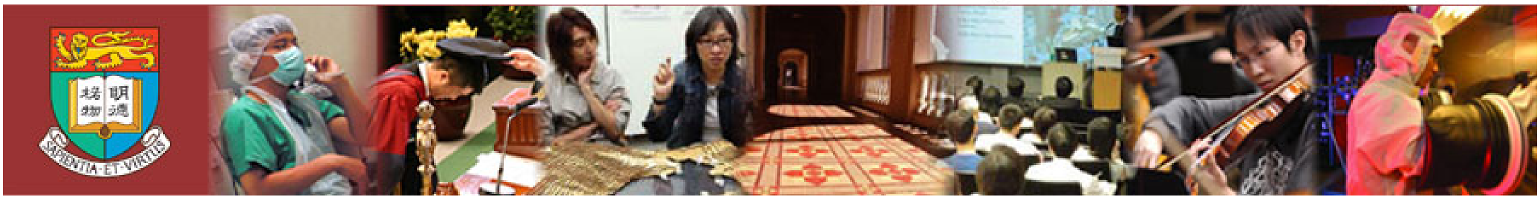

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A New Green's Function Formulation for Modeling Homogeneous Objects in Layered Medium

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***Abstract*—A new Green's function formulation is developed systematically for modeling general homogeneous (dielectric or magnetic) objects in a layered medium. The dyadic form of the Green's function is first derived based on the pilot vector potential approach. The matrix representation in the moment method implementation is then derived by applying integration by parts and vector identities. The line integral issue in the matrix representation is investigated, based on the continuity property of the propagation factor and the consistency of the primary term and the secondary term. The extinction theorem is then revisited in the inhomogeneous background and a surface integral equation for general homogeneous objects is set up. Different from the popular mixed potential integral equation formulation, this method avoids the artificial definition of scalar potential. The singularity of the matrix representation of the Green's function can be made as weak as possible. Several numerical results are demonstrated to validate the formulation developed in this paper. Finally, the duality principle of the layered medium Green's function is discussed in the appendix to make the formulation succinct.**

***Index Terms*—Dyadic form, homogeneous objects, layered medium Green's function, matrix representation, surface integral equation.**

I. Introduction

SURFACE integral equation (SIE) method is one of the most powerful methods in analyzing electromagnetic radiation and scattering problems. By introducing an "impulse response" Green's function, and invoking Green's theorem, the Helmholtz equation can be cast into an equivalent surface integral equation, where the unknowns are pushed to the boundary of the scatterers [1]. Though the derivation is elegant, the Green's functions for arbitrary inhomogeneous medium is not trivial and can only be obtained numerically, which may be as complex as solving the original problem. For this reason, most research on surface integral equation methods are focused on homogeneous environment, such as aerospace applications. However, if the inhomogeneity is one dimensional piecewise, or so called layered medium, the Green's function can be determined analytically in the spectral (Fourier) domain, and the spatial domain counterpart can be obtained by simply inverse Fourier transforming it. In fact, this scenario consists of a broad class of applications in both microwave and optical regimes, such as microstrip antennas and microwave circuits, geophysical exploration, ground-penetrating radar (GPR), solar cell, light-emitting diode (LED), and lithography, etc.

Sommerfeld first investigated a dipole radiating above a half space using Hertzian potential [2]. This problem was then further studied and extended to general multilayered medium by various researchers [1], [3]–[6]. In numerical simulation, the two-dimensional (2D) analysis was developed in [7], where scattering from a conducting cylinder partially buried in a half space is analyzed. For more practical applications, the three-dimensional (3D) analysis was carried out and applied to many problems on radiation and scattering from perfect electric conducting (PEC) objects in the background of layered medium [8]–[13]. Among them, the layered medium Green's function based on transmission line analog and the corresponding mixed potential integral equation (MPIE) developed in [9] becomes one of most popular formulations. A correction vector is introduced to make the definition of a "scalar potential" possible, which is due to the fact that a scalar potential of a point source in layered medium do not in general exist [14]. Different choice of the vector potential and correction term leads to three different formulations: Formulation A, B, and C.

For analysis of general scatterers in layered medium, the volume integral equation (VIE) method is usually applied [15], [16]. Though VIE can handle inhomogeneous objects, the number of unknowns is typically large and the equation should be reformulated if there are contrast in both permittivity and permeability. On the other hand, the SIE is favorable for homogeneous objects due to its unknown scale and elegant form. Different from the PEC case, both surface electric current and magnetic current are required and more types of Green's functions are needed in the SIE formulation for general homogeneous (dielectric or magnetic) objects [17]–[19]. A dielectric body of revolution (BOR) buried in soil covered with a layer of snow is solved by SIE in [20], where the "Formulation C" in [9] is applied. The SIE is further extended for general dielectric target in a multilayered medium in [21], with application to radar-based sensing of plastic land mines. In [22], the SIE is applied to analyze a dielectric resonator in layered medium coupled to a microstrip circuit. Though the MPIE is also applied, the Green's function is different from previous one. The vector potential is kept in the simple form as is developed by

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Sommerfeld [2], while the scalar potential is manipulated in a "dyadic way". Other attempts are also made in solving this problem, for example, a spectral integral method is developed for 2D PEC and dielectric objects with a closed boundary in layered medium [23].

To avoid the artificial definition of the scalar potential in layered medium, a new formulation of layered medium Green's function is developed in electric field integral equation (EFIE) for modeling PEC objects [24]. By applying integration by parts, the singularity can be reduced as weak as possible. It is shown in [25] that this formulation is as convenient as the popular MPIE method developed in [9].

In this paper, we formulate the Green's function in a systematic way and extend it to model general homogeneous objects. The rest of the paper is organized as follows: Section II first reviews the integral operators and introduces the dyadic form of the Green's function. The matrix representation in moment method [26] implementation is then derived in Section III, followed by the line integral issue investigation, which is based on the continuity property of the propagation factor and the consistency analysis. After that, in Section IV, the extinction theorem is revisited for an inhomogeneous environment and the surface integral equation is set up. Finally, in Section V, several numerical results are presented to validate the formulation developed in this paper. To make the formulation succinct, only the electric-type Green's function ($\bar{\mathbf{G}}_e$ and $\nabla\times\bar{\mathbf{G}}_e$) is discussed in this paper, and the magnetic-type Green's function ($\bar{\mathbf{G}}_m$ and $\nabla\times\bar{\mathbf{G}}_m$) can be obtained by applying the duality principle of the layered medium Green's function shown in the Appendix.

