



<b>Title</b>	<b>Robust Recursive Steering Vector Estimation and Adaptive Beamforming under Sensor Uncertainties</b>
<b>Author(s)</b>	<b>Liao, B; Chan, SC; Tsui, KM</b>
<b>Citation</b>	<b>IEEE Transactions on Aerospace and Electronic Systems, 2013, v. 49 n. 1, p. 489-501</b>
<b>Issued Date</b>	<b>2013</b>
<b>URL</b>	<b><a href="http://hdl.handle.net/10722/189086">http://hdl.handle.net/10722/189086</a></b>
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# Recursive Steering Vector Estimation and Adaptive Beamforming under Uncertainties

BIN LIAO, Student Member, IEEE  
SHING-CHOW CHAN, Member, IEEE  
KAI-MAN TSUI  
The University of Hong Kong

**The accurate determination of the steering vector of a sensor array that corresponds to a desired signal is often hindered by uncertainties due to array imperfections, such as the presence of a direction-of-arrival (DOA) estimation error, mutual coupling, array sensor gain/phase uncertainties, and sensor position perturbations. Consequently, the performance of conventional beamforming algorithms that use the nominal steering vector may be significantly degraded. A new method for recursively correcting possible deterministic errors in the estimated steering vector is proposed here. It employs the subspace principle and estimates the desired steering vector by using a convex optimization approach. We show that the solution can be obtained in closed form by using the Lagrange multiplier method. As the proposed method is based on an extended version of the conventional orthonormal PAST (OPAST) algorithm, it has low implementation complexity, and moving sources can be handled. In addition, a robust beamformer with a new error bound that uses the proposed steering vector estimate is derived by optimizing the worst case performance of the array after taking the uncertainties of the array covariance matrix into account. This gives a diagonally loaded Capon beamformer, where the loading level is related to the bound of the uncertainty in the array covariance matrix. Numerical results show that the proposed algorithm performs well, especially at high signal-to-noise ratio (SNR) and in the presence of deterministic sensor uncertainties.**

Manuscript received March 4, 2011; revised September 14, 2011; released for publication May 29, 2012.

IEEE Log No. T-AES/49/1/944366.

Refereeing of this contribution was handled by S. Watts.

Authors' address: Department of Electrical and Electronic Engineering, The University of Hong Kong, Room 722, Chow Yei Ching Building, Pokfulam Road, Hong Kong, E-mail: (scchan@eee.hku.hk).

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## I. INTRODUCTION

Adaptive beamforming using sensor arrays has been widely used in various fields such as radar, sonar, wireless communication, and microphone array processing [1]. Basically, adaptive beamforming aims to enhance the desired signal received while suppressing the undesirable noise and interference. Adaptive beamforming can be achieved by embedding known training signals in the source signal transmitted or, blindly, by utilizing the estimated steering vectors of the sources. The steering vector is the signal gain vector of the emitting source at a given coordinate with respect to the array. It is therefore a function of the source coordinate and the geometry of the array. For a known array geometry, one can estimate the steering vectors of the far-field sources and therefore determine their directions-of-arrival (DOAs). Based on the estimated steering vector of the desired signal, the interference can be efficiently suppressed by conventional adaptive beamforming algorithms, such as the Capon beamformer [2]. However, the steering vector in real systems may not be determined accurately from the array geometry alone due to the presence of uncertainties, such as sensor gain/phase uncertainties, position variations, and mutual coupling [3, 4]. Previous works show that these distortions may dramatically degrade the performance of the conventional beamforming methods. Therefore, robust beamforming methods to address these uncertainties have received great attention over the last decades [5–10]. For instance, additional linear constraints on the beam pattern have been proposed to better attenuate the interference and broaden the response around the nominal look direction [5, 6]. Unfortunately, these constraints may reduce the degree of freedom for suppressing undesired interference. This effect is especially significant for arrays with a small number of sensors. Another problem is that these constraints are not explicitly related to the uncertainty of the array steering vector [7, 8]. In [9] and [10] quadratic constraint on the Euclidean norm of the beamformer weight vector or the uncertainty of the array steering vector has also been exploited. This leads to another popular class of robust beamforming techniques called diagonal loading (DL). In these methods the array covariance matrix is loaded with an appropriate multiple, called the loading level, of the identity matrix in order to satisfy the imposed quadratic constraint. However, it is somewhat difficult to relate the loading level with the uncertainty bounds of the array steering vector, which may not be available in practice.

In this paper, instead of relying completely on the norm constraints in the beamforming algorithm, we focus on the problem of robust steering vector estimation for beamforming. A new algorithm for correcting possible deterministic errors in the steering

vector is proposed. Though the steering vector of the desired signal may be distorted by the imperfections of the array, it is shown that the proposed algorithm is capable of estimating the deterministic error in the steering vector that results from, say, array gain/phase uncertainties. In order to estimate this error, a convex problem is formulated based on the subspace principle. We show that the problem can be solved in closed form, and, hence, an explicit expression of the robust steering vector can be derived. A sensitivity analysis of the derived robust beamformer to errors in steering vector is also performed. It is found that the variance of the beamformer weight vector is extremely sensitive to the eigenvalues of the array covariance matrix for a given error variance of the steering vector. Thus, an approach to determine the loading level of the robust Capon beamformer given the proposed steering vector estimation and perturbation bound of the array covariance matrix is proposed. The resultant robust beamformer is obtained by minimizing its worst case performance. The proposed algorithm has an arithmetic complexity of  $O(N^3)$ , which is comparable to the conventional diagonally loaded Capon beamformer.

Another recent approach in [11] is to estimate the mismatch using sequential quadratic programming. The proposed method differs from this approach in that it focuses on adaptive and recursive implementations and provides an analytic solution of the steering vector error with the help of the subspace principle. Moreover, it is able to handle dynamic cases with moving sources because it is developed based on an extended orthonormal PAST (OPAST) algorithm [12–14]. Alternatively, other efficient algorithms such as [22]–[25] may also be used. However, we only focus on the OPAST algorithms due to page limitation. Finally, computer simulation experiments are conducted to demonstrate the excellent performance and effectiveness of the proposed method over the conventional methods, especially at high signal-to-noise ratio (SNR) and in the presence of deterministic gain/phase uncertainties.

The paper is organized as follows. The problem formulation and standard Capon beamforming are briefly introduced in Section II. The proposed robust steering vector estimation for beamforming is given in Section III. In Section IV numerical examples are conducted to demonstrate the excellent performance and effectiveness of the proposed methods, and finally, Section V concludes the paper.

## II. PROBLEM FORMULATION

Consider an antenna array with  $N$  sensors impinged by  $K + 1$  narrowband uncorrelated signals, which include one desired signal and  $K$  interferences. Here we assume that  $K + 1 < N$ . The  $N \times 1$  array output  $\mathbf{x}(t)$  observed at the  $t$ th snapshot

consists of the outputs of the  $N$  sensors, i.e.,  $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T$  with  $[\cdot]^T$  denoting the matrix transpose. More precisely the array output can be written as

$$\mathbf{x}(t) = \mathbf{s}(t) + \mathbf{i}(t) + \mathbf{n}(t) \quad (1)$$

where  $\mathbf{s}(t) = \mathbf{a}(\theta_0)s_0(t)$ ,  $\mathbf{i}(t) = \sum_{k=1}^K \mathbf{a}(\theta_k)s_k(t)$ ,  $\mathbf{n}(t)$  are the desired signal, interference, and noise components, respectively. Moreover,  $\mathbf{a}(\theta_0)$  and  $\{\mathbf{a}(\theta_k)\}_{k=1}^K$  are, respectively, the steering vectors of the desired signal and interferences. For an ideal uniform linear array,  $\mathbf{a}(\theta) = [1, e^{j2\pi\lambda^{-1}d\sin\theta}, \dots, e^{j2\pi\lambda^{-1}(N-1)d\sin\theta}]^T$  with  $\lambda$ ,  $d$ , and  $\theta$  denoting the carrier wavelength, inter-sensor spacing, and DOA, respectively. In this paper the noise is considered to be an additive white Gaussian noise (AWGN) with zero mean and covariance matrix  $\sigma^2\mathbf{I}$ , where  $\mathbf{I}$  is an identity matrix. The sensor outputs are linearly combined by a beamformer to form the desired output:

$$y(t) = \mathbf{w}^H \mathbf{x}(t) \quad (2)$$

where  $[\cdot]^H$  denotes Hermitian transposition and  $\mathbf{w}$  is the  $N \times 1$  complex weight vector of the beamformer. The objective is to maximize the signal-to-interference-plus-noise ratio (SINR)

$$\text{SINR} = \frac{\sigma_0^2 |\mathbf{w}^H \mathbf{a}_0|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \quad (3)$$

where  $\mathbf{a}_0$  denotes  $\mathbf{a}(\theta_0)$  for simplicity,  $\sigma_0^2 = E[\mathbf{s}(t)\mathbf{s}^H(t)]$  is the power of the desired signal,  $\mathbf{R}_{i+n} = E[(\mathbf{i}(t) + \mathbf{n}(t))(\mathbf{i}(t) + \mathbf{n}(t))^H]$  is the covariance matrix of interference-plus-noise, and  $E[\cdot]$  denotes the statistical expectation. Alternatively, the optimal weight vector is obtained by solving the following optimization problem [2]

$$\begin{aligned} \min \quad & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{a}_0 = 1 \end{aligned} \quad (4)$$

where  $\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)]$  is the covariance matrix of the array output. It is known that the solution of (4) is given by

$$\mathbf{w}_{\text{MVDR}} = \frac{\mathbf{R}^{-1} \mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}^{-1} \mathbf{a}_0} \quad (5)$$

which is called minimum variance distortionless response (MVDR) beamformer or the Capon beamformer. It should be noted that this beamformer is obtained based on the assumption that the array response or the steering vector of the desired signal, i.e.,  $\mathbf{a}_0$ , is known accurately. However, as mentioned earlier,  $\mathbf{a}_0$  is subject to uncertainties due to various imperfections of the array and DOA estimation error. Hence, the true steering vector  $\mathbf{a}$  of the desired signal should be written as

$$\mathbf{a} = \mathbf{a}_0 + \Delta \quad (6)$$

where  $\Delta$  denotes the uncertainty in  $\mathbf{a}_0$ . Once  $\Delta$  is known, one just needs to replace  $\mathbf{a}_0$  in (5) by  $\mathbf{a}$  to get the optimal MVDR beamformer. In practice,

the uncertainty  $\Delta$  is generally unknown to users, and the performance of the beamformer (5) will degrade considerably when  $\Delta$  is simply ignored. Hence, a number of robust methods have been proposed to take this uncertainty into account. For instance, by assuming that the true steering vector lies within an ellipsoid centered at  $\mathbf{a}_0$ , the robust Capon beamforming (RCB) [7] or the robust minimum variance beamforming (RMVB) [8] algorithm can be employed to solve for  $\mathbf{a}$ . However, both of these methods require a priori knowledge of the ellipsoid, such as its norm bound.

