+1)} I \right\}$$

then a solution $\varepsilon^{(m+1)*} \leq \varepsilon^{(m)*}$ can be obtained. As a result, through Steps 2 and 3, one can conclude that Algorithm ILMI is convergent although we may not achieve a feasible solution for the stabilization problem.

Remark 5: Like other iterative approaches, Algorithm ILMI involves local optimization due to the dependence of the choice of the initial values. Therefore, if the algorithm fails to arrive at a feasible solution, we may select other Y_i (randomly) and run Algorithm ILMI again. In addition, the initial delay bound \bar{d} in Step 1 of Algorithm ILMI can be chosen to be a sufficiently small positive scalar. If a negative $\varepsilon^{(m)*}$ is found, increase \bar{d} with a prescribed small increment $\mu > 0$ as $\bar{d} = \bar{d} + \mu$, then run the algorithm again till the algorithm has no feasible result. On the contrary, if no feasible d is found, the system may not be stabilizable via a static output feedback control.

VI. NUMERICAL EXAMPLES

This section presents several numerical examples to verify the validity of the results obtained. In the sequel, the question mark symbol ? denotes an unknown element in the transition rate matrix Π .

Example 1 (Stability): Consider the underlying MJTDS Σ with two operation modes:

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.29 & 0.75 \\ -2.65 & -2.33 \end{bmatrix}, & A_{d1} &= \begin{bmatrix} -1.24 & 0.00 \\ 0.15 & 1.15 \end{bmatrix}; \\ A_2 &= \begin{bmatrix} -2.50 & 2.19 \\ -1.46 & -1.93 \end{bmatrix}, & A_{d2} &= \begin{bmatrix} -1.57 & 0.00 \\ 2.24 & -1.12 \end{bmatrix}. \end{aligned}$$

TABLE I
DIFFERENT TRANSITION RATE MATRICES FOR EXAMPLE 1

Completely unknown (Case I)	Partially unknown (Case II)	Partially unknown (Case III)
$\Pi = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$	$\Pi = \begin{bmatrix} ? & ? \\ 2.11 & -2.11 \end{bmatrix}$	$\Pi = \begin{bmatrix} -2.11 & 2.11 \\ ? & ? \end{bmatrix}$

TABLE II
COMPARISON ON \bar{d} IN EXAMPLE 1

π_l	-1	-10	-50	-100	-200	-1000/-2000/-5000
\bar{d} (Case I)	0.8615	0.6957	0.6699	0.6680	0.6675	0.6673
\bar{d} (Case II)	0.9085	0.7358	0.7118	0.7089	0.7074	0.7063/0.7062/0.7061
\bar{d} (Case III)	$\sim \infty$	0.7603	0.7398	0.7362	0.7342	0.7332

We now apply the proposed approach to estimate the upper bound \bar{d} such that MJTDS Σ given in this example is stochastically stable for any delay satisfying $0 < d \leq \bar{d}$. Here, three cases for the transition rate matrix Π shown in Table I will be separately discussed. In Case I, all the elements in each row of Π are unknown. For different diagonal element lower bounds π_l of the diagonal elements π_{ii} ($i = 1, 2$) given in Table II, by Theorem 2, the upper bounds \bar{d} are obtained. It can be seen from the second row in Table II that the smaller the assigned value of π_l , the smaller the delay upper bound obtained, which tends to a constant value.

If we select another Π as in Case II, where the elements in the first row are completely unknown and those in the second row are fully known, based on Theorems 1 and 2, the MJTDS Σ is stochastically stable if

$$F_1^{\perp T} [\Phi^1 + \langle \pi_l(P_1 - P_j) \rangle] F_1^\perp < 0, \quad j = 1, 2$$

$$F_2^{\perp T} [\Phi^2 + \langle 2.11(P_1 - P_2) \rangle] F_2^\perp < 0.$$

To calculate using MATLAB toolbox, the possible upper bounds \bar{d} satisfy the above inequalities are listed in Table II for π_l assigned as -1 , -10 , -100 , and -500 , respectively. If the transition rate matrix Π becomes

$$\Pi = \begin{bmatrix} -2.11 & 2.11 \\ ? & ? \end{bmatrix}$$

in Case III, the upper bounds on the delay are obtained in the last row in Table II. There is an interesting phenomenon in such case that \bar{d} becomes an effectively “infinite” value (1.0×10^{21}) when π_l is assigned as -1 . Within the numerical accuracy of computation, we may conclude that MJTDS Σ is delay-independent if the uncertainty bound of the unknown diagonal element π_{ii} is not less than -1 .