II. Dyadic Form of Layered Medium Green's Function

The dyadic form of the layered medium Green's function describes the electric or magnetic field generated by an electric or magnetic dipole in a layered medium. In other existing literature, one usually defines four dyadic Green's functions, $\bar{\mathbf{G}}^{EJ}(\mathbf{r},\mathbf{r}')$, $\bar{\mathbf{G}}^{HJ}(\mathbf{r},\mathbf{r}')$, $\bar{\mathbf{G}}^{EM}(\mathbf{r},\mathbf{r}')$ and $\bar{\mathbf{G}}^{HM}(\mathbf{r},\mathbf{r}')$. In this paper, however, we will apply the notation slightly differently, namely $\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')$ and $\bar{\mathbf{G}}_m(\mathbf{r},\mathbf{r}')$ and their curls [19]. It is convenient to introduce the integral operators with kernels of different Green's functions to express the electromagnetic fields generated by arbitrary electric and magnetic current sources. So we will first introduce different sets of integral operators and then derive the expressions of their kernels.

A. The Integral Operators

There are generally four integral operators [19].

$$\mathbf{E}(\mathbf{r})=\mathcal{L}_{\rm E}(\mathbf{r},\mathbf{r}')\cdot \mathrm{J}(\mathbf{r}')+\mathcal{K}_{\rm E}(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}') \tag{1}$$

$$\mathbf{H}(\mathbf{r})=\mathcal{L}_{\rm H}(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}')+\mathcal{K}_{\rm H}(\mathbf{r},\mathbf{r}')\cdot \mathrm{J}(\mathbf{r}') \tag{2}$$

where $\mathbf{J}$ denotes the electric current and $\mathbf{M}$ is the magnetic current, $\mathbf{r}$ is the observation point and $\mathbf{r}'$ is the source point. These expressions are valid for any inhomogeneous medium, however, we will focus on the planarly layered medium here. The operator $\mathcal{L}_{\rm E}$ is defined as [19]

$$\mathcal{L}_{\rm E}(\mathbf{r},\mathbf{r}')\cdot=i\omega\int d\mathbf{r}'\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')\mu(\mathbf{r}')\cdot \tag{3}$$

where $\bar{\mathbf{G}}_e$ is the electric-type dyadic Green's function. According to the Maxwell's equations, the operator $\mathcal{K}_{\rm H}$ can then be defined as

$$\mathcal{K}_{\rm H}(\mathbf{r},\mathbf{r}')\cdot=\mu^{-1}(\mathbf{r})\int d\mathbf{r}'\nabla\times\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')\mu(\mathbf{r}')\cdot \tag{4}$$

Due to the duality principle of the Green's function shown in the Appendix, we can further define the other two operators as,

$$\mathcal{L}_{\rm H}(\mathbf{r},\mathbf{r}')\cdot=i\omega\int d\mathbf{r}'\bar{\mathbf{G}}_m(\mathbf{r},\mathbf{r}')\epsilon(\mathbf{r}') \tag{5}$$

$$\mathcal{K}_{\rm E}(\mathbf{r},\mathbf{r}')\cdot=-\epsilon^{-1}(\mathbf{r})\int d\mathbf{r}'\nabla\times\bar{\mathbf{G}}_m(\mathbf{r},\mathbf{r}')\epsilon(\mathbf{r}'). \tag{6}$$

It should be noted that in homogeneous space, usually two operators are involved ($\mathcal{L}_{\rm E}$ and $\mathcal{K}_{\rm E}$, or simply $\mathcal{L}$ and $\mathcal{K}$), since $\mathcal{L}_{\rm H}$ and $\mathcal{K}_{\rm H}$ can be easily obtained by $\mathcal{L}_{\rm E}$ and $\mathcal{K}_{\rm E}$ explicitly, either by a factor of wave impedance $(1/\eta^2)$ or a negative sign [19], due to the fact that for homogeneous medium Green's function, we have $\bar{\mathbf{G}}_e=\bar{\mathbf{G}}_m$ (see Appendix). However, in layered medium, the relation can only be determined by the duality principle implicitly, so we prefer to stay with the four operators in this paper.

It should also be noted that the dyadic Green's function can typically be expressed in a uniform manner, where the direct term is absorbed in the total expression in the spectral domain, as is done for example in [17]. However, from implementation point of view, we will always separate the Green's function into the primary term (direct interaction in homogeneous environment) and the secondary term (due to the reflection and transmission of the layered medium) in the following sections, since the primary term can be determined in closed form and be calculated separately. In the following, we only focus on $\bar{\mathbf{G}}_e$ and $\nabla\times\bar{\mathbf{G}}_e$, since $\bar{\mathbf{G}}_m$ and $\nabla\times\bar{\mathbf{G}}_m$ can be easily obtained by the duality principle of the Green's function shown in the Appendix.

B. Primary Term of the Green's Function

The primary term is the same as the homogeneous medium Green's function. It is nonzero only when the observation point and the source point are in the same layer. For the sake of completeness, we will also briefly list the expressions of the primary term.

1) Expression of $\bar{\mathbf{G}}_e$*:* The $\bar{\mathbf{G}}_e$ of the primary term in (3) is

$$\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')=\left(\bar{\mathbf{I}}+\frac{\nabla\nabla}{k_m^2}\right)g(\mathbf{r},\mathbf{r}') \tag{7}$$

with the scalar Green's function

$$g(\mathbf{r},\mathbf{r}')=\frac{e^{ik_m|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}=\frac{e^{ik_mR}}{4\pi R} \tag{8}$$

where the subscript m is the layer index of the source point (and n for observation layer, here $m=n$), and k_m is the wave number in layer m. In Cartesian coordinates, the dyadic Green's function can be further written as,

$$\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}')=\left[(\bar{\mathbf{I}}-\hat{R}\hat{R})+(\bar{\mathbf{I}}-3\hat{R}\hat{R})\frac{ik_mR-1}{k_m^2R^2}\right]g(\mathbf{r},\mathbf{r}') \tag{9}$$

where $\bar{\mathbf{I}}$ is the identity dyad and

$$\hat{R} = \frac{\mathbf{R}}{R} = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{R}\left[(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}\right]. \tag{10}$$