In general the uncertainty  $\Delta$  in (6) consists of two error components, namely 1) the deterministic error which changes only slowly with time as a result of sensor gain/phase uncertainties and location errors, etc., and 2) the stochastic error which results from other stochastic variations, such as sensor noise on the initial DOA estimation. In the following section a new approach is introduced to estimate a correction to  $\mathbf{a}_0$  by taking advantage of the subspace principle.

### III. ROBUST STEERING VECTOR ESTIMATION FOR BEAMFORMING

#### A. Robust Steering Vector Estimation

In practice the nominal steering vector  $\mathbf{a}_0$  is usually obtained by a DOA estimation algorithm given the array geometry. Due to uncertainties of the array, such as aforementioned sensor locations or gain/phase uncertainties, the steering vector computed from the given array geometry may deviate from the true one. Therefore, the nominal steering vector  $\mathbf{a}_0$  may be subject to a deterministic error  $\Delta$  from the true steering vector  $\mathbf{a}$ .

Since the subspace principle is an effective approach in high-resolution DOA estimation, we propose a new method to determine this deterministic correction  $\Delta$  by using the subspace approach. In general, the uncertainty  $\Delta$  should lie inside a hypersphere with radius  $\varepsilon$

$$\|\mathbf{a} - \mathbf{a}_0\|^2 = \|\Delta\|^2 \leq \varepsilon. \quad (7)$$

Conventionally, the error bound parameter  $\varepsilon$  is assumed to be known, e.g., [7]. Since this knowledge may not be accurately available in practice, we propose to estimate the uncertainty  $\Delta$  directly without the prior knowledge of  $\varepsilon$ .

Based on the subspace principle, we know that the true steering vector  $\mathbf{a}$  is orthogonal to the  $N \times (N - K - 1)$  noise subspace  $\mathbf{U}_n$ , i.e.,

$$\mathbf{U}_n^H \mathbf{a} = \mathbf{U}_n^H (\mathbf{a}_0 + \Delta) = 0. \quad (8)$$

Generally, it is assumed that  $K + 1 < N$  and the ambient noise is AWGN so that  $\mathbf{U}_n$  can be obtained from the eigenvalue decomposition (EVD) of the covariance matrix  $\mathbf{R}$  and so that  $\mathbf{U}_n$  consists of the  $N - K - 1$  eigenvectors corresponding to the  $N - K$

$-1$  smallest eigenvalues. However, the computational complexity of EVD may be prohibitive for some real-time applications. Therefore, the subspace tracking algorithm is employed in this paper to reduce the arithmetic complexity and to handle scenarios involving moving sources.

Though the true steering vector is unknown, it usually lies within a small region around the nominal steering vector. Therefore, it is natural to choose the smallest  $\Delta$  such that (8) is satisfied. On the other hand, since the noise subspace is estimated, say by subspace tracking algorithms, slight tracking errors are inevitable, and it will depend on the speed of the moving sources and other stochastic errors, such as sensor noises. To address this issue, it is assumed that true subspace is given by  $\mathbf{U}_n = \hat{\mathbf{U}}_n + \delta\mathbf{U}_n$ , where  $\hat{\mathbf{U}}_n$  is the estimated noise subspace and  $\delta\mathbf{U}_n$  is the estimation error due to sensor noise or other stochastic errors. Consequently, (8) becomes

$$(\hat{\mathbf{U}}_n + \delta\mathbf{U}_n)^H (\mathbf{a}_0 + \Delta) = 0. \quad (9)$$

It should be noted that  $\Delta$  represents the deterministic part of the errors which arise from, say gain/phase mismatch and location errors, which are assumed to be invariant. On the other hand, the estimation error  $\delta\mathbf{U}_n$ , which may arise from sensor noise, etc., is a random matrix. Though its exact value is unknown, for well-designed systems, it is reasonable to assume that it is zero mean. Moreover, we show that the determination of  $\Delta$  is benefited from the knowledge of its covariance

$$\mathbf{C}_{\delta U} = E[\delta\mathbf{U}_n \delta\mathbf{U}_n^H]. \quad (10)$$

Furthermore, since  $\mathbf{U}_n$  is estimated by the subspace tracking algorithm, the stochastic error appears as an instantaneous variation of the subspace, and hence, its covariance matrix can be approximately estimated from the subspace tracking algorithm. This is explained in detail in Section III-B when the tracking of the subspace is investigated. Therefore, the proposed method is particularly useful for the DOA tracking scenario, where the subspace can be continuously tracked. This is, however, different from conventional robust beamforming methods, which usually do not take subspace tracking into account.

Next, we focus on the determination of  $\Delta$ . First of all, we rearrange (9) as

$$\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta) = -\delta\mathbf{U}_n^H (\mathbf{a}_0 + \Delta). \quad (11)$$

By taking the Euclidean norm on both sides of (11), one gets

$$\|\hat{\mathbf{U}}_n^H (\mathbf{a}_0 + \Delta)\|^2 = (\mathbf{a}_0 + \Delta)^H (\delta\mathbf{U}_n \delta\mathbf{U}_n^H) (\mathbf{a}_0 + \Delta). \quad (12)$$

Moreover, by taking expectation over  $\delta\mathbf{U}_n$  on both sides of (12), we have

$$\|\hat{\mathbf{U}}_n^H(\mathbf{a}_0 + \Delta)\|^2 = (\mathbf{a}_0 + \Delta)^H \mathbf{C}_{\delta U} (\mathbf{a}_0 + \Delta). \quad (13)$$

Since both  $\mathbf{C}_{\delta U}$  and  $\Delta$  are typically small, the right hand side can be approximated by omitting the terms that involve the product of  $\mathbf{C}_{\delta U}$  and  $\Delta$  as follows

$$(\mathbf{a}_0 + \Delta)^H \mathbf{C}_{\delta U} (\mathbf{a}_0 + \Delta) \approx \mathbf{a}_0^H \mathbf{C}_{\delta U} \mathbf{a}_0 \triangleq \zeta. \quad (14)$$

As a result, the linear equality (8) is modified to  $\|\hat{\mathbf{U}}_n^H(\mathbf{a}_0 + \Delta)\|^2 \approx \zeta$ , which is then relaxed to the following quadratic inequality

$$\|\hat{\mathbf{U}}_n^H(\mathbf{a}_0 + \Delta)\|^2 \leq \zeta \quad (15)$$

since typically only the bounds on the uncertainties are required. Consequently, the problem at hand is to minimize the Euclidean norm of  $\Delta$  while satisfying (15):

$$\begin{aligned} \min \|\Delta\|^2 \\ \text{s.t. } \|\hat{\mathbf{U}}_n^H(\mathbf{a}_0 + \Delta)\|^2 \leq \zeta. \end{aligned} \quad (16)$$

It is noted that (16) is a convex quadratically constrained quadratic programming problem, and hence, an optimal solution does exist. We now employ the Lagrange multiplier method to solve for this solution. The Lagrangian  $L$  associated with (16) is given by

$$L(\Delta, \lambda) = \|\Delta\|^2 + \lambda(\|\hat{\mathbf{U}}_n^H(\mathbf{a}_0 + \Delta)\|^2 - \zeta) \quad (17)$$

where  $\lambda > 0$  is the Lagrange multiplier and we have excluded the trivial solution  $\Delta = 0$ . By setting the partial derivative of (17) with respect to  $\Delta$  to zero, one gets the first-order necessary condition for optimality as follows

$$\Delta + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \Delta + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 = 0.$$

On the other hand, since the problem is convex and the objective function is differentiable, any stationary point is also the global solution. Hence, the optimal solution  $\hat{\Delta}$  to (16) is given by

$$\hat{\Delta} = -\lambda(\mathbf{I} + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H)^{-1} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0. \quad (18)$$

A common way to determine  $\lambda$  is to substitute (18) back to the equation  $\|\hat{\mathbf{U}}_n^H(\mathbf{a}_0 + \Delta)\|^2 = \zeta$ , and it, in general, gives rise to a nonlinear equation in  $\lambda$ .

