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A Generalized Algorithm for Fast Two-Dimensional Angle Estimation of a Single Source With Uniform Circular Arrays

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Abstract—We reported an algorithm for two-dimensional (2-D) angle estimation of a single source with uniform circular arrays (UCAs) in our previous work. However, it is worth noting that this method is derived based on the assumption that the number of the sensors is even. This may limit its application in practice. To this end, we extend our previous work and propose a generalized algorithm that is applicable to UCAs with the sensor number being even as well as odd. Similar to our previous work, the generalized algorithm offers closed-form estimates of azimuth and elevation angles. Thus, it is computationally efficient. Numerical examples are provided to show the effectiveness and good performance of the proposed generalized approach.

Index Terms—Array signal processing, two-dimensional (2-D) angle estimation, uniform circular array (UCA).

I. INTRODUCTION

IT IS well known that uniform circular arrays (UCAs) are attractive configurations due to their ability of providing 360° azimuthal converge as well as the information of source elevation angles. Moreover, directional patterns synthesized with UCAs can be electronically rotated in the plane of the array without significant change of beam shape. Hence, UCAs have been widely applied to radar, sonar, and mobile communications, and much effort has been devoted to two-dimensional (2-D) direction-of-arrival (DOA) estimation with UCAs [1]–[4].

In [1], two algorithms, i.e., UCA-RB-MUSIC and UCA-ESPRIT, were developed for DOA estimation with UCAs. In these two methods, phase-mode excitation-based beamforming in conjunction with subspace techniques is employed to obtain high-resolution DOA estimates. In [2], the symmetric configuration of UCAs was utilized to form a centrosymmetric array and construct the covariance matrix in the form of the Hermitian persymmetric matrix. Then, the spatial averaging algorithm and spectrum search were applied to estimate the DOAs. In [3], the model-fitting approach was proposed to apply to a small number of uniformly spaced beams, and a method called global matched filter was developed for DOA estimation. Note that the eigenvalue decomposition

and/or 2-D spectrum search are required in the above-mentioned algorithms.

To avoid the high complexity of eigenvalue decomposition, especially the 2-D spectrum search, we proposed an algorithm for 2-D angle estimation of a single source with UCAs in [4]. This method is free of eigenvalue decomposition and 2-D spectrum search, and the azimuth and elevation angles can be obtained in closed form from a least-squared (LS) problem. As a result, it is computationally efficient. However, this method is limited to the specific case with the sensor number being even. To this end, a generalized algorithm that is applicable to cases with the sensor number being even as well as odd is proposed in this letter. Similar to our previous work in [4], neither eigenvalue decomposition nor 2-D spectrum search is needed in this generalized approach. Thus, the complexity is very low. Simulation results also demonstrate the good performance of the proposed method.

II. ANGLE ESTIMATION ALGORITHMS

Consider a UCA with M isotropic sensors impinged by a single narrowband source signal from far field. The array output observed at the t th snapshot consists of the outputs of the M sensors and can be written as

$$\begin{aligned} \mathbf{x}(t) &= [x_1(t), x_2(t), \dots, x_M(t)]^T \\ &= \mathbf{a}(\theta, \phi)s(t) + \mathbf{n}(t) \end{aligned} \quad (1)$$

where $\mathbf{a}(\theta, \phi)$ is the $M \times 1$ steering vector corresponding to the DOA of the source, and $\theta \in [-\pi, \pi)$ and $\phi \in [0, \pi/2)$ are the azimuth and elevation angles, respectively. $s(t)$ is the signal waveform and assumed to be zero-mean and power σ_s^2 . $\mathbf{n}(t)$ is the $M \times 1$ sensor noise vector that is commonly assumed to be an additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}$, where σ_n^2 and \mathbf{I} denote the noise variance and $M \times M$ identity matrix, respectively. According to the configuration of the UCA, the steering vector is given by

$$\mathbf{a}(\theta, \phi) = [a_1(\theta, \phi), a_2(\theta, \phi), \dots, a_M(\theta, \phi)]^T \quad (2)$$

where $a_m(\theta, \phi) = e^{j(2\pi r/\lambda) \cos(\theta - \varphi_m) \sin(\phi)}$, $\varphi_m = 2\pi(m - 1)/M$, and λ and r denote the wavelength and radius, respectively. The covariance matrix is then given by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \sigma_s^2 \mathbf{a}(\theta, \phi)\mathbf{a}^H(\theta, \phi) + \sigma_n^2 \mathbf{I}. \quad (3)$$