2) Expression of $\nabla \times \bar{\mathbf{G}}_e$*:* For $\nabla \times \bar{\mathbf{G}}_e$ of the primary term in (4), we have

$$\nabla \times \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') = \nabla \times \bar{\mathbf{I}} g(\mathbf{r},\mathbf{r}'). \tag{11}$$

It can also be obtained in closed form as

$$\begin{aligned}\nabla \times \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') &= \begin{bmatrix} 0 & -\partial_z & \partial_y \\ \partial_z & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{bmatrix} g(\mathbf{r},\mathbf{r}') \\ &= \begin{bmatrix} 0 & -(z-z') & y-y' \\ z-z' & 0 & -(x-x') \\ -(y-y') & x-x' & 0 \end{bmatrix} \\ &\quad \cdot \frac{1}{R}\left(ik_m - \frac{1}{R}\right) g(\mathbf{r},\mathbf{r}'). \end{aligned} \tag{12}$$

C. Secondary Term of the Green's Function

The secondary term of the Green's function is from the reflection and transmission of the layered medium, it can be expressed as [24]

$$\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') = \bar{\mathbf{G}}_e^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') + \frac{1}{k_{nm}^2}\bar{\mathbf{G}}_e^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') \tag{13}$$

where $k_{nm}^2 = \omega^2\epsilon_n\mu_m$, and

$$\bar{\mathbf{G}}_e^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') = (\nabla \times \hat{z})(\nabla' \times \hat{z}) g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') \tag{14}$$

$$\bar{\mathbf{G}}_e^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') = (\nabla \times \nabla \times \hat{z})(\nabla' \times \nabla' \times \hat{z}) g^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') \tag{15}$$

where

$$\begin{aligned} g^{\alpha}(\mathbf{r},\mathbf{r}') &= \frac{i}{8\pi^2}\int\int_{-\infty}^{+\infty} \frac{d\mathbf{k}_s}{k_{mz}k_s^2} e^{i\mathbf{k}_s\cdot(\mathbf{r}_s-\mathbf{r}'_s)} F^{\alpha}(k_s,z,z') \\ &= \frac{i}{4\pi}\int_0^{+\infty} \frac{dk_\rho}{k_{mz}k_\rho} J_0(k_\rho\rho) F^{\alpha}(k_\rho,z,z') \end{aligned} \tag{16}$$

here α represents TE or TM wave, $F^{\alpha}(k_\rho,z,z')$ is the propagation factor [1], [27], and $k_{mz} = \sqrt{k_m^2 - k_\rho^2}$. In Cartesian coordinates, we have $\mathbf{k}_s = k_x\hat{x} + k_y\hat{y}$, $\mathbf{r}_s = x\hat{x} + y\hat{y}$, while in the cylindrical coordinates, we further have $k_x = k_\rho\cos\alpha$, $k_y = k_\rho\sin\alpha$, and $x - x' = \rho\cos\phi$, $y - y' = \rho\sin\phi$. If we denote a 2D inverse Fourier transform and a Sommerfeld integral as [9]

$$\begin{aligned}\mathcal{F}^{-1}\left\{\tilde{f}(k_x,k_y)\right\} &= \frac{1}{(2\pi)^2}\int\int_{-\infty}^{+\infty} dk_x dk_y \tilde{f}(k_x,k_y) \\ &\quad \cdot e^{ik_x(x-x')+ik_y(y-y')} \end{aligned} \tag{17}$$

$$S_n\left\{\tilde{f}(k_\rho)\right\} = \frac{1}{2\pi}\int_0^{+\infty} dk_\rho \tilde{f}(k_\rho) J_n(k_\rho\rho) k_\rho^{n+1} \tag{18}$$

we have

$$g^{\alpha}(\mathbf{r},\mathbf{r}') = \frac{i}{2}\mathcal{F}^{-1}\left\{\frac{F^{\alpha}(k_s,z,z')}{k_{mz}k_s^2}\right\} = \frac{i}{2}S_0\left\{\frac{F^{\alpha}(k_\rho,z,z')}{k_{mz}k_\rho^2}\right\}. \tag{19}$$

The spatial derivatives in Cartesian coordinates can be transformed to be higher order Sommerfeld integrals in the cylindrical system by applying the integral representation of the Bessel function [1]

$$J_n(k_\rho\rho) = \frac{1}{2\pi}\int_0^{2\pi} d\alpha e^{ik_\rho\rho\cos\alpha + in\alpha - in\frac{\pi}{2}}. \tag{20}$$

The expressions have been derived in [9] (Appendix) and will not be repeated here. Notice that the second-order Sommerfeld integrals are further simplified to lower-order counterparts by using the recursive property of the Bessel function [18], [29]

$$J_2(k_\rho\rho) = \frac{2}{k_\rho\rho}J_1(k_\rho\rho) - J_0(k_\rho\rho). \tag{21}$$

All the relations will be utilized in deriving the following Green's function components (the time convention in this paper is $e^{-i\omega t}$ and is different from that of [9]).

1) Expression of $\bar{\mathbf{G}}_e$*:* We first consider the TE wave, since

$$\nabla \times \hat{z} = \partial_y\hat{x} - \partial_x\hat{y} \tag{22}$$

we have

$$\bar{\mathbf{G}}_e^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') = \begin{bmatrix} \partial_y\partial_{y'} & -\partial_y\partial_{x'} & 0 \\ -\partial_x\partial_{y'} & \partial_x\partial_{x'} & 0 \\ 0 & 0 & 0 \end{bmatrix} g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') \tag{23}$$

where

$$\begin{aligned} G_{e,xx}^{\mathrm{TE}} &= \frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{F^{\mathrm{TE}}}{k_{mz}k_\rho^2}\right\} \\ &\quad + \frac{i}{4}(1-\cos 2\phi)S_0\left\{\frac{F^{\mathrm{TE}}}{k_{mz}}\right\} \end{aligned} \tag{24}$$

$$G_{e,xy}^{\mathrm{TE}} = \frac{i}{2\rho}\sin 2\phi S_1\left\{\frac{F^{\mathrm{TE}}}{k_{mz}k_\rho^2}\right\} - \frac{i}{4}\sin 2\phi S_0\left\{\frac{F^{\mathrm{TE}}}{k_{mz}}\right\} \tag{25}$$

$$G_{e,yx}^{\mathrm{TE}} = G_{e,xy}^{\mathrm{TE}} \tag{26}$$

$$\begin{aligned} G_{e,yy}^{\mathrm{TE}} &= -\frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{F^{\mathrm{TE}}}{k_{mz}k_\rho^2}\right\} \\ &\quad + \frac{i}{4}(1+\cos 2\phi)S_0\left\{\frac{F^{\mathrm{TE}}}{k_{mz}}\right\}. \end{aligned} \tag{27}$$