Fortunately, we show below that a closed-form solution of  $\lambda$  can be obtained. First, we assume that the noise subspace  $\hat{\mathbf{U}}_n$  is orthogonally obtained, i.e.,  $\hat{\mathbf{U}}_n$  satisfies

$$\hat{\mathbf{U}}_n^H \hat{\mathbf{U}}_n = \mathbf{I}.$$

Then, the term  $(\mathbf{I} + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H)^{-1}$  on the right side of (18) can be simplified to

$$\begin{aligned} (\mathbf{I} + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H)^{-1} &= \mathbf{I} - \lambda \hat{\mathbf{U}}_n (\mathbf{I} + \lambda \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H)^{-1} \hat{\mathbf{U}}_n^H \\ &= \mathbf{I} - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \end{aligned} \quad (19)$$

with the help of the matrix inverse lemma  $(\mathbf{I} + \mathbf{AB})^{-1} = \mathbf{I} - \mathbf{A}(\mathbf{I} + \mathbf{BA})^{-1}\mathbf{B}$ . Substituting (19) into (18) one gets

$$\begin{aligned} \hat{\Delta} &= -\lambda \left( \mathbf{I} - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \right) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 \\ &= -\lambda \left( \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \right) \mathbf{a}_0 \\ &= -\frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0. \end{aligned} \quad (20)$$

Then, substituting (20) into the constraint of the problem in (16), one gets the following equation on  $\lambda$

$$\begin{aligned} \left\| \hat{\mathbf{U}}_n^H \left( \mathbf{a}_0 - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 \right) \right\|^2 \\ = \left\| \hat{\mathbf{U}}_n^H \mathbf{a}_0 - \frac{\lambda}{1 + \lambda} \hat{\mathbf{U}}_n^H \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0 \right\|^2 \\ = \left\| \frac{1}{1 + \lambda} \hat{\mathbf{U}}_n^H \mathbf{a}_0 \right\|^2 = \zeta. \end{aligned} \quad (21)$$

Consequently,  $\lambda$  is given by

$$\lambda = \alpha^{-1} - 1 \quad (22)$$

where  $\alpha$  is defined as

$$\alpha = (\zeta^{-1} \mathbf{a}_0^H \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0)^{-1/2}. \quad (23)$$

Finally, by substituting (23) into (21), we obtain the following closed-form solution to the problem in (16) as

$$\hat{\Delta} = (\alpha - 1) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0. \quad (24)$$

From (10) and (14) it can be seen that when  $\delta\mathbf{U}_n \rightarrow 0$ , we have  $\mathbf{C}_{\delta U} \rightarrow 0$  and  $\zeta \rightarrow 0$ . Consequently, the value of  $\alpha$  is approximately zero, and the solution in (24) reduces to  $\hat{\Delta} = -\hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{a}_0$ . Careful examination shows that this is the solution of the conventional projection approach [20, 21]. Therefore, the proposed approach offers an alternative interpretation to the conventional projection approach and extends it further to include possible uncertainties that arise from tracking or other stochastic errors. One of its main advantages is that a simple analytical solution is available that greatly simplifies the implementation. We now discuss the recursive tracking of the subspace and the determination of  $\mathbf{C}_{\delta U}$ .

## B. Noise Subspace Tracking and Robust Beamforming

As mentioned previously the noise subspace  $\mathbf{U}_n$  can be estimated by EVD of the array covariance

matrix  $\mathbf{R}$ . However, the complexity of EVD may be prohibitive in some practical implementations, especially for antenna arrays with a large number of elements. More importantly, EVD may not be feasible for dynamic environments where moving sources are involved. A number of subspace tracking algorithms have been proposed to deal with this problem in the last decades [12–14].

Though most of these algorithms focus on signal subspace tracking, they can also be extended to noise subspace tracking since the noise and signal subspaces are related by

$$\tilde{\mathbf{U}}_n(t) = \mathbf{I} - \hat{\mathbf{U}}_s(t)[\hat{\mathbf{U}}_s^H(t)\hat{\mathbf{U}}_s(t)]^{-1}\hat{\mathbf{U}}_s^H(t) \quad (25)$$

where  $\hat{\mathbf{U}}_s(t)$  is the estimated signal subspace. It can be seen that the subspaces are now functions of the time index  $t$  since the subspaces are tracked continuously. Despite the simple relationship the complexity of (25) is still  $O(N^3)$  due to the matrix inversion operation. Fortunately, with the use of the OPAST algorithm [14], the signal subspace  $\hat{\mathbf{U}}_s(t)$  when estimated is orthogonal, and hence,  $\hat{\mathbf{U}}_s^H(t)\hat{\mathbf{U}}_s(t) = \mathbf{I}$ . Consequently, (25) can be reduced to

$$\tilde{\mathbf{U}}_n(t) = \mathbf{I} - \hat{\mathbf{U}}_s(t)\hat{\mathbf{U}}_s^H(t) \quad (26)$$

which provides a more efficient mean for computing the noise subspace. Furthermore, it is known that  $\hat{\mathbf{U}}_n(t)\hat{\mathbf{U}}_n^H(t) + \hat{\mathbf{U}}_s(t)\hat{\mathbf{U}}_s^H(t) = \mathbf{I}$ . This implies that  $\tilde{\mathbf{U}}_n(t) = \hat{\mathbf{U}}_n(t)\hat{\mathbf{U}}_n^H(t)$ . Therefore, if the noise subspace is obtained as (26), the uncertainty of the steering vector at time  $t$  can be estimated according to (24), as follows:

$$\hat{\Delta}(t) = (\alpha(t) - 1)\tilde{\mathbf{U}}_n(t)\mathbf{a}_0(t) \quad (27)$$

where  $\alpha(t) = (\zeta^{-1}(t)\mathbf{a}_0^H(t)\tilde{\mathbf{U}}_n(t)\mathbf{a}_0(t))^{-1/2}$  and  $\zeta(t) = \mathbf{a}_0^H(t)\mathbf{C}_{\delta U}(t)\mathbf{a}_0(t)$ .

We now extend the OPAST algorithm to recursively track the noise subspace  $\tilde{\mathbf{U}}_n(t)$  and the covariance  $\mathbf{C}_{\delta U}$  required by our robust steering vector estimation algorithm. According to the extended OPAST algorithm shown in Table I, the orthogonal signal subspace  $\hat{\mathbf{U}}_s(t)$  is recursively updated as

$$\hat{\mathbf{U}}_s(t) = \hat{\mathbf{U}}_s(t-1) + \tilde{\mathbf{e}}(t)\mathbf{g}^H(t) \quad (28)$$

where  $\tilde{\mathbf{e}}(t)$  and  $\mathbf{g}(t)$  are defined in Table I. Substituting  $\hat{\mathbf{U}}_s(t)$  into (26) one gets

$$\tilde{\mathbf{U}}_n(t) = \tilde{\mathbf{U}}_n(t-1) + \delta\tilde{\mathbf{U}}_n(t) \quad (29)$$

where

$$\begin{aligned} \delta\tilde{\mathbf{U}}_n(t) &= -\hat{\mathbf{U}}_s(t-1)\mathbf{g}(t)\tilde{\mathbf{e}}^H(t) - \tilde{\mathbf{e}}(t)\mathbf{g}(t)\hat{\mathbf{U}}_s^H(t) \\ &\quad - \|\mathbf{g}(t)\|^2\tilde{\mathbf{e}}(t)\tilde{\mathbf{e}}^H(t). \end{aligned} \quad (30)$$

It can be seen that the noise subspace can now be estimated recursively from the signal subspace with low arithmetic complexity. We also notice that (30)

TABLE I  
The Extended OPAST Algorithm for Noise Subspace and its Covariance Tracking

Initialize $\mathbf{P}(0)$ , $\mathbf{U}_s(0)$ , $\mathbf{U}_n(0)$ and $\mathbf{C}_{\delta U}(t)$
For $t = 1, 2, \dots$ , do
$\mathbf{y}(t) = \hat{\mathbf{U}}_s^H(t-1)\mathbf{x}(t)$
$\mathbf{h}(t) = \mathbf{P}(t-1)\mathbf{y}(t)$
$\mathbf{g}(t) = \mathbf{h}(t)/[\beta + \mathbf{y}^H(t)\mathbf{h}(t)]$
$\mathbf{P}(t) = \beta^{-1}Tr\{[\mathbf{P}(t-1) - \mathbf{g}(t)\mathbf{h}^H(t)]\}$
$\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{U}}_s(t-1)\mathbf{y}(t)$
$\tau(t) = \ \mathbf{g}(t)\ ^{-2}((1 + \ \mathbf{e}(t)\ ^2\ \mathbf{g}(t)\ ^2)^{-2} - 1)$
$\tilde{\mathbf{e}}(t) = \tau(t)\tilde{\mathbf{U}}_s(t-1)\mathbf{g}(t) + (1 + \tau(t)\ \mathbf{g}(t)\ ^2)\mathbf{e}(t)$
$\hat{\mathbf{U}}_s(t) = \hat{\mathbf{U}}_s(t-1) + \tilde{\mathbf{e}}(t)\mathbf{g}^H(t)$
$\delta\tilde{\mathbf{U}}_n(t) = -\hat{\mathbf{U}}_s(t-1)\mathbf{g}(t)\tilde{\mathbf{e}}^H(t) - \mathbf{e}(t)\mathbf{g}(t)\hat{\mathbf{U}}_s^H(t) - \ \mathbf{g}(t)\ ^2\tilde{\mathbf{e}}(t)\tilde{\mathbf{e}}^H(t)$
$\tilde{\mathbf{U}}_n(t) = \tilde{\mathbf{U}}_n(t-1) + \delta\tilde{\mathbf{U}}_n(t)$
$\mathbf{C}_{\delta U}(t) = \beta\mathbf{C}_{\delta U}(t-1) + (1 - \beta)\delta\tilde{\mathbf{U}}_n(t)\delta\tilde{\mathbf{U}}_n^H(t)$
end $t$

provides us with the instantaneous perturbation of the noise subspace from which its covariance can be efficiently estimated. More precisely we propose to estimate the covariance of the noise subspace, i.e.,  $\mathbf{C}_{\delta U}(t)$ , recursively as follows

$$\mathbf{C}_{\delta U}(t) = \beta\mathbf{C}_{\delta U}(t-1) + (1 - \beta)\delta\tilde{\mathbf{U}}_n(t)\delta\tilde{\mathbf{U}}_n^H(t) \quad (31)$$

where  $0 < \beta \leq 1$  is a forgetting factor. Once the noise subspace  $\tilde{\mathbf{U}}_n(t)$  and the covariance  $\mathbf{C}_{\delta U}(t)$  are obtained, the value of the bound  $\zeta(t)$  and the uncertainty of steering vector  $\hat{\Delta}(t)$  can be estimated according to (14) and (27), respectively. Accordingly, the steering vector can be updated as  $\hat{\mathbf{a}}(t) = \mathbf{a}_0(t) + \hat{\Delta}(t)$ . The conventional MVDR beamformer can thus be invoked to obtain a new robust beamformer by replacing  $\mathbf{a}_0(t)$  in (5) by  $\hat{\mathbf{a}}(t)$ . Hence, the following robust MVDR (R-MVDR) beamformer is proposed as

$$\mathbf{w}_{\text{R-MVDR}}(t) = \frac{\mathbf{R}^{-1}(t)(\mathbf{a}_0(t) + \hat{\Delta}(t))}{(\mathbf{a}_0(t) + \hat{\Delta}(t))\mathbf{R}^{-1}(t)(\mathbf{a}_0(t) + \hat{\Delta}(t))}. \quad (32)$$