It is clearly observed from Table II that, when all or part of the transition rates are unable to obtain, the maximum time delay tolerated by MJTDS Σ for maintaining stability becomes smaller as the transition rates knowledge reduces and the uncertainty of the unknown diagonal element increases.

For the last two cases, the computed values of \bar{d} obtained in [37] are 0.7041 and 0.6962, respectively, and no allowable delay upper bound can be found. Moreover, compared with the stability conditions in Theorem 4 [37], Theorem 2 in our paper not only gives larger upper bound on the time delay, but also has significantly fewer variables. When MJTDSs have N modes and $A_i, A_{di} \in \mathbb{R}^{n \times n}$, the number of decision variables to be determined in our approach is $((N)/(2) + 1)(n^2 + n)$, while

that in [37] is $((13N)/(2) + 1)n^2 + ((5N)/(2) + 1)n$. In other words, the total computation variables involved in [37] are a dozen times more than those in Theorem 2 for large N or n .

Example 2: Consider the following MJTDS Σ with three operation modes:

$$A_1 = \begin{bmatrix} 0.00 & -0.75 \\ 0.00 & -3.00 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 1.00 & 0.00 \\ 0.00 & -1.00 \end{bmatrix};$$

$$A_2 = \begin{bmatrix} -2.50 & -0.43 \\ 5.00 & -1.00 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1.00 & 0.00 \\ 0.00 & 1.00 \end{bmatrix};$$

$$A_3 = \begin{bmatrix} -3.00 & 0.00 \\ 5.00 & -3.00 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} -1.00 & 0.00 \\ 0.00 & 1.00 \end{bmatrix};$$

and

$$B_1 = \begin{bmatrix} 1.00 \\ 0.00 \end{bmatrix}, \quad B_{d1} = \begin{bmatrix} 1.00 \\ 0.00 \end{bmatrix};$$

$$B_2 = \begin{bmatrix} 0.00 \\ -1.00 \end{bmatrix}, \quad B_{d2} = \begin{bmatrix} 0.00 \\ 1.00 \end{bmatrix};$$

$$B_3 = \begin{bmatrix} 1.00 \\ 1.00 \end{bmatrix}, \quad B_{d3} = \begin{bmatrix} 0.00 \\ 1.00 \end{bmatrix}.$$

Suppose the transition rate matrix Π is given by

$$\Pi = \begin{bmatrix} -1.30 & ? & ? \\ ? & -0.7 & ? \\ ? & 0.5 & ? \end{bmatrix}$$

The purpose of this example is to verify the efficiency of the proposed approach for stabilizability analysis. Note that the transition rate matrix Π involves both known and unknown diagonal elements. We assign *a priori* a lower bound of the unknown diagonal elements as $\pi_l = -200$. Based on Theorem 4 and Algorithm ILMI with $Y_1 = Y_2 = Y_3 = I$, system Σ is stabilizable with $\bar{d} = 54.3189$ (the largest value of the delay ensuring stability based on the feasibility of inequalities derived in Theorem 5) via the state feedback controllers whose gains given in Table III, and the other corresponding matrix variables are as follows:

$$X_1 = \begin{bmatrix} 1.5830 & -0.2876 & 0.7400 & 2.0950 \\ -0.0301 & 1.5311 & -0.0712 & 7.8717 \\ -383.0951 & 35.5506 & 607.1956 & 42.7209 \\ 15.6777 & -348.5555 & 54.3213 & 479.3590 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 1.1896 & -0.9115 & 2.3598 & 1.2808 \\ -0.4590 & 3.3291 & -4.2556 & 0.9696 \\ -388.6283 & -59.2718 & 402.9296 & -6.9125 \\ 89.4565 & -381.5922 & -170.5630 & 474.7880 \end{bmatrix},$$