Similarly, for TM wave, we have

$$\nabla \times \nabla \times \hat{z} = \partial_x\partial_z\hat{x} + \partial_y\partial_z\hat{y} + k_s^2\hat{z} \tag{28}$$

then

$$\bar{\mathbf{G}}_e^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') = \begin{bmatrix} \partial_x\partial_{x'}\partial_z\partial_{z'} & \partial_x\partial_{y'}\partial_z\partial_{z'} & \partial_x\partial_z k_\rho^2 \\ \partial_y\partial_{x'}\partial_z\partial_{z'} & \partial_y\partial_{y'}\partial_z\partial_{z'} & \partial_y\partial_z k_\rho^2 \\ \partial_{x'}\partial_{z'}k_\rho^2 & \partial_{y'}\partial_{z'}k_\rho^2 & k_\rho^4 \end{bmatrix} \cdot g^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') \tag{29}$$

where

$$G_{e,xx}^{\mathrm{TM}} = -\frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{\partial_z\partial_{z'}F^{\mathrm{TM}}}{k_{mz}k_\rho^2}\right\} + \frac{i}{4}(1+\cos 2\phi)S_0\left\{\frac{\partial_z\partial_{z'}F^{\mathrm{TM}}}{k_{mz}}\right\} \quad (30)$$

$$G_{e,xy}^{\mathrm{TM}} = -\frac{i}{2\rho}\sin 2\phi S_1\left\{\frac{\partial_z\partial_{z'}F^{\mathrm{TM}}}{k_{mz}k_\rho^2}\right\} + \frac{i}{4}\sin 2\phi S_0\left\{\frac{\partial_z\partial_{z'}F^{\mathrm{TM}}}{k_{mz}}\right\} \quad (31)$$

$$G_{e,xz}^{\mathrm{TM}} = -\frac{i}{2}\cos\phi S_1\left\{\frac{\partial_z F^{\mathrm{TM}}}{k_{mz}}\right\} \quad (32)$$

$$G_{e,yx}^{\mathrm{TM}} = G_{e,xy}^{\mathrm{TM}} \quad (33)$$

$$G_{e,yy}^{\mathrm{TM}} = \frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{\partial_z\partial_{z'}F^{\mathrm{TM}}}{k_{mz}k_\rho^2}\right\} + \frac{i}{4}(1-\cos 2\phi)S_0\left\{\frac{\partial_z\partial_{z'}F^{\mathrm{TM}}}{k_{mz}}\right\} \quad (34)$$

$$G_{e,yz}^{\mathrm{TM}} = -\frac{i}{2}\sin\phi S_1\left\{\frac{\partial_z F^{\mathrm{TM}}}{k_{mz}}\right\} \quad (35)$$

$$G_{e,zx}^{\mathrm{TM}} = \frac{i}{2}\cos\phi S_1\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}}\right\} \quad (36)$$

$$G_{e,zy}^{\mathrm{TM}} = \frac{i}{2}\sin\phi S_1\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}}\right\} \quad (37)$$

$$G_{e,zz}^{\mathrm{TM}} = \frac{i}{2}S_0\left\{\frac{k_\rho^2 F^{\mathrm{TM}}}{k_{mz}}\right\}. \quad (38)$$

There are 6 Sommerfeld integrals in the expression of $\bar{\mathbf{G}}_e$.

2) Expression of $\nabla\times\bar{\mathbf{G}}_e$*:* For $\nabla\times\bar{\mathbf{G}}_e$, from (13)–(15), we simply have

$$\nabla\times\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') = \nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') + \frac{1}{k_{nm}^2}\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') \quad (39)$$

where

$$\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') = (\nabla\times\nabla\times\hat{z})(\nabla'\times\hat{z})g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') \quad (40)$$

$$\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') = k_n^2(\nabla\times\hat{z})(\nabla'\times\nabla'\times\hat{z})g^{\mathrm{TM}}(\mathbf{r},\mathbf{r}'). \quad (41)$$

Here (41) is derived due to the fact that for the secondary field, we have

$$\left(\nabla^2 + k_n^2\right)g^{\alpha}(\mathbf{r},\mathbf{r}') = 0. \quad (42)$$

Again we first consider the TE wave, from (22), (28), and (40), we have

$$\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') = \begin{bmatrix} \partial_x\partial_z\partial_{y'} & -\partial_x\partial_z\partial_{x'} & 0 \\ \partial_y\partial_z\partial_{y'} & -\partial_y\partial_z\partial_{x'} & 0 \\ k_\rho^2\partial_{y'} & -k_\rho^2\partial_{x'} & 0 \end{bmatrix} g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') \quad (43)$$

where

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}\right]_{xx} = -\frac{i}{2\rho}\sin 2\phi S_1\left\{\frac{\partial_z F^{\mathrm{TE}}}{k_{mz}k_\rho^2}\right\} + \frac{i}{4}\sin 2\phi S_0\left\{\frac{\partial_z F^{\mathrm{TE}}}{k_{mz}}\right\} \quad (44)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}\right]_{xy} = \frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{\partial_z F^{\mathrm{TE}}}{k_{mz}k_\rho^2}\right\} - \frac{i}{4}(1+\cos 2\phi)S_0\left\{\frac{\partial_z F^{\mathrm{TE}}}{k_{mz}}\right\} \quad (45)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}\right]_{yx} = \frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{\partial_z F^{\mathrm{TE}}}{k_{mz}k_\rho^2}\right\} + \frac{i}{4}(1-\cos 2\phi)S_0\left\{\frac{\partial_z F^{\mathrm{TE}}}{k_{mz}}\right\} \quad (46)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}\right]_{yy} = -\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}\right]_{xx} \quad (47)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}\right]_{zx} = \frac{i}{2}\sin\phi S_1\left\{\frac{F^{\mathrm{TE}}}{k_{mz}}\right\} \quad (48)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TE}}\right]_{zy} = -\frac{i}{2}\cos\phi S_1\left\{\frac{F^{\mathrm{TE}}}{k_{mz}}\right\}. \quad (49)$$