It should be noted that for online implementations and moving sources, the covariance matrix  $\mathbf{R}(t)$  should also be recursively estimated, say by the popular formula

$$\mathbf{R}(t) = \beta\mathbf{R}(t-1) + (1 - \beta)\mathbf{x}(t)\mathbf{x}^H(t). \quad (33)$$

### C. Sensitivity Analysis and Modification of the R-MVDR Beamformer

So far, it has been shown that a new R-MVDR beamformer can be obtained by exploiting the OPAST algorithm. In this section, we briefly analyze its sensitivity to the error in the steering vector, and we show that the proposed beamformer can be extended further to take the error of the array covariance matrix into account. To begin with, we assume that  $\mathbf{R}(t)$  is nonsingular and denote its EVD by  $\mathbf{U}(t)\Lambda(t)\mathbf{U}^H(t)$ , where  $\Lambda(t)$  and  $\mathbf{U}(t)$  compose the eigenvalues and

eigenvectors, respectively. Moreover,  $\mathbf{U}(t)$  is an orthogonal matrix that satisfies  $\mathbf{U}(t)\mathbf{U}^H(t) = \mathbf{I}$ . Here, it is considered that the stochastic error in  $\mathbf{a}(t)$  is  $\delta\mathbf{a}(t)$  with zero mean, then (32) can be rewritten as

$$\begin{aligned}\tilde{\mathbf{w}}_{\mathbf{R}\text{-MVDR}}(t) &= \frac{\mathbf{U}(t)\Lambda^{-1}(t)\mathbf{U}^H(t)(\mathbf{a}_0(t) + \Delta(t))}{(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))^H\mathbf{U}(t)\Lambda^{-1}(t)\mathbf{U}^H(t)(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))} \\ &= \frac{\alpha_0(t)\mathbf{w}_{\mathbf{R}\text{-MVDR}}(t) + \mathbf{U}(t)\Lambda^{-1}(t)\mathbf{U}^H(t)\delta\mathbf{a}(t)}{(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))^H\mathbf{U}(t)\Lambda^{-1}(t)\mathbf{U}^H(t)(\hat{\mathbf{a}}_0(t) + \delta\mathbf{a}(t))}\end{aligned}\quad (34)$$

where  $\hat{\mathbf{a}}_0(t) = \mathbf{a}_0(t) + \hat{\Delta}(t)$  and  $\alpha_0(t) = \hat{\mathbf{a}}_0^H(t)\mathbf{R}^{-1}(t)\hat{\mathbf{a}}_0(t)$ . Define

$$\tilde{\mathbf{W}}_{\mathbf{R}\text{-MVDR}}(t) = \mathbf{U}^H(t)\tilde{\mathbf{w}}_{\mathbf{R}\text{-MVDR}}(t) \quad (35a)$$

$$\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t) = \mathbf{U}^H(t)\mathbf{w}_{\mathbf{R}\text{-MVDR}}(t) \quad (35b)$$

$$\hat{\mathbf{A}}_0(t) = \mathbf{U}^H(t)\hat{\mathbf{a}}_0(t) \quad (35c)$$

$$\delta\mathbf{A}(t) = \mathbf{U}^H(t)\delta\mathbf{a}(t). \quad (35d)$$

We have

$$\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t) = \frac{\alpha_0(t)\mathbf{W}_0(t) + \Lambda^{-1}(t)\delta\mathbf{A}(t)}{(\mathbf{A}_0(t) + \delta\mathbf{A}(t))^H\Lambda^{-1}(t)(\mathbf{A}_0(t) + \delta\mathbf{A}(t))}. \quad (36)$$

Hence, the mean of the weight vector is approximately given by

$$\begin{aligned}E[\tilde{\mathbf{W}}_{\mathbf{R}\text{-MVDR}}(t)] &\approx \frac{\alpha_0(t)\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t)}{E[(\hat{\mathbf{A}}_0(t) + \delta\mathbf{A}(t))^H\Lambda^{-1}(t)(\hat{\mathbf{A}}_0(t) + \delta\mathbf{A}(t))]} \\ &= \frac{\alpha_0(t)\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t)}{\hat{\mathbf{A}}_0^H(t)\Lambda^{-1}(t)\hat{\mathbf{A}}_0(t) + \text{tr}(\Lambda^{-1}(t)\mathbf{C}_{\delta\mathbf{A}})}\end{aligned}\quad (37)$$

where  $\mathbf{C}_{\delta\mathbf{A}}(t) = E[\delta\mathbf{A}(t)\delta\mathbf{A}^H(t)]$  and we have truncated the higher order terms in the expansion of the delta method [27], so that  $E[\tilde{\mathbf{W}}_{\mathbf{R}\text{-MVDR}}(t)]$  can be approximated by evaluating the expectation of its numerator and denominator separately. Furthermore, if  $\mathbf{C}_{\delta\mathbf{A}}(t)$  is small so that the second term in the denominator is small compared with the first term, which is the usual case, then

$$E[\tilde{\mathbf{W}}_{\mathbf{R}\text{-MVDR}}(t)] \approx \frac{\alpha_0(t)\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t)}{\hat{\mathbf{A}}_0^H(t)\Lambda^{-1}(t)\hat{\mathbf{A}}_0(t)} = \mathbf{W}_{\mathbf{R}\text{-MVDR}}(t) \quad (38)$$

since  $\alpha_0(t) = \hat{\mathbf{a}}_0^H(t)\mathbf{R}^{-1}(t)\hat{\mathbf{a}}_0(t) = \hat{\mathbf{A}}_0^H(t)\Lambda^{-1}(t)\hat{\mathbf{A}}_0(t)$ . The perturbation of  $\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t)$  due to  $\delta\mathbf{A}(t)$  is thus

$$\delta\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t) \approx \alpha_0^{-1}(t)\Lambda^{-1}(t)\delta\mathbf{A}(t). \quad (39)$$

Let  $\mathbf{C}_{\delta\mathbf{W}}(t) = E[\delta\mathbf{W}_{\mathbf{R}\text{-MVDR}}(t)\delta\mathbf{W}_{\mathbf{R}\text{-MVDR}}^H(t)]$ , we have

$$\begin{aligned}\text{tr}(\mathbf{C}_{\delta\mathbf{W}}(t)) &\approx \text{tr}(E[\alpha_0^{-2}(t)\Lambda^{-1}(t)\delta\mathbf{A}(t)\delta\mathbf{A}^H(t)\Lambda^{-1}(t)]) \\ &= \text{tr}(\alpha_0^{-2}(t)\Lambda^{-1}(t)\mathbf{C}_{\delta\mathbf{A}}\Lambda^{-1}(t)) \\ &= \alpha_0^{-2}(t)\sum_{i=1}^N\lambda_i^{-2}(t)\mathbf{C}_{\delta\mathbf{A},i}(t)\end{aligned}\quad (40)$$

where  $\mathbf{C}_{\delta\mathbf{A},i}(t)$  and  $\lambda_i(t)$  are the  $i$ th diagonal entry of  $\mathbf{C}_{\delta\mathbf{A}}(t)$  and  $\Lambda^{-1}(t)$ , respectively. It is noted that  $\text{tr}(\mathbf{C}_{\delta\mathbf{W}}(t))$  increases with the variance  $\mathbf{C}_{\delta\mathbf{A}}(t)$ . More importantly it can be seen that  $\mathbf{w}_{\mathbf{R}\text{-MVDR}}(t)$  is extremely sensitive to eigenvalues of  $\mathbf{R}(t)$ , especially when  $\mathbf{R}(t)$  is ill-conditioned. Hence, even if  $\mathbf{C}_{\delta\mathbf{A}}(t)$  is not very large, the perturbation in  $\mathbf{R}(t)$  results in significant variation of  $\mathbf{w}_{\mathbf{R}\text{-MVDR}}(t)$ . In this case one cannot obtain a proper beamformer even though the true steering vector is known.