TABLE III
STATE FEEDBACK STABILIZING CONTROLLERS FOR EXAMPLE 2 WITH
 $d = 54.3189$

	Controller gains: K^S for (C_S)
Theorem 4 ($\pi_l = -200$)	$K_1^S = [-1.3009 \quad -0.0636]$ $K_2^S = [0.7870 \quad -0.8607]$ $K_3^S = [0.3845 \quad -0.5009]$

$$X_3 = \begin{bmatrix} 1.3248 & -0.1061 & 3.4500 & 1.5200 \\ 0.4189 & 1.7038 & -7.6297 & 8.0053 \\ -393.6532 & 85.7045 & 479.5951 & -44.6333 \\ -26.2936 & -348.0110 & 66.6904 & 350.4067 \end{bmatrix},$$

and

$$\hat{P}_1 = \begin{bmatrix} 0.6011 & -0.0229 \\ -0.0229 & 0.3332 \end{bmatrix}, \quad \hat{P}_2 = \begin{bmatrix} 0.8867 & 0.1560 \\ 0.1560 & 0.3072 \end{bmatrix},$$

$$\hat{P}_3 = \begin{bmatrix} 0.6727 & -0.0663 \\ -0.0663 & 0.3601 \end{bmatrix},$$

$$\hat{Q} = \begin{bmatrix} 0.2104 & -0.0164 \\ -0.0164 & 0.0895 \end{bmatrix},$$

$$\hat{R} = 10^{-3} \times \begin{bmatrix} 0.4674 & 0.1024 \\ 0.1024 & 1.2311 \end{bmatrix}.$$

Given the output measurements in each subsystem as

$$C_1 = [-1.6254 \quad 1.2743],$$

$$C_2 = [2.3800 \quad 1.8525],$$

$$C_3 = [-1.9296 \quad 1.1239],$$

by Theorem 3 and Algorithm ILMI, with the same initial matrices $Y_i = I$ ($i = 1, 2, 3$), MJTDS Σ in this example is MSS with $d = 11.5999$ (the largest value on the delay to guarantee stability based on Algorithm ILMI) via static output feedback with gains presented in Table IV.

The associated matrix variables are

$$X_1 = \begin{bmatrix} 1.6045 & 0.5282 & 0.3330 & -2.3129 \\ 0.1080 & 2.7821 & -0.2046 & 4.8160 \\ -129.6448 & 6.2239 & 202.9774 & 11.3830 \\ 2.0023 & -83.9309 & 10.1199 & 194.2531 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 1.1098 & -0.7928 & 2.2999 & 2.6023 \\ -0.3192 & 3.3511 & -4.3283 & 0.4178 \\ -133.1967 & -22.3472 & 135.1486 & 1.3352 \\ 13.7300 & -82.7930 & -40.9890 & 179.9389 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 1.4613 & -0.0020 & 3.8522 & 0.7118 \\ 0.2536 & 2.9987 & -7.9822 & 3.6162 \\ -136.1142 & 8.3824 & 179.4264 & -22.3756 \\ -2.1352 & -83.7732 & 3.8018 & 196.9411 \end{bmatrix},$$

and

$$\hat{P}_1 = \begin{bmatrix} 0.6263 & -0.0259 \\ -0.0259 & 0.2845 \end{bmatrix}, \quad \hat{P}_2 = \begin{bmatrix} 0.9022 & 0.1093 \\ 0.1093 & 0.3047 \end{bmatrix},$$

$$\hat{P}_3 = \begin{bmatrix} 0.6274 & -0.0263 \\ -0.0263 & 0.2842 \end{bmatrix},$$

$$\hat{Q} = \begin{bmatrix} 0.2354 & -0.0964 \\ -0.0964 & 0.5159 \end{bmatrix},$$

$$\hat{R} = 10^{-3} \times \begin{bmatrix} 0.8359 & 0.1353 \\ 0.1353 & 3.8116 \end{bmatrix}.$$

TABLE IV
STATIC OUTPUT FEEDBACK STABILIZING CONTROLLERS FOR EXAMPLE 2
WITH $d = 11.5999$

	Controller gains: K^O for (C_O)
Theorem 3 ($\pi_l = -200$)	$K_1^O = 0.7721$ $K_2^O = 0.2198$ $K_3^O = -0.0451$

TABLE V
STATE FEEDBACK STABILIZING CONTROLLERS FOR EXAMPLE 3 WITH
 $d = 3.8640$

	Controller gains: K^S for (C_S)
Corollary 1 ($\pi_l = -200$)	$K_1^S = [-1.6111 \quad 0.3378]$ $K_2^S = [0.0186 \quad -1.8501]$

It is worth mentioning that in this example to design the state feedback and the static output feedback controllers, the maximum iteration number in Algorithm ILMI is assigned as 30. According to the authors' numerical experience, the feasible delay upper bound \bar{d} will often increase to a limiting value as the maximum number of iterations increases when the same decision variables are used. In other words, the feasible solution obtained by Algorithm ILMI not only depends on the choice of initial matrices Y_i but also depends on the maximal number of iterations fixed in advance.