Letting $R_{p,q}$ denote the (p, q) th element of \mathbf{R} , we have

$$\begin{aligned} R_{p,q} &= \sigma_s^2 a_p(\theta, \phi)a_q^*(\theta, \phi) \\ &= \sigma_s^2 e^{j(2\pi r/\lambda)(\cos(\theta - \varphi_p) \sin(\phi) - \cos(\theta - \varphi_q) \sin(\phi))} \end{aligned} \quad (4)$$

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where $*$ denotes the complex conjugation operator. It can be seen that the phase angle of $R_{p,q}$ is

$$w_{p,q} = (2\pi r/\lambda)(\cos(\theta - \varphi_p) - \cos(\theta - \varphi_q)) \sin(\phi). \quad (5)$$

Since $|\cos(\theta - \varphi_p) - \cos(\theta - \varphi_q)| \leq 2$, to guarantee that there is no phase ambiguity in $w_{p,q}$, we assume that $r/\lambda \leq 1/4$, and hence we have $w_{p,q} \in [-\pi, \pi)$. It should be noted that if $r/\lambda > 1/4$, various techniques can be adopted for solving the problem of phase ambiguity [4]–[6]. The interested reader is referred to these works, and we shall not consider this problem for simplicity. In all, the problem we are interested in is to estimate the azimuth and elevation angles θ and ϕ from a set of array observations.

A. Previous Method [4]

Assume that the sensor number M is even, we have $\varphi_{m+M/2} = \varphi_m + \pi$, and $a_m(\theta, \phi) = a_{m+M/2}^*(\theta, \phi)$, for any $m = 1, 2, \dots, M/2$. Moreover, according to (4), we have

$$R_{m,m+M/2} = \sigma_s^2 e^{j(4\pi r/\lambda) \cos(\theta - \varphi_m) \sin(\phi)}. \quad (6)$$

As we assume that there is no phase ambiguity, the phase of $R_{m,m+M/2}$ is then given by

$$\begin{aligned} w_{m,m+M/2} &= (4\pi r/\lambda) \cos(\theta - \varphi_m) \sin(\phi) \\ &= \frac{4\pi r}{\lambda} [\cos(\varphi_m) \sin(\varphi_m)] \begin{bmatrix} \sin(\phi) \cos(\theta) \\ \sin(\phi) \sin(\theta) \end{bmatrix}. \end{aligned} \quad (7)$$

Define \mathbf{w} , \mathbf{A} , and \mathbf{b} as

$$\mathbf{w} = [w_{1,1+M/2} \quad w_{2,2+M/2} \quad \cdots \quad w_{M/2,M}]^T \quad (8a)$$

$$\mathbf{A} = \begin{bmatrix} \cos(\varphi_1) & \sin(\varphi_1) \\ \cos(\varphi_2) & \sin(\varphi_2) \\ \vdots & \vdots \\ \cos(\varphi_{M/2}) & \sin(\varphi_{M/2}) \end{bmatrix} \quad (8b)$$

$$\mathbf{b} = \frac{4\pi r}{\lambda} \begin{bmatrix} \sin(\phi) \cos(\theta) \\ \sin(\phi) \sin(\theta) \end{bmatrix}. \quad (8c)$$

One gets

$$\mathbf{w} = \mathbf{A}\mathbf{b}. \quad (9)$$

In practice, the phase vector \mathbf{w} can be estimated from the estimates of $R_{m,m+M/2}$, $m = 1, 2, \dots, M/2$, which are usually obtained as

$$\hat{R}_{m,m+M/2} = \frac{1}{K} \sum_{t=1}^K x_m(t)x_{m+M/2}^*(t) \quad (10)$$

where K is the number of snapshots. Hence, we have $\hat{w}_{m,m+M/2} = \text{angle}(\hat{R}_{m,m+M/2})$. According to (9), the LS estimate of \mathbf{b} is given by

$$\hat{\mathbf{b}} = [\hat{b}_1, \hat{b}_2]^T = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \hat{\mathbf{w}}. \quad (11)$$

As a result, the estimates of ϕ and θ are obtained as

$$\begin{aligned} \hat{\theta} &= \text{angle}(\hat{b}_1 + j\hat{b}_2) \\ \hat{\phi} &= \sin^{-1} \left(\frac{\lambda}{4\pi r} \sqrt{\hat{b}_1^2 + \hat{b}_2^2} \right). \end{aligned} \quad (12)$$