An effective method is to employ robust beamforming approaches. For instance, the DL method, which is closely related to ridge regression in reducing the variance of the estimator while scarifying slightly the bias, is commonly used. The regularized solution is given by adding a small diagonal matrix to  $\mathbf{R}(t)$ :

$$\bar{\mathbf{w}}_{\mathbf{R}\text{-MVDR}}(t) = \frac{(\mathbf{R}(t) + \mu\mathbf{I})^{-1}\hat{\mathbf{a}}(t)}{\hat{\mathbf{a}}^H(t)(\mathbf{R}(t) + \mu\mathbf{I})^{-1}\hat{\mathbf{a}}(t)} \quad (41)$$

where  $\mu \geq 0$  is the DL level. Though the beamformer is now biased, the variance is

$$\text{tr}(\mathbf{C}_{\mathbf{R}\text{-}\delta\mathbf{W}}(t)) \approx \sum_{i=1}^N \frac{\mathbf{C}_{\delta\mathbf{A},i}(t)(\lambda_i(t) + \mu)^{-2}}{(\mathbf{A}_0^H(t)(\Lambda(t) + \mu\mathbf{I})^{-1}(t)\mathbf{A}_0(t))^2} \quad (42)$$

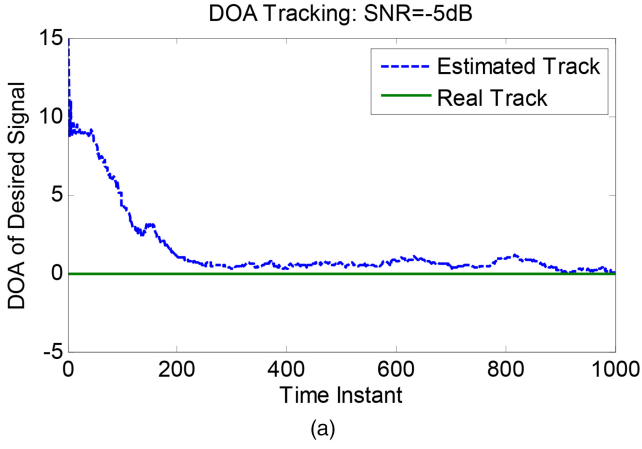
which decreases with increasing value of  $\mu$ . However, it is well known that the DL level is somewhat difficult to determine in practice. One conventional way to deal with this problem is to estimate the noise power  $\sigma_n^2$  and to select the regularization parameter  $\mu$  as  $\min(\kappa\sigma_n^2, \sigma_{\min}^2)$ , where  $\kappa$  is a user-defined constant and  $\sigma_{\min}^2$  is the minimum loading level. Usually,  $\kappa$  is chosen as 10 to combat the uncertainties of the steering vector and covariance matrix. Unfortunately, it is shown in much of the literature that such a fixed diagonal level cannot provide satisfactory performance. A number of robust beamforming algorithms have, therefore, been proposed by assuming that the steering vector and/or covariance matrix are known imprecisely and that they lie within certain bounds [7–9], [15–17].

In this paper, we estimate the perturbation bound of the array covariance matrix  $\mathbf{R}(t)$  so that it can be utilized in these robust beamforming algorithms. Moreover, it is adopted to determine the loading level of the conventional DL method, which yields a simple but robust beamformer. First, let the true and mismatched array covariance matrices be  $\tilde{\mathbf{R}}(t)$  and  $\mathbf{R}(t)$ , respectively. Hence, we have

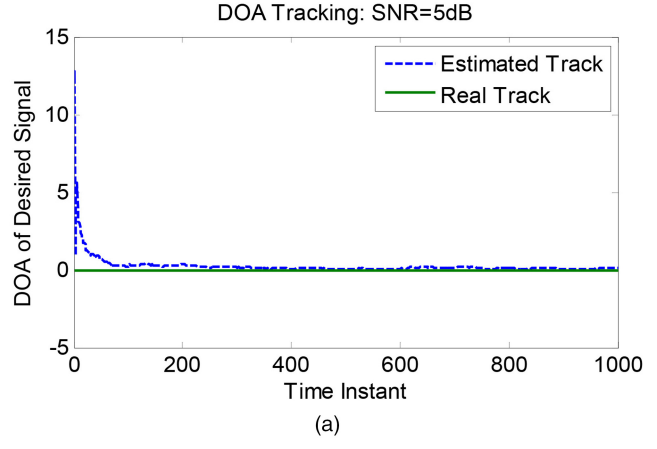
$$\tilde{\mathbf{R}}(t) = \mathbf{R}(t) + \delta\mathbf{R}(t) \quad (43)$$

where  $\delta\mathbf{R}(t)$  is the error matrix due to the perturbation of  $\mathbf{R}(t)$  and it is assumed to be bounded by a certain known or estimated parameter  $\gamma(t)$ , i.e.,

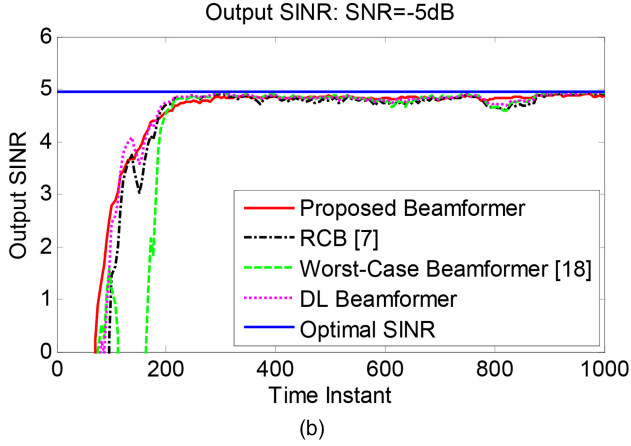
$$\|\delta\mathbf{R}(t)\| \leq \gamma(t). \quad (44)$$



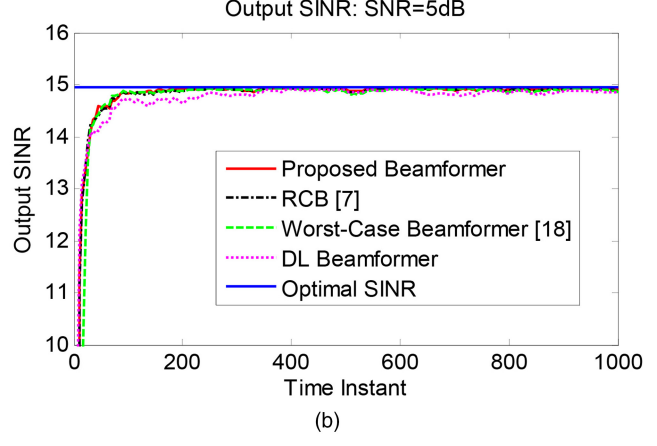
(a)



(a)



(b)



(b)

Fig. 1. (a) DOA tracking using extended OPAST in stationary case with SNR = -5 dB. (b) Output SINR of various beamformers in stationary case with SNR = -5 dB.

Fig. 2. (a) DOA tracking using extended OPAST in stationary case with SNR = 5 dB. (b) Output SINR of various beamformers in stationary case with SNR = 5 dB.

Consequently, the problem in (4) can be rewritten as

$$\begin{aligned} \min \mathbf{w}^H(t)(\mathbf{R}(t) + \delta\mathbf{R}(t))\mathbf{w}(t) \\ \text{s.t. } \mathbf{w}^H(t)\hat{\mathbf{a}}(t) = 1, \quad \|\delta\mathbf{R}(t)\| \leq \gamma(t) \end{aligned} \quad (45)$$

which can further be rewritten as the following problem of minimizing the worst case output power

$$\begin{aligned} \min_{\mathbf{w}} \max_{\|\delta\mathbf{R}(t)\| \leq \gamma} \mathbf{w}^H(t)(\mathbf{R}(t) + \delta\mathbf{R}(t))\mathbf{w}(t) \\ \text{s.t. } \mathbf{w}^H(t)\hat{\mathbf{a}}(t) = 1. \end{aligned} \quad (46)$$

In order to solve (46), we first solve the problem

$$\begin{aligned} \max_{\delta\mathbf{R}(t)} \mathbf{w}^H(t)(\mathbf{R}(t) + \delta\mathbf{R}(t))\mathbf{w}(t) \\ \text{s.t. } \|\delta\mathbf{R}(t)\| \leq \gamma(t) \end{aligned} \quad (47)$$

whose solution is given by [17]

$$\delta\mathbf{R}(t) = \gamma(t) \frac{\mathbf{w}(t)\mathbf{w}^H(t)}{\|\mathbf{w}(t)\|^2}. \quad (48)$$

After some manipulation the problem (46) can finally be reformulated as

$$\begin{aligned} \min \mathbf{w}^H(t)(\mathbf{R}(t) + \gamma(t)\mathbf{I})\mathbf{w}(t) \\ \text{s.t. } \mathbf{w}^H(t)\hat{\mathbf{a}}(t) = 1. \end{aligned} \quad (49)$$

Apparently, the solution of the problem in (49) is given by

$$\mathbf{w}_{\text{R-MVDR-WC}}(t) = \frac{(\mathbf{R}(t) + \gamma(t)\mathbf{I})^{-1}\hat{\mathbf{a}}(t)}{\hat{\mathbf{a}}^H(t)(\mathbf{R}(t) + \gamma(t)\mathbf{I})^{-1}\hat{\mathbf{a}}(t)}. \quad (50)$$

Comparing the worst case solution (50) with that in (41), the loading level  $\mu$  is directly related to the perturbation bound of the covariance matrix  $\gamma(t)$  in (44). In real systems, it may be able to select the perturbation bound  $\gamma(t)$  based on prior information. In this paper the instantaneous variation of the array covariance matrix, i.e.,  $\mathbf{R}(t) - \mathbf{R}(t-1)$ , is adopted to estimate the perturbation bound. More precisely, such a bound is assumed to be proportional to the norm of the instantaneous variation  $\gamma(t) = k\|\mathbf{R}(t) - \mathbf{R}(t-1)\|$ . In fact, it is found experimentally that  $\gamma(t)$  can be chosen from a wide range, with  $k$  between 1% to 20%, without significantly affecting the performance. Hence, the choice of  $\gamma(t)$  is not a crucial problem if the value of  $\gamma(t)$  is not too large. For illustrative purposes we choose  $\gamma(t) = k\|\mathbf{R}(t) - \mathbf{R}(t-1)\|$  with  $k = 10\%$  in Section IV. Finally, the proposed robust steering vector estimation and diagonally loaded MVDR beamformer based on worst case performance