Similarly, for the TM wave, we have

$$\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') = \begin{bmatrix} \partial_y\partial_{x'}\partial_{z'} & \partial_y\partial_{y'}\partial_{z'} & \partial_y k_\rho^2 \\ -\partial_x\partial_{x'}\partial_{z'} & -\partial_x\partial_{y'}\partial_{z'} & -\partial_x k_\rho^2 \\ 0 & 0 & 0 \end{bmatrix} \cdot k_n^2 g^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') \quad (50)$$

where

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}\right]_{xx} = -\frac{i}{2\rho}\sin 2\phi S_1\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}k_\rho^2}\right\}k_n^2 + \frac{i}{4}\sin 2\phi S_0\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}}\right\}k_n^2 \quad (51)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}\right]_{xy} = \frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}k_\rho^2}\right\}k_n^2 + \frac{i}{4}(1-\cos 2\phi)S_0\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}}\right\}k_n^2 \quad (52)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}\right]_{xz} = -\frac{i}{2}\sin\phi S_1\left\{\frac{F^{\mathrm{TM}}}{k_{mz}}\right\}k_n^2 \quad (53)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}\right]_{yx} = \frac{i}{2\rho}\cos 2\phi S_1\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}k_\rho^2}\right\}k_n^2 - \frac{i}{4}(1+\cos 2\phi)S_0\left\{\frac{\partial_{z'} F^{\mathrm{TM}}}{k_{mz}}\right\}k_n^2 \quad (54)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}\right]_{yy} = -\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}\right]_{xx} \quad (55)$$

$$\left[\nabla\times\bar{\mathbf{G}}_e^{\mathrm{TM}}\right]_{yz} = \frac{i}{2}\cos\phi S_1\left\{\frac{F^{\mathrm{TM}}}{k_{mz}}\right\}k_n^2. \quad (56)$$

There are 5 Sommerfeld integrals in the expression of $\nabla\times\bar{\mathbf{G}}_e$.

Due to the natural decomposition of TE and TM waves, several rows and columns of the dyads in the above expressions are zeros. The zero in the 3rd row of (23) is because E_z of TE waves is always zero, regardless whether it is excited by J_x, J_y, or J_z. Due to reciprocity, the 3rd column is also zero. Since the vertical electric dipole can only generate TM waves, J_z does not contribute to the TE waves in (43), hence the 3rd column is zero. In (50), again by the definition of TM waves, H_z is always zero, hence the 3rd row is zero.

To validate the dyadic form of the Green's function derived in this section, an electric dipole radiating in a seven-layer medium is investigated. The layered medium is shown in Fig. 2, and the working frequency is $f = 300$ MHz. The layered medium is both dielectric and magnetic. The source is at $(x = 0, y =$

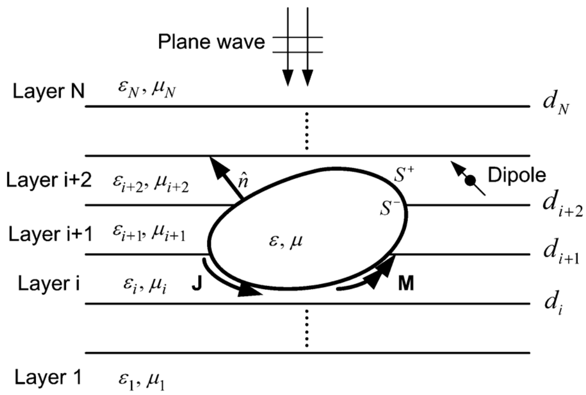


Fig. 1. A homogeneous (dielectric or magnetic) object is embedded in a layered medium. The external excitation is either a plane wave or a Hertzian dipole. Equivalent electric and magnetic currents are induced on the boundary and then generate the scattered field.

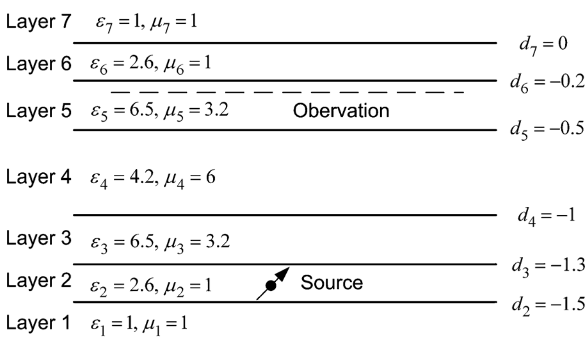


Fig. 2. An electric or magnetic dipole is radiating in a seven-layer medium (unit: m). The layered medium is both dielectric and magnetic, and the layer constants are shown in the figure.

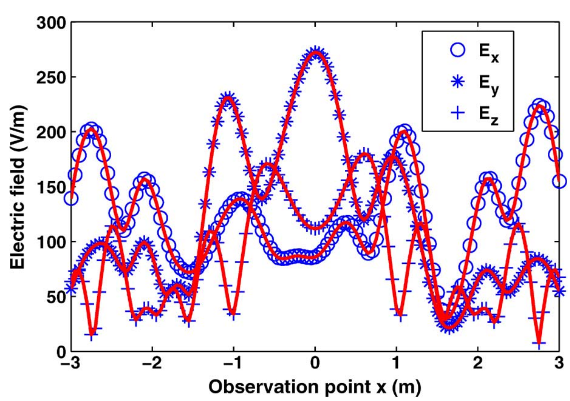


Fig. 3. The magnitude of electric field generated by an electric dipole. The result is validated by FEKO (solid line). The normalized root-mean-square deviation for E_x, E_y and E_z are 1.77×10^{-4}, 1.97×10^{-4}, and 3.82×10^{-4}.