B. Proposed Generalized Method

It is worth noting that the above method is derived based on the assumption that the number of the sensors is even. In this section, we shall introduce a new method that is free of this assumption, and hence it is applicable to UCAs with the sensor number being either even or odd. Careful examination shows that the phase angle (5) can be reformulated as

$$w_{p,q} = \frac{2\pi r}{\lambda} \begin{bmatrix} \cos(\varphi_p) - \cos(\varphi_q) \\ \sin(\varphi_p) - \sin(\varphi_q) \end{bmatrix}^T \begin{bmatrix} \sin(\phi) \cos(\theta) \\ \sin(\phi) \sin(\theta) \end{bmatrix}. \quad (13)$$

Assume that $q - p = n$, and define the following terms

$$\bar{\mathbf{w}} = [w_{1,1+n} \quad w_{2,2+n} \quad \cdots \quad w_{M-n,M}]^T \quad (14a)$$

$$\bar{\mathbf{A}} = \begin{bmatrix} \cos(\varphi_1) - \cos(\varphi_{1+n}) & \sin(\varphi_1) - \sin(\varphi_{1+n}) \\ \cos(\varphi_2) - \cos(\varphi_{2+n}) & \sin(\varphi_2) - \sin(\varphi_{2+n}) \\ \vdots & \vdots \\ \cos(\varphi_{M-n}) - \cos(\varphi_M) & \sin(\varphi_{M-n}) - \sin(\varphi_M) \end{bmatrix} \quad (14b)$$

$$\bar{\mathbf{b}} = \frac{2\pi r}{\lambda} \begin{bmatrix} \sin(\phi) \cos(\theta) \\ \sin(\phi) \sin(\theta) \end{bmatrix}. \quad (14c)$$

Obviously, according to (13), we have

$$\bar{\mathbf{w}} = \bar{\mathbf{A}}\bar{\mathbf{b}} \quad (15)$$

which is similar to (9). Therefore, a similar idea in [4] can be adopted for angle estimation based on (15) and (14c). More precisely, given the estimate $\hat{R}_{i,i+n}$ and its corresponding angle estimate $\hat{w}_{i,i+n} = \text{angle}(\hat{R}_{i,i+n})$, $i = 1, 2, \dots, M - n$, we can get the estimate of $\bar{\mathbf{w}}$ as $\hat{\bar{\mathbf{w}}} = [\hat{w}_{1,1+n} \quad \hat{w}_{2,2+n} \quad \cdots \quad \hat{w}_{M-n,M}]^T$. Then, the LS estimate of $\bar{\mathbf{b}}$ is given by

$$\hat{\bar{\mathbf{b}}} = [\hat{b}_1, \hat{b}_2]^T = (\bar{\mathbf{A}}^T \bar{\mathbf{A}})^{-1} \bar{\mathbf{A}}^T \hat{\bar{\mathbf{w}}}. \quad (16)$$

From (14c), it is known that the estimates of ϕ and θ can be obtained as

$$\begin{aligned} \hat{\theta} &= \text{angle}(\hat{b}_1 + j\hat{b}_2) \\ \hat{\phi} &= \sin^{-1} \left(\frac{\lambda}{2\pi r} \sqrt{\hat{b}_1^2 + \hat{b}_2^2} \right). \end{aligned} \quad (17)$$

Interestingly, it can be noted that if the sensor number M is even and we choose $n = M/2$, then we have $\varphi_{i+n} = \varphi_i + \pi$, $\cos(\varphi_i) - \cos(\varphi_{i+n}) = 2\cos(\varphi_i)$, $\sin(\varphi_i) - \sin(\varphi_{i+n}) = 2\sin(\varphi_i)$. Therefore, (9) and (15) are equivalent, and the proposed method is reduced to our previous work [4]. Moreover, in the proposed generalized algorithm, n can be chosen to be $\lfloor M/2 \rfloor$ or values around $\lfloor M/2 \rfloor$, where $\lfloor \cdot \rfloor$ denotes the floor function. Finally, we can notice that, in the case of M sensors and K snapshots, the computational complexity of the generalized method is $nK + O(4M)$. However, in the conventional eigenstructure-based algorithms, e.g., UCA-ESPRIT technique, the complexity is $M^2K + O(M^3)$.