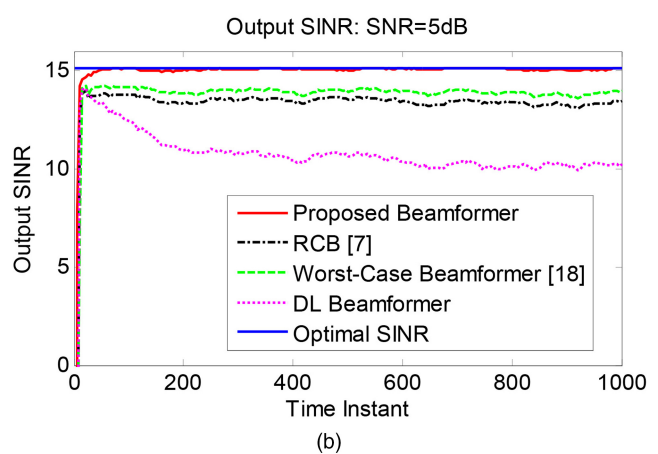
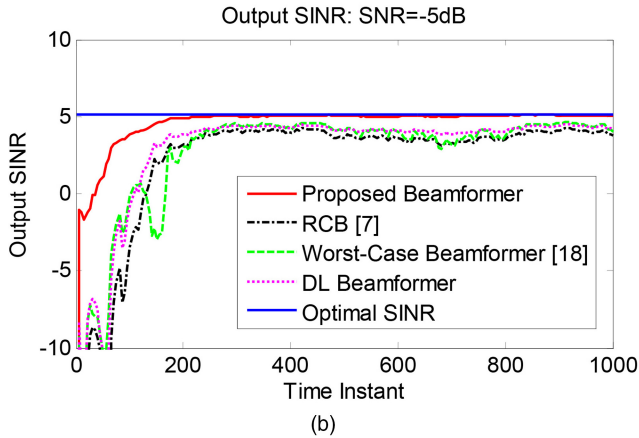
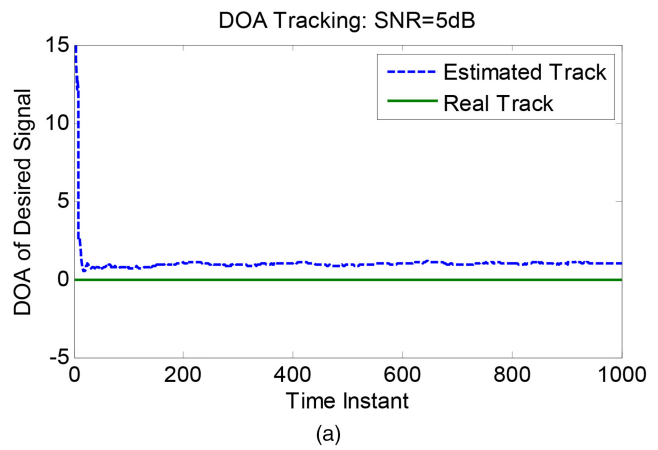
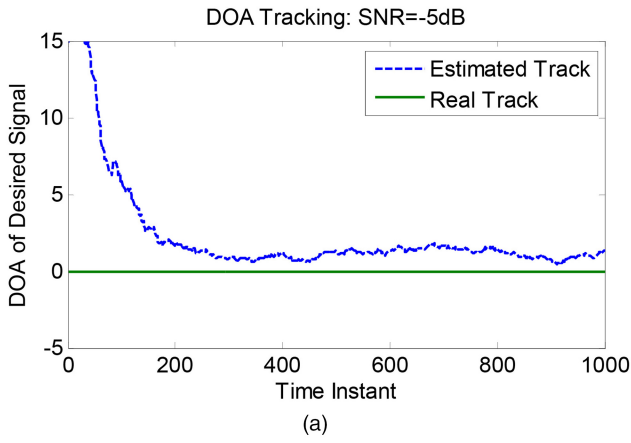


Fig. 3. (a) DOA tracking using extended OPAST in stationary case with gain/phase uncertainties and SNR = -5 dB. (b) Output SINR of various beamformers in stationary case with gain/phase uncertainties and SNR = -5 dB.

Fig. 4. (a) DOA tracking using extended OPAST in stationary case with gain/phase uncertainties and SNR = 5 dB. (b) Output SINR of various beamformers in stationary case with gain/phase uncertainties and SNR = 5 dB.

TABLE II  
Main Steps of the Proposed Algorithm

Step 1)	Update the covariance matrix $\mathbf{R}(t)$ recursively as $\mathbf{R}(t) = \beta\mathbf{R}(t-1) + (1-\beta)\mathbf{x}(t)\mathbf{x}^H(t)$ .
Step 2)	Update the signal subspace using the OPAST algorithm as $\tilde{\mathbf{U}}_s(t) = \hat{\mathbf{U}}_s(t-1) + \hat{\mathbf{e}}(t)\mathbf{g}^H(t)$ .
Step 3)	Compute the noise subspace with the estimated signal subspace as $\tilde{\mathbf{U}}_n(t) = \mathbf{I} - \tilde{\mathbf{U}}_s(t)\tilde{\mathbf{U}}_s^H(t)$ and the covariance matrix $\mathbf{C}_{\delta U}(t)$ based on the extended OPAST algorithm as $\mathbf{C}_{\delta U}(t) = \beta\mathbf{C}_{\delta U}(t-1) + (1-\beta)\delta\tilde{\mathbf{U}}_n(t)\delta\tilde{\mathbf{U}}_n^H(t)$ . Calculate $\zeta(t)$ as (14).
Step 4)	Use the estimated noise subspace $\tilde{\mathbf{U}}_n(t)$ , $\zeta(t)$ and the given nominal steering vector $\mathbf{a}_0(t)$ to compute the uncertainty in the steering vector as (27).
Step 5)	Update the steering vector as $\hat{\mathbf{a}}(t) = \mathbf{a}_0(t) + \hat{\Delta}(t)$ .
Step 6)	Compute the beamformer as (50) with the estimated $\hat{\mathbf{a}}(t)$ and $\gamma(t)$ .

optimization (R-MVDR-WC) is summarized in Table II.

We now briefly discuss the arithmetic complexity of the proposed algorithm. In step 1, the covariance matrix can be efficiently updated in  $O(N^2)$  complexity. In step 2, the signal subspace can be updated in  $O(N(K+1))$  complexity. The complexity in step 3 is

$O(N^3)$ , which is larger than the previous two steps due to the matrix product  $\delta\tilde{\mathbf{U}}_n(t)\delta\tilde{\mathbf{U}}_n^H(t)$ . The complexity in step 6 is also  $O(N^3)$  FLOPs due to the required matrix inversion process. Hence, the proposed method is of the same order as other conventional algorithms, such as Capon beamforming, RCB, and DL.

#### IV. NUMERICAL EXAMPLE

In order to evaluate the performance of the proposed algorithm, a uniform linear array (ULA) with  $N = 10$  sensors separated by a half-wavelength is considered. The noise is assumed to be AWGN with a power of 0 dB. One desired signal and two interferences are assumed to impinge on the array from far-field. In the first two examples, the DOA of the desired signal is assumed to be fixed at  $0^\circ$ , whereas in the last two examples, the DOA of the desired signal is considered to be time varying and is given by  $10^\circ \times 10^{-3}t$ ,  $0 \leq t \leq 1000$ , where  $t$  is the index of snapshots. For all simulation, the DOAs of the two interferences are fixed to be  $40^\circ$  and  $60^\circ$ . The powers of the interferences are fixed to be 30 dB, i.e., the interference-to-noise ratio (INR) is 30 dB. The noise subspace is obtained by using the

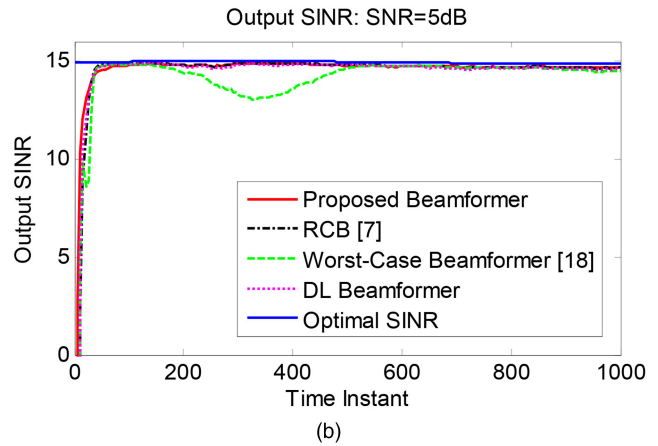
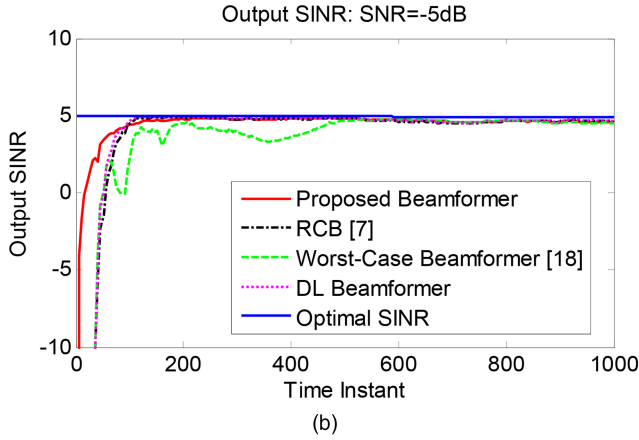
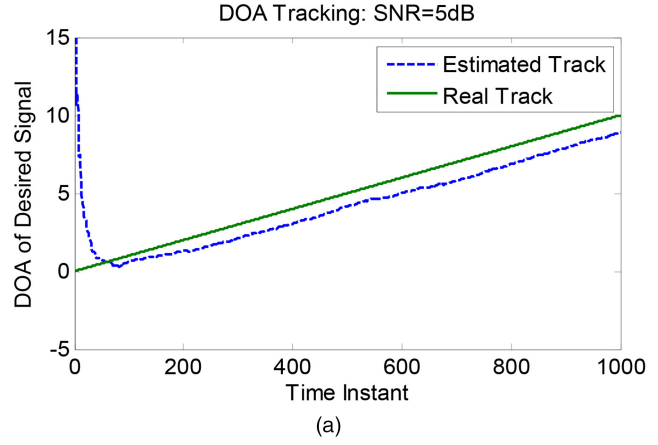
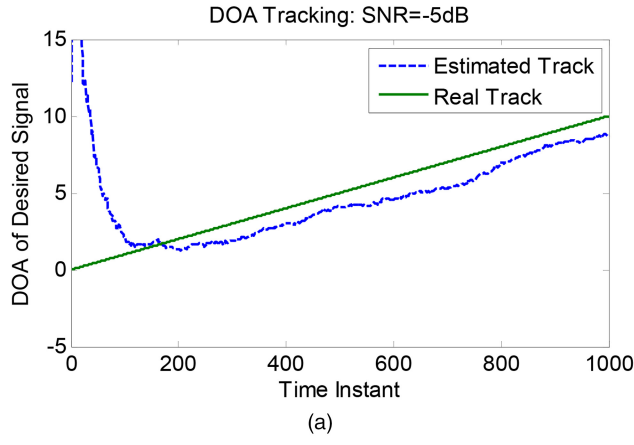


Fig. 5. (a) DOA tracking using extended OPAST in dynamic case with SNR = -5 dB. (b) Output SINR of various beamformers in dynamic case with SNR = -5 dB.