Example 3: Consider deterministic switched MJTDS Σ under arbitrary switching which has two subsystems as follows:

$$A_1 = \begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{d1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

$$A_2 = \begin{bmatrix} -5 & 0 \\ 5 & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{d2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Based on Corollary 1, a feasible upper bound on delay $\bar{d} = 3.8640$ can be reached. With such a delay value, the system is stochastically stable via state feedback with gains given in Table V.

Other matrix variables are

$$X_1 = \begin{bmatrix} 2.5385 & 2.6862 & 2.1550 & -8.9408 \\ -3.3518 & -1.6149 & 4.4913 & 9.1123 \\ -0.8428 & -47.9064 & -11.9590 & 54.5843 \\ -22.4126 & 25.9375 & 40.4167 & -40.6922 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} -1.1451 & -5.1643 & 9.0988 & 4.6827 \\ -0.7836 & 2.9517 & -5.8854 & 5.7179 \\ 5.5240 & -42.4781 & -20.2731 & 50.6897 \\ -27.1522 & 15.9373 & 45.0984 & -31.7518 \end{bmatrix},$$

and

$$\hat{P}_1 = \begin{bmatrix} 0.5678 & -0.0445 \\ -0.0445 & 0.2550 \end{bmatrix}, \quad \hat{P}_2 = \begin{bmatrix} 0.5678 & -0.0445 \\ -0.0445 & 0.2550 \end{bmatrix},$$

$$\hat{Q} = \begin{bmatrix} 0.6023 & -0.1288 \\ -0.1288 & 0.1725 \end{bmatrix},$$

$$\hat{R} = \begin{bmatrix} 0.0311 & 0.0247 \\ 0.0247 & 0.0255 \end{bmatrix}.$$

In this example, the initial matrices Y_i ($i = 1, 2$) in Algorithm ILMI are chosen to be a random matrix

$$Y_1 = Y_2 = Y_3 = \begin{bmatrix} 9.2169 & 1.1977 & -8.3018 & -0.6000 \\ 0.2977 & 10.0484 & -0.6088 & 1.6939 \\ -7.0000 & -0.2088 & 12.8659 & 1.3282 \\ -0.8881 & 1.6939 & 1.6282 & -4.8728 \end{bmatrix}.$$

In addition, it is interesting to see that \hat{P}_1 is approximately equal to \hat{P}_2 in the result with

$$\hat{P}_1 - \hat{P}_2 = 10^{-8} \times \begin{bmatrix} 0.2076 & -0.0728 \\ -0.0728 & 0.0275 \end{bmatrix}.$$

When all the transition rates are unknown, for the i th row of Π , the stabilizability criteria in Corollary 1 are

$$\begin{bmatrix} \hat{\Phi}^i + \langle \pi_l(\hat{P}_i - \hat{P}_i) \rangle - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} < 0, \text{ when } j = i$$

$$\begin{bmatrix} \hat{\Phi}^i + \langle \pi_l(\hat{P}_i - \hat{P}_j) \rangle - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} < 0, \forall j \text{ when } j \neq i$$

equivalently, that is,

$$\begin{bmatrix} \hat{\Phi}^j - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} < 0, \text{ when } j = i \quad (12)$$

$$\begin{bmatrix} \hat{\Phi}^i - \hat{I}_1 & * \\ X_i \hat{I}_2 + F_i & -X_i X_i^T \end{bmatrix} + \langle \pi_l^i(\hat{P}_i - \hat{P}_j) \rangle < 0, \forall j \text{ when } j \neq i \quad (13)$$

Intuitively, the more negative the assigned value of π_l , the closer \hat{P}_1 and \hat{P}_2 get. For instance, assigning $\pi_l = -20$ and $\pi_l = -50$, we obtain, respectively,

$$\hat{P}_1 - \hat{P}_2 = 10^{-2} \times \begin{bmatrix} 0.6324 & -0.2856 \\ -0.2856 & 0.1289 \end{bmatrix}$$

$$\hat{P}_1 - \hat{P}_2 = 10^{-7} \times \begin{bmatrix} 0.1990 & -0.0781 \\ -0.0781 & 0.0311 \end{bmatrix}$$

Therefore, the two inequalities in (12) and (13) naturally lead to a sufficient condition by having the first inequality holds with $\hat{P}_i = \hat{P}_j$.