$0, z = -1.4)$ m and the observation line is at $(-3 \leq x \leq 3, y = 1, z = -0.3)$ m. The polarization of the dipole is $(\theta = 20°, \phi = 30°)$. The electric field of an electric dipole is first evaluated, where $\bar{\mathbf{G}}_e$ is involved. The result is validated by FEKO and is shown in Fig. 3. The normalized root-mean-square deviation for E_x, E_y and E_z are 1.77×10^{-4}, 1.97×10^{-4}, and 3.82×10^{-4}. To further test the magnetic-type Green's function, the electric field of a magnetic dipole is also calculated, where $\nabla \times \bar{\mathbf{G}}_m$ is involved. The result is shown in Fig. 4 and again validated by FEKO. The normalized root-mean-square deviation for E_x, E_y and E_z are 8.21×10^{-5}, 2.12×10^{-4}, and 7.14×10^{-5}.

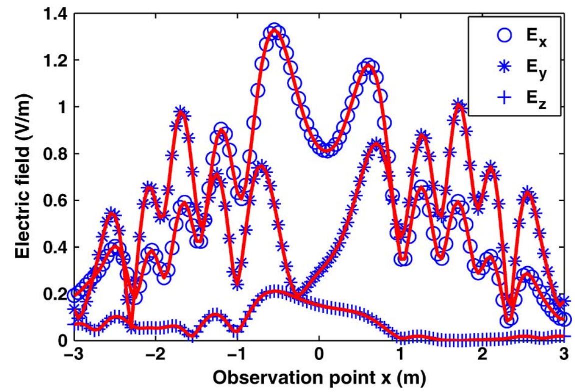


Fig. 4. The magnitude of electric field generated by a magnetic dipole. The result is validated by FEKO (solid line). The normalized root-mean-square deviation for E_x, E_y and E_z are 8.21×10^{-5}, 2.12×10^{-4}, and 7.14×10^{-5}.

III. Matrix Representation of Layered Medium Green's Function

The dyadic form of the Green's function in the last section can map the current source to the field directly. However, the singularity of the Green's function is high in spatial domain due to the spatial derivatives, or it has high frequency component in the spectral domain, which leads to slow convergence of the Sommerfeld integral. Usually, in moment method implementation [26], the mixed potential form of the integral equation (MPIE) is applied, where the Green's function has lower singularity. As has already been mentioned in the introduction, the scalar potential in layered medium is ambiguous, so careful manipulation should be made to obtain a proper MPIE. Due to this fact, a matrix-friendly formulation for PEC objects without defining any artificial potential is developed in [24]. In this formulation, integration by parts is applied wherever possible to transfer the partial derivative from the Green's function to the basis functions, where the Rao-Wilton-Glisson (RWG) basis [30] is applied as both expansion and testing functions. Hence, the singularity of the Green's function in spatial domain can be made as weak as possible. In other words, the Sommerfeld integral can be made rapidly convergent.

In this section, we will extend the matrix representation for general homogeneous (dielectric or magnetic) objects. Again we only discuss the $\mathcal{L}_{\mathrm{E}}$ and $\mathcal{K}_{\mathrm{H}}$ where $\bar{\mathbf{G}}_e$ and $\nabla \times \bar{\mathbf{G}}_e$ are involved since the $\mathcal{L}_{\mathrm{H}}$ and $\mathcal{K}_{\mathrm{E}}$ can be easily obtained by the duality principle of the Green's function shown in the Appendix.

A. Matrix Representation of Primary Term

The primary term is from the direct interaction in a homogeneous environment, hence the conventional MPIE form can be applied since the auxiliary potentials can be easily determined if the Lorentz gauge is utilized [30]. The matrix representation is

$$\begin{aligned}\langle \mathbf{f}_j(\mathbf{r}), \mathcal{L}_{\mathrm{E}}(\mathbf{r}, \mathbf{r}'), \mathbf{f}_i(\mathbf{r}') \rangle &= i\omega\mu_m \left\langle \mathbf{f}_j(\mathbf{r}), \bar{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}'), \mathbf{f}_i(\mathbf{r}') \right\rangle \\ &= i\omega\mu_m \left\langle \mathbf{f}_j(\mathbf{r}), g(\mathbf{r}, \mathbf{r}'), \mathbf{f}_i(\mathbf{r}') \right\rangle \\ &\quad + \frac{1}{i\omega\epsilon_m} \left\langle \nabla \cdot \mathbf{f}_j(\mathbf{r}), g(\mathbf{r}, \mathbf{r}'), \nabla' \cdot \mathbf{f}_i(\mathbf{r}') \right\rangle \end{aligned} \quad (57)$$

$$\begin{aligned}&\langle \mathbf{f}_j(\mathbf{r}), \mathcal{K}_{\rm H}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\rangle\\&\quad=\frac{\mu_m}{\mu_n}\left\langle \mathbf{f}_j(\mathbf{r}), \nabla\times\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\right\rangle\\&\quad=\frac{\mu_m}{\mu_n}\left\langle \mathbf{f}_j(\mathbf{r}), \nabla g(\mathbf{r},\mathbf{r}')\times\bar{\mathbf{I}}, \mathbf{f}_i(\mathbf{r}')\right\rangle\end{aligned}\tag{58}$$

where the inner product is defined as $\langle \mathbf{f}(\mathbf{r}), \mathbf{X}(\mathbf{r})\rangle = \int_s d\mathbf{r}\mathbf{f}(\mathbf{r})\cdot\mathbf{X}(\mathbf{r})$ for real function $\mathbf{f}(\mathbf{r})$. Also $\mathbf{f}_j(\mathbf{r})$ is the j-th testing function and $\mathbf{f}_i(\mathbf{r}')$ is the i-th basis function, $g(\mathbf{r},\mathbf{r}')$ is the scalar homogeneous medium Green's function shown in (8).