Fig. 6. (a) DOA tracking using extended OPAST in dynamic case with SNR = 5 dB. (b) Output SINR of various beamformers in dynamic case with SNR = 5 dB.

extended OPAST subspace tracking method as shown in Table I, where the forgetting factor is  $\beta = 0.99$ . The perturbation bound of the array covariance matrix is estimated as  $\gamma(t) = \|\mathbf{R}(t) - \mathbf{R}(t-1)\| \times 10\%$ . In all examples the signal-plus-interference number and hence the signal subspace rank used in the extended OPAST are assumed to be known, and they are equal to 3. In practice the subspace rank can be estimated by using, say, the minimum description length (MDL) algorithm.

For comparison the following conventional algorithms are also tested: 1) the conventional DL beamformer with a fixed loading level of 10; 2) the RCB [7] with the error bound equal to  $\varepsilon = 3.2460$ , which corresponds to a  $2^\circ$  DOA mismatch when the DOA of the desired signal is  $0^\circ$ , and 3) the worst case method [18]. In the simulations the DOA of the desired signal is first estimated using the conventional ESPRIT algorithm [26] with the tracked signal subspace  $\hat{\mathbf{U}}_s(t)$ . Then, the proposed robust beamforming, as well as other conventional algorithms, is invoked based on the estimated DOA. The performances of all these methods are compared in terms of the output SINR.

*Example 1. Stationary Case:* In the first example we test the performance of the proposed method in a stationary case, i.e., the desired signal has a fixed DOA, which is assumed to be  $0^\circ$ . We assume that there are no other uncertainties except the DOA mismatch due to the accuracy of the DOA tracking algorithm. The output SINR at each time instant is calculated according to (3). Figures 1 and 2 show the tracked DOA of the desired signal and the output SINR of various beamformers with a low SNR of -5 dB and a relatively high SNR of 5 dB, respectively. From these two figures it can be seen that when the SNR is low, the tracking algorithm converges slowly. Hence, there is a large DOA mismatch before convergence. Also, it can be seen that the proposed beamformer gives a better performance when there is a large DOA mismatch. On the other hand, when the tracking algorithm converges, the DOA of the desired signal can be estimated with high accuracy. Therefore, all beamformers can give excellent performance, which is almost identical to the optimal one.

*Example 2. Stationary Case with Array Gain/Phase Uncertainties:* In order to test the robustness of the proposed method against array imperfections,

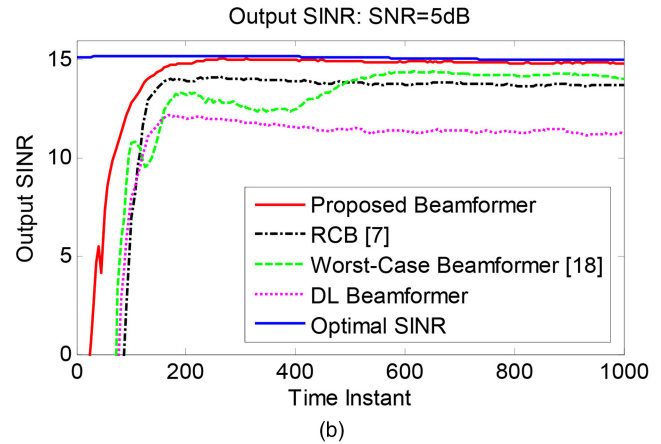
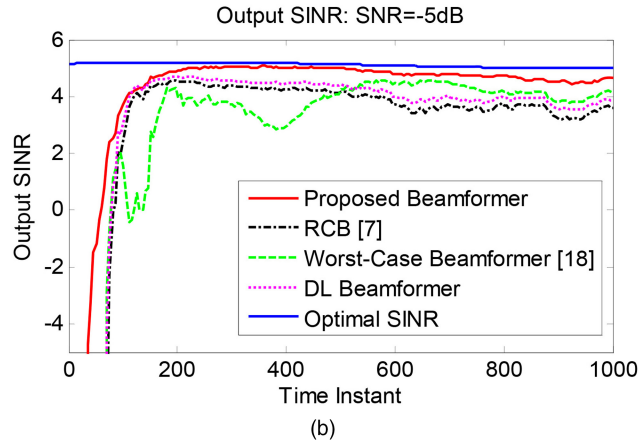
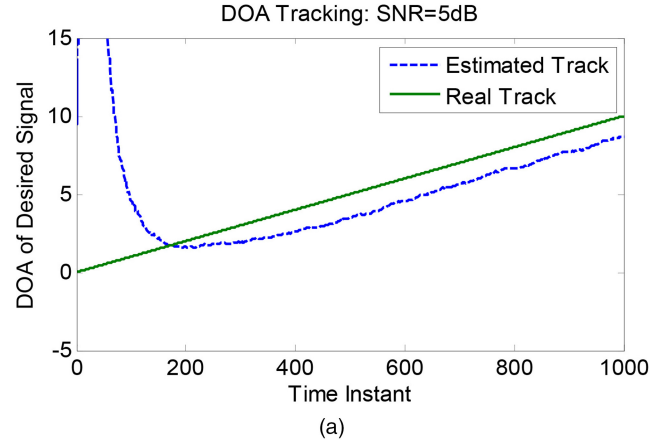
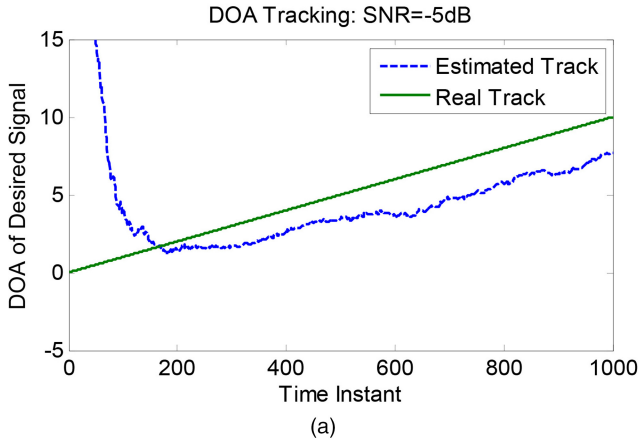


Fig. 7. (a) DOA tracking using extended OPAST in dynamic case with gain/phase uncertainties and SNR = -5 dB. (b) Output SINR of various beamformers in dynamic case with gain/phase uncertainties and SNR = -5 dB.

Fig. 8. (a) DOA tracking using extended OPAST in dynamic case with gain/phase uncertainties and SNR = 5 dB. (b) Output SINR of various beamformers in dynamic case with gain/phase uncertainties and SNR = 5 dB.

in this example, the array gain/phase uncertainties are considered. It is known that these uncertainties usually lead to a degradation of the DOA estimation and beamforming performance. Following the last example, in this simulation, each sensor (except the first reference sensor) is further assumed to suffer from a gain/phase uncertainty of the form  $\rho_i e^{j\phi_i}$ ,  $2 \leq i \leq N$ . Both the gain and phase uncertainties are assumed to be uniformly distributed as  $\rho_i \sim U(0.8, 1.2)$  and  $\phi_i \sim U(-\pi/5, \pi/5)$ . For simulation a fixed set of the gain/phase uncertainties is taken as:  $\{\rho_i\}_{i=2}^N = \{1.0369, 0.9695, 1.0033, 1.0176, 1.0560, 1.0309, 0.9665, 1.0718, 1.0690\}$  and  $\{\phi_i\}_{i=2}^N = \{-0.2916, -0.2947, 0.2547, 0.4015, 0.1534, 0.3667, -0.3193, -0.4652, -0.1343\}$ . The resultant DOA tracking and output SINR are shown in Fig. 3 and Fig. 4. Obviously, we can notice that the accuracy of DOA tracking is considerably degraded due to the existence of array gain/phase uncertainties. As can be seen in Fig. 3(a) and Fig. 4(a), there is a larger DOA mismatch even after the convergence of the tracking algorithm compared with the case without array gain/phase uncertainties. However, it can be seen that the proposed method outperforms the conventional

ones and nearly achieves optimal performance. Careful examination also shows that the performance of the conventional RCB deteriorates due to such uncertainties. Since the worst case beamformer [18] takes the uncertainties in the array covariance matrix into account, it is able to achieve a better performance than that of RCB.