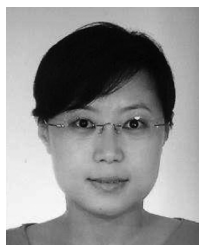
VII. CONCLUSION

This paper has investigated the analysis and synthesis problems of MJTDSs with incomplete knowledge of the transition rate matrix. In the analysis aspect, an LMI approach has been developed to test the stability property. The information of the transition rates has been fully used in the criteria. The decoupling of the Lyapunov matrices and the system matrices can also allow robust stability result for polytopic-type uncertainties to be developed. In the synthesis aspect, both the state feedback and the static output feedback controllers have been designed in a similar framework which guarantee the stochastic stability of the closed-loop systems. The desired controllers can be constructed through a convex optimization problem.

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Baozhu Du was born in Liaoning Province, China, in 1981. She received the B.S. degree in information and computing science, and M.S. degrees in operational research and cybernetics from Northeastern University, Shenyang, Liaoning Province, China, in 2003 and 2006, respectively. She received the Ph.D. degree in mechanical engineering from The University of Hong Kong in 2010.

She joined Nanjing University of Science and Technology in April 2011, taking a lectureship in School of Automation. Her current research interests

include stability analysis and robust control/filter theory of time-delay systems, positive systems, Markovian jump systems, and networked control systems.



James Lam (F'12) received the B.Sc. (1st Hons.) degree in mechanical engineering from the University of Manchester, U.K., and was awarded the Ashbury Scholarship, the A.H. Gibson Prize, and the H. Wright Baker Prize for his academic performance. He received the M.Phil. and Ph.D. degrees from the University of Cambridge, U.K. His doctoral and post-doctoral research projects were supported by the Croucher Foundation Scholarship and Fellowship. He was a recipient of the Outstanding Researcher Award of the University of Hong Kong and a Distinguished Visiting Fellow of the Royal Academy of Engineering. Prior to joining the University of Hong Kong in 1993, he held lectureships at the City University of Hong Kong and the University of Melbourne. He has research interests in model reduction, robust control and filtering, delay, singular systems, Markovian jump systems, multidimensional systems, networked control systems, vibration control, and biological networks.

Prof. Lam is a Chartered Mathematician, Chartered Scientist, Fellow of Institute of Electrical and Electronic Engineers, Fellow of Institution of Engineering and Technology, and Fellow of Institute of Mathematics and Its Applications. He is Editor-in-Chief of *IET Control Theory and Applications*, Subject Editor of *Journal of Sound and Vibration*, Associate Editor of *Automatica*, *Asian Journal of Control*, *International Journal of Systems Science*, *International Journal of Applied Mathematics and Computer Science*, *Journal of the Franklin Institute*, *Multidimensional Systems and Signal Processing*, and is editorial member of *Dynamics of Continuous, Discrete and Impulsive Systems: Series B (Applications & Algorithms)*, and *Proc. IMechE Part I: Journal of Systems and Control Engineering*. He was an Associate Member of the *IEEE Transactions on Signal Processing*, and a member of the IFAC Technical Committee on Control Design. Professor Lam has served the Engineering Panel of the Research Grants Council, HKSAR. He is a co-recipient of the International Journal of Systems Science Prize Paper Award.



Yun Zou was born in 1962. He received the B.S. degree in mathematics from Northwestern University in 1983 and the M.S. and Ph.D. degrees in control theory and control engineering from the Automatic Institute of Nanjing University of Science and Technology, China, in 1987 and in 1990, respectively.

Now he is a Professor of the Automatic Institute at Nanjing University of Science and Technology. His research interests include differential-algebraic equation systems and two dimensional systems.



Zhan Shu (M'09) was born in Nanchang, Jiangxi province, China, in 1982. He received the B.Eng. degree in automation from Huazhong University of Science and Technology (HUST), China, in 2003, and the Ph.D. degree in control engineering from The University of Hong Kong (HKU) in 2008.

He was a research assistant/associate in HKU from 2008 to 2009, and a postdoctoral researcher at the Hamilton Institute, National University of Ireland, Maynooth, from 2009 to 2011. He joined the University of Southampton, U.K., in Sep. 2011, taking a lectureship in Faculty of Engineering and the Environment. His current research interests include hybrid systems, stochastic systems, positive systems, robust control, estimation and filtering, decentralized control, systems biology with emphasis on cell regulating networks, immunology, and physiology.