B. Matrix Representation of Secondary Term

For the secondary term, the matrix representation of the $\mathcal{L}_{\rm E}$ operator is [24]

$$\begin{aligned}&\langle \mathbf{f}_j(\mathbf{r}), \mathcal{L}_{\rm E}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\rangle\\&\quad= i\omega\mu_m\langle \mathbf{f}_{js}(\mathbf{r}), g_{e,ss}(\mathbf{r},\mathbf{r}'), \mathbf{f}_{is}(\mathbf{r}')\rangle\\&\qquad+i\omega\mu_m\langle \hat{z}\cdot\mathbf{f}_j(\mathbf{r}), g_{e,zz}(\mathbf{r},\mathbf{r}'), \hat{z}\cdot\mathbf{f}_i(\mathbf{r}')\rangle\\&\qquad+i\omega\mu_m\langle \hat{z}\cdot\mathbf{f}_j(\mathbf{r}), g_{e,zd}(\mathbf{r},\mathbf{r}'), \nabla'\cdot\mathbf{f}_i(\mathbf{r}')\rangle\\&\qquad+i\omega\mu_m\langle \nabla\cdot\mathbf{f}_j(\mathbf{r}), g_{e,dz}(\mathbf{r},\mathbf{r}'), \hat{z}\cdot\mathbf{f}_i(\mathbf{r}')\rangle\\&\qquad+i\omega\mu_m\langle \nabla\cdot\mathbf{f}_j(\mathbf{r}), g_{e,dd}(\mathbf{r},\mathbf{r}'), \nabla'\cdot\mathbf{f}_i(\mathbf{r}')\rangle\end{aligned}\tag{59}$$

where $\mathbf{f}_s = -\hat{z}\times\hat{z}\times\mathbf{f}$ is the horizontal projection of the basis function and

$$g_{e,ss} = \frac{i}{2}S_0\left\{\frac{F^{\rm TE}}{k_{mz}}\right\}\tag{60}$$

$$g_{e,zz} = \frac{i}{2}S_0\left\{\left(-\partial_z\partial_{z'}F^{\rm TE}+k_{mn}^2F^{\rm TM}\right)\frac{1}{k_{mz}k_\rho^2}\right\}\tag{61}$$

$$g_{e,zd} = \frac{i}{2}S_0\left\{\left(-\partial_zF^{\rm TE}-\frac{\mu_n}{\mu_m}\partial_{z'}F^{\rm TM}\right)\frac{1}{k_{mz}k_\rho^2}\right\}\tag{62}$$

$$g_{e,dz} = \frac{i}{2}S_0\left\{\left(-\partial_{z'}F^{\rm TE}-\frac{\epsilon_m}{\epsilon_n}\partial_zF^{\rm TM}\right)\frac{1}{k_{mz}k_\rho^2}\right\}\tag{63}$$

$$g_{e,dd} = \frac{i}{2}S_0\left\{\left(-F^{\rm TE}+\frac{\partial_z\partial_{z'}}{k_{nm}^2}F^{\rm TM}\right)\frac{1}{k_{mz}k_\rho^2}\right\}.\tag{64}$$

One should note that for RWG basis function straddling the interface of two adjacent layers, the coefficient μ_m should be modified accordingly. Since in real implementation the matrix element is accounted for by half RWG-half RWG (triangle-triangle) interactions, the expression in (59) is convenient and shall not cause confusion. The derivation details can be found in [24] and will not be repeated here.

We pay more attention to the matrix representation of the $\mathcal{K}_H$ operator with the kernel of $\nabla\times\bar{\mathbf{G}}_e$ here. For the TE wave, from (40), (42), we have

$$\nabla\times\bar{\mathbf{G}}_e^{\rm TE}(\mathbf{r},\mathbf{r}') = \left(\nabla\nabla\cdot\hat{z}+k_n^2\hat{z}\right)(\nabla'\times\hat{z})g^{\rm TE}(\mathbf{r},\mathbf{r}').\tag{65}$$

It can be divided into two parts and the matrix representation can be obtained by applying integration by parts

$$\begin{aligned}&\left\langle \mathbf{f}_j(\mathbf{r}), \nabla\times\bar{\mathbf{G}}_e^{\rm TE}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\right\rangle\\&\quad=-\left\langle \nabla\cdot\mathbf{f}_j(\mathbf{r}), \partial_z\nabla'\times\hat{z}g^{\rm TE}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\right\rangle\\&\qquad+\int_c dl\mathbf{f}_j(\mathbf{r})\cdot\hat{n}\left\langle\partial_z\nabla'\times\hat{z}g^{\rm TE}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\right\rangle\\&\qquad+\left\langle \hat{z}\cdot\mathbf{f}_j(\mathbf{r}), k_n^2\nabla'\times\hat{z}g^{\rm TE}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\right\rangle\end{aligned}\tag{66}$$

where contour c is the inner edge of the RWG basis function and $\hat{n}$ is the unit vector normal to this edge in the plane of the triangle.

Similarly from (41), we have

$$\nabla\times\bar{\mathbf{G}}_e^{\rm TM}(\mathbf{r},\mathbf{r}') = k_n^2(\nabla\times\hat{z})\left(\nabla'\nabla'\cdot\hat{z}+k_m^2\hat{z}\right)g^{\rm TM}(\mathbf{r},\mathbf{r}')\tag{67}$$

then the matrix representation is

$$\begin{aligned}&\left\langle \mathbf{f}_j(\mathbf{r}), \nabla\times\bar{\mathbf{G}}_e^{\rm TM}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\right\rangle\\&\quad=-\left\langle \mathbf{f}_j(\mathbf{r}), k_n^2\nabla\times\hat{z}\partial_{z'}g^{\rm TM}(\mathbf{r},\mathbf{r}'), \nabla'\cdot\mathbf{f}_i(\mathbf{r}')\right\rangle\\&\qquad+\left\langle \mathbf{f}_j(\mathbf{r}), \int_{c'} dl'k_n^2\nabla\times\hat{z}\partial_{z'}g^{\rm TM}(\mathbf{r},\mathbf{r}')\mathbf{f}_i(\mathbf{r}')\cdot\hat{n}'\right\rangle\\&\qquad+\left\langle \mathbf{f}_j(\mathbf{r}), k_n^2k_m^2\nabla\times\hat{z}g^{\rm TM}(\mathbf{r},\mathbf{r}'), \hat{z}\cdot\mathbf{f}_i(\mathbf{r}')\right\rangle.\end{aligned}\tag{68}$$