*Example 3. Dynamic Case:* The settings in this example are identical to those in Example 1, except that the DOA of the desired signal is time varying and given by  $10^\circ \times 10^{-3}t$ ,  $0 \leq t \leq 1000$ , where  $t$  is the index of snapshots. Figure 5 and Fig. 6 show the DOA tracking results and output SINR with SNR of -5 dB and 5 dB, respectively. Compared with the stationary case, it can be noticed that there is a much larger DOA mismatch due to the dynamic of the desired signal. However, we can find that after convergence, all the methods can still successfully suppress the undesired interference and achieve excellent performance.

*Example 4. Dynamic Case with Array Gain/Phase Uncertainties:* It has been shown in Example 2 that when there are array gain/phase uncertainties, the DOA cannot be well tracked even in a stationary

case. In this example, we show the performance of the proposed method in a time-varying case with array gain/phase uncertainties. Again, the dynamic model of the desired signal is assumed to be the same as that in Example 3. The DOA tracking results and output SINRs for the SNRs at  $-5$  dB and  $5$  dB are shown, respectively, in Fig. 7 and Fig. 8. As expected the DOA tracking performance degrades due to the array gain/phase uncertainties. Furthermore, it can be seen that the conventional methods are significantly influenced by such uncertainties, especially at higher SNRs. On the contrary, the proposed method can still achieve an excellent performance.

## V. CONCLUSIONS

A new method for correcting possible deterministic errors in the steering vector due to sensor uncertainties is presented. It uses the subspace principle, and the resulting problem can be formulated as a convex problem and solved in closed form. Using an extended OPAST algorithm, the algorithm is further extended to handle scenarios involving moving sources while requiring low complexity. An analysis on the perturbation of beamforming weights due to DOA estimation errors is also performed, and it suggests that the former is also highly sensitive to the eigenvalues of the estimated covariance matrix. Hence, a new adaptive beamformer, which minimizes the worst case performance of the array subject to covariance matrix uncertainties, is also presented. The resultant beamformer resembles the diagonally loaded Capon beamformer with the loading level given by a bound on the uncertainties in the array covariance matrix, which can be estimated recursively. Simulation results show that the proposed algorithm can offer satisfactory performance, especially at high SNR levels and in the presence of deterministic sensor uncertainties.

## REFERENCES

- [1] Li, J. and Stoica, P.  
*Robust Adaptive Beamforming*.  
Hoboken, NJ: Wiley, 2006.
- [2] Capon, J.  
High-resolution frequency wavenumber spectrum analysis.  
*Proceedings of the IEEE*, **57**, 8 (1969), 1408–1418.
- [3] Ye, Z. and Liu, C.  
Non-sensitive adaptive beamforming against mutual coupling.  
*IET Signal Processing*, **3**, 1 (Jan. 2009), 1–6.
- [4] Ng, B. P., Er, M. H., and Kot, C.  
Array gain/phase calibration techniques for adaptive beamforming and direction finding.  
*IEE Proceedings—Radar, Sonar and Navigation*, **141**, 1 (Feb. 1994), 25–29.
- [5] Er, M. H. and Cantoni, A.  
Derivative constraints for broadband element space antenna array processors.  
*IEEE Transactions on Acoustics, Speech and Signal Processing*, **ASSP-31**, 6 (Dec. 1983), 1378–1393.
- [6] Zou, Q., Yu, Z. L., and Lin, Z.  
A robust algorithm for linearly constrained adaptive beamforming.  
*IEEE Signal Processing Letters*, **11**, 1 (Jan. 2004), 26–29.
- [7] Li, J., Stoica, P., and Wang, Z.  
On robust Capon beamforming and diagonal loading.  
*IEEE Transactions on Signal Processing*, **51**, 7 (July 2003), 1702–1715.
- [8] Lorenz, R. and Boyd, S.  
Robust minimum variance beamforming.  
*IEEE Transactions on Signal Processing*, **53**, 5 (May 2005), 1684–1696.
- [9] Hudson, J. E.  
*Adaptive Array Principles*.  
London: Peter Peregrinus, 1981.
- [10] Tian, Z., Bell, K. L., and Van Trees, H. L.  
A recursive least squares implementation for LCMP beamforming under quadratic constraints.  
*IEEE Transactions on Signal Processing*, **49**, 6 (June 2001), 1138–1145.
- [11] Hassanien, A., Vorobyov, S. A., and Wong, K. M.  
Robust adaptive beamforming using sequential quadratic programming: An iterative solution to the mismatch problem.  
*IEEE Signal Processing Letters*, **15** (2008), 733–736.
- [12] Yang, B.  
Projection approximation subspace tracking.  
*IEEE Transactions on Signal Processing*, **43**, 1 (Jan. 1995), 95–107.
- [13] Sanchez-Araujo, J. and Marcos, S.  
An efficient PASTd-algorithm implementation for multiple direction of arrival tracking.  
*IEEE Transactions on Signal Processing*, **47**, 8 (Aug. 1999), 2321–2324.
- [14] Abed-Meraim, K., Chkeif, A., and Hua, Y.  
Fast orthonormal PAST algorithm.  
*IEEE Signal Processing Letters*, **7**, 3 (Mar. 2000), 60–62.
- [15] Vorobyov, S. A., et al.  
Adaptive beamforming with joint robustness against mismatched signal steering vector and interference nonstationary.  
*IEEE Signal Processing Letters*, **11**, 2 (Feb. 2004), 108–111.
- [16] Cox, H., Zeskind, R., and Owen, M.  
Robust adaptive beamforming.  
*IEEE Transactions on Acoustics, Speech and Signal Processing*, **ASSP-35**, 10 (Oct. 1987), 1365–1376.
- [17] Shahbazpanahi, S., et al.  
Robust adaptive beamforming for general-rank signal models.  
*IEEE Transactions on Signal Processing*, **51**, 9 (Sept. 2003), 2257–2268.
- [18] Kim, S. J., et al.  
Robust beamforming via worst-case SINR maximization.  
*IEEE Transactions on Signal Processing*, **56**, 4 (Apr. 2008), 1539–1545.
- [19] Vorobyov, S. A., Gershman, A. B., and Luo, Z. Q.  
Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem.  
*IEEE Transactions on Signal Processing*, **51**, 2 (Feb. 2003), 313–324.
- [20] Feldman, D. D. and Griffiths, L. J.  
A projection approach for robust adaptive beamforming.  
*IEEE Transactions on Signal Processing*, **42**, 4 (Apr. 1994), 867–876.

- [21] Chang, L. and Yeh, C. C.  
Performance of DMI and eigenspace-based beamformers.  
*IEEE Transactions on Antennas and Propagation*, **40**, 11  
(Nov. 1992), 1336–1347.
- [22] Honig, M. L. and Goldstein, J. S.  
Adaptive reduced-rank interference suppression based on  
the multistage Wiener filter.  
*IEEE Transactions on Communications*, **50**, 6 (June 2002),  
986–994.
- [23] Pados, D. A. and Karystinos, G. N.  
An iterative algorithm for the computation of the MVDR  
filter.  
*IEEE Transactions on Signal Processing*, **49**, 2 (Feb.  
2001), 290–300.
- [24] de Lamare, R. C., Wang, L., and Fa, R.  
Adaptive reduced-rank LCMV beamforming algorithms  
based on joint iterative optimization of filters: Design and  
analysis.  
*Signal Processing*, **90**, 2 (Feb. 2010), 640–652.
- [25] de Lamare, R. C. and Sampaio-Neto, R.  
Adaptive reduced-rank processing based on joint and  
iterative interpolation, decimation, and filtering.  
*IEEE Transactions on Signal Processing*, **57**, 7 (July  
2009), 2503–2514.
- [26] Roy, R. and Kailath, T.  
ESPRIT—Estimation of signal parameters via rotational  
invariance techniques.  
*IEEE Transactions on Acoustics, Speech and Signal  
Processing*, **37**, 7 (July 1989), 984–995.
- [27] Oehlert, G. W.  
A note on the delta method.  
*The American Statistician*, **46**, 1 (Feb. 1992), 27–29.



**Bin Liao** (S'09) received B.Eng. and M.Eng. degrees from Xidian University, Xi'an, China in 2006 and 2009, respectively.

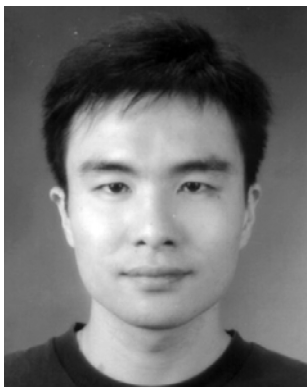
He is currently pursuing a Ph.D degree in the Department of Electrical and Electronic Engineering, University of Hong Kong. His main research interests are array signal processing and adaptive filtering.



**Shing-Chow Chan** (S'87—M'92) received B.S. (Eng.) and Ph.D. degrees from the University of Hong Kong, Pokfulam, Hong Kong in 1986 and 1992, respectively.

Since 1994 he has been with the University of Hong Kong and currently is an associate professor. He was a visiting researcher with Microsoft Corporation, Redmond, WA and Microsoft, Beijing, China in 1998 and 1999, respectively. His research interests include fast transform algorithms, filter design and realization, multirate signal processing, and imagebased rendering.

Dr. Chan is currently an associate editor of the *IEEE Transactions on Circuits and Systems—I: Regular Papers* and the *Journal of Very Large Scale Integration Signal Processing and Video Technology*, and a member of the Digital Signal Processing Technical Committee of the IEEE Circuits and Systems Society. He was the Chairman of the IEEE Hong Kong Chapter of Signal Processing from 2000 to 2002.



**Kai-Man Tsui** received B.Eng., M.Phil., and Ph.D. degrees in electrical and electronic engineering from the University of Hong Kong, in 2001, 2004, and 2008, respectively.

He is currently working as a postdoctoral fellow in the Department of Electrical and Electronic Engineering at the University of Hong Kong. His main research interests are in array signal processing; high speed AD converter architecture; biomedical signal processing, digital signal processing; multirate filter bank and wavelet design; and digital filter design, realization, and application.