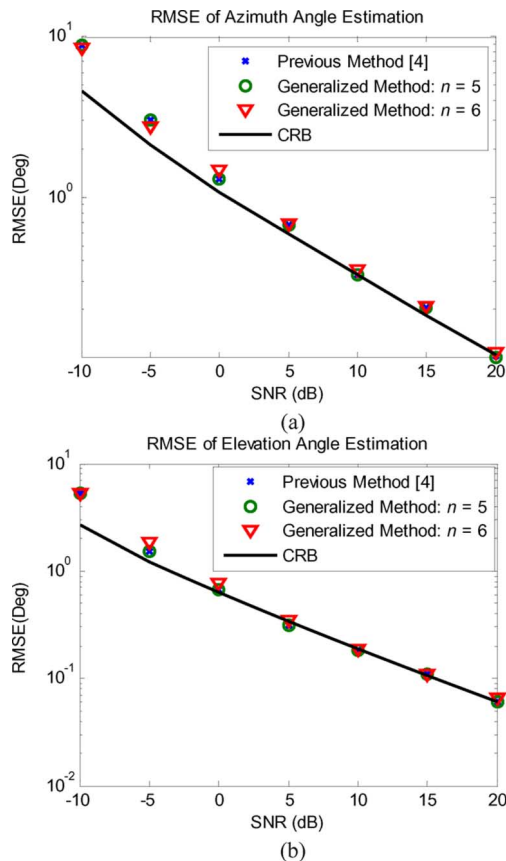


Fig. 1. RMSE of angle estimation when $M = 10$. (a) Azimuth angle. (b) Elevation angle.

III. SIMULATION RESULTS

First, a UCA with $M = 10$ sensors with $r/\lambda = 1/4$ is considered. The signal impinges on the array from angle pair $(\theta = 120^\circ, \phi = 30^\circ)$. The number of snapshots is $K = 500$. The root mean squared errors (RMSEs) of azimuth and elevation angle estimation are measured for performance evaluation. The number of independent runs for calculating the RMSE is 100. In all examples, n is chosen to be $\lfloor M/2 \rfloor$ or $\lfloor M/2 \rfloor + 1$ for the proposed generalized method. Since the sensor number is even in the first example, the results of the previous method [4] are shown for comparison. Moreover, the Cramér-Rao bounds (CRBs) for azimuth and elevation angle estimation are shown. From the results shown in Fig. 1, it can be noticed that the proposed method performs very well both in the cases of $n = 5$ and 6. Furthermore, if $n = 5$, the performance of the proposed method is identical to that of the previous method [4]. This is because the proposed method is reduced to the previous one when $n = M/2$. $M = 10$. Next, we change the sensor number to be $M = 11$. In this case, the previous method [4] is not applicable since the sensor number is odd. However, as can be seen from Fig. 2, the proposed generalized method offers satisfactory performance. It should be noted that the eigenstructure-based methods may also offer good performance at the expense of computational complexity.

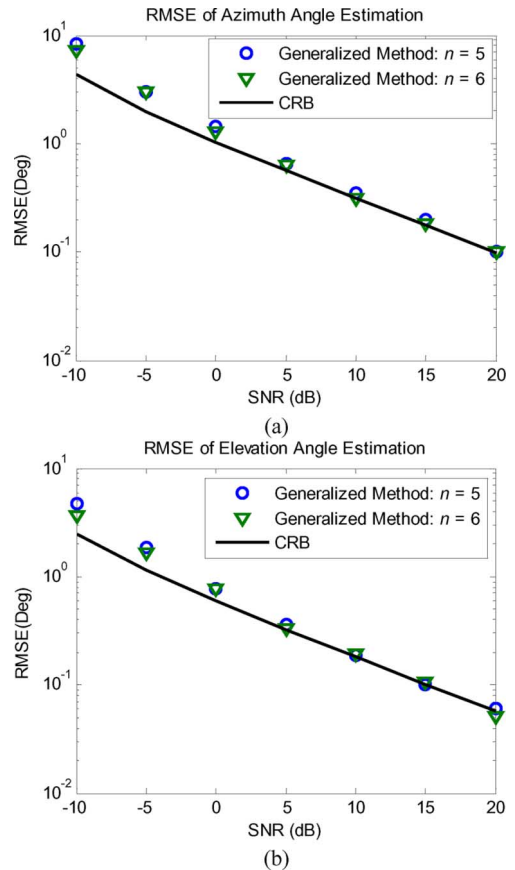


Fig. 2. RMSE of angle estimation when $M = 11$. (a) Azimuth angle. (b) Elevation angle.

IV. CONCLUSION

A generalized algorithm for 2-D angle estimation of a single source with UCAs is proposed in this letter. It is applicable to cases with the sensor number being both even and odd. Similar to our previous work, the generalized algorithm offers closed-form estimates of azimuth and elevation angles. Hence, it is computationally efficient. Moreover, it is shown that the proposed method can be reduced to the previous method when the sensor number is even. Simulation results show that the proposed method performs very well both in the case with the sensor number being even and odd.

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