The curl operator can be approximated by the finite difference method in spatial domain. For accurate evaluation, the curl can also be implemented in the spectral domain, as is done in Section II. In summary, from (4), (22), (66), and (68), the matrix representation of the $\mathcal{K}_{\rm H}$ operator is

$$\begin{aligned}&\langle \mathbf{f}_j(\mathbf{r}), \mathcal{K}_{\rm H}(\mathbf{r},\mathbf{r}'), \mathbf{f}_i(\mathbf{r}')\rangle\\&\quad=\frac{\mu_m}{\mu_n}\langle \nabla\cdot\mathbf{f}_j(\mathbf{r}), \mathbf{g}_{ce,ds}(\mathbf{r},\mathbf{r}'), \mathbf{f}_{is}(\mathbf{r}')\rangle\\&\qquad+\frac{\mu_m}{\mu_n}\int_c dl\mathbf{f}_j(\mathbf{r})\cdot\hat{n}\langle -\mathbf{g}_{ce,ds}(\mathbf{r},\mathbf{r}'), \mathbf{f}_{is}(\mathbf{r}')\rangle\\&\qquad+\frac{\mu_m}{\mu_n}\langle \hat{z}\cdot\mathbf{f}_j(\mathbf{r}), \mathbf{g}_{ce,zs}(\mathbf{r},\mathbf{r}'), \mathbf{f}_{is}(\mathbf{r}')\rangle\\&\qquad+\frac{\mu_m}{\mu_n}\langle \mathbf{f}_{js}(\mathbf{r}), \mathbf{g}_{ce,sd}(\mathbf{r},\mathbf{r}'), \nabla'\cdot\mathbf{f}_i(\mathbf{r}')\rangle\\&\qquad+\frac{\mu_m}{\mu_n}\left\langle \mathbf{f}_{js}(\mathbf{r}), -\int_{c'} dl'\mathbf{g}_{ce,sd}(\mathbf{r},\mathbf{r}')\mathbf{f}_i(\mathbf{r}')\cdot\hat{n}'\right\rangle\\&\qquad+\frac{\mu_m}{\mu_n}\langle \mathbf{f}_{js}(\mathbf{r}), \mathbf{g}_{ce,sz}(\mathbf{r},\mathbf{r}'), \hat{z}\cdot\mathbf{f}_i(\mathbf{r}')\rangle\end{aligned}\tag{69}$$

where

$$\mathbf{g}_{ce,ds} = \begin{bmatrix}-\sin\phi\\ \cos\phi\end{bmatrix}\frac{i}{2}S_1\left\{\frac{\partial_zF^{\rm TE}}{k_{mz}k_\rho^2}\right\}\tag{70}$$

$$\mathbf{g}_{ce,zs} = k_n^2\begin{bmatrix}\sin\phi\\ -\cos\phi\end{bmatrix}\frac{i}{2}S_1\left\{\frac{F^{\rm TE}}{k_{mz}k_\rho^2}\right\}\tag{71}$$

$$\mathbf{g}_{ce,sd} = \frac{\mu_n}{\mu_m}\begin{bmatrix}\sin\phi\\ -\cos\phi\end{bmatrix}\frac{i}{2}S_1\left\{\frac{\partial_{z'}F^{\rm TM}}{k_{mz}k_\rho^2}\right\}\tag{72}$$

$$\mathbf{g}_{ce,sz} = k_{mn}^2\begin{bmatrix}-\sin\phi\\ \cos\phi\end{bmatrix}\frac{i}{2}S_1\left\{\frac{F^{\rm TM}}{k_{mz}k_\rho^2}\right\}\tag{73}$$

where $\mathbf{g}_{ce}$ is a 2D vector (x and y components).

C. Line Integral Issue

Although the line integral is absent in MPIE in homogeneous environment (namely only the matrix representation of the primary term is involved), it is always an issue in layered medium when the object is penetrating more than one layer and the RWG basis defined on two adjacent triangles has to straddle the interface of two layers. However, if the kernel is continuous across the interface, such line integral from positive and negative triangles of a RWG basis function can be canceled.

In fact, this is usually the case. The continuity of the kernel can be verified by applying the continuity of the propagation factor $F^{\alpha}(z,z')$ in (16). When the observation point $\mathbf{r}$ comes across the interface, we have [1]

$$F_1^{\alpha}(z_+,z') = F_2^{\alpha}(z_-,z') \tag{74}$$

$$p_1^{-1}(z_+)\partial_z F_1^{\alpha}(z_+,z') = p_2^{-1}(z_-)\partial_z F_2^{\alpha}(z_-,z') \tag{75}$$

where $p=\mu$ for TE wave and $p=\epsilon$ for TM wave. To analyze the source point $\mathbf{r}'$, the symmetry relation of the $F^{\alpha}(z,z')$ can be applied [31]

$$\frac{p(z')}{k_z(z')}F(z,z') = \frac{p(z)}{k_z(z)}F(z',z). \tag{76}$$

Hence we have the following relations

$$\frac{p_1\left(z'_+\right)}{k_{1z}\left(z'_+\right)}F_1^{\alpha}\left(z,z'_+\right) = \frac{p_2\left(z'_-\right)}{k_{2z}\left(z'_-\right)}F_2^{\alpha}\left(z,z'_-\right) \tag{77}$$

$$k_{1z}^{-1}\left(z'_+\right)\partial_{z'}F_1^{\alpha}\left(z,z'_+\right) = k_{2z}^{-1}\left(z'_-\right)\partial_{z'}F_2^{\alpha}\left(z,z'_-\right). \tag{78}$$

The continuity of the kernel of the $\mathcal{L}_{\mathrm{E}}$ in (59) is shown in [25]. The one of $\mathcal{K}_{\mathrm{H}}$ in (69) can be verified similarly since the horizontal partial derivatives $(\nabla(')\times\hat{z})$ in (66) and (68) do not affect the vertical continuity due to the phase matching condition.

D. Consistency Analysis

However, the propagation factor shown in (74)–(78) contains all fields information (including the primary field if necessary), the continuity is valid only when all the fields are expressed as the Sommerfeld integral of the propagation factor. Since we have separated the primary term in closed-form to avoid ill-convergent self-term Sommerfeld integral, and also to gain the convenience of singularity treatment, we have to make sure that the matrix representation of the two terms are consistent before we can surely remove the line integral. By setting the basis function and the testing function in different layers, and then making the layers to be homogeneous, we can easily test the consistency. The secondary term is consistent with the primary term in the matrix representation of $\mathcal{L}_{\mathrm{E}}$ [28], hence there is always no line integral in (59). However, for the $\mathcal{K}_{\mathrm{H}}$, we find that the secondary term is not consistent with the primary term. In the primary term, the "∇" operator acts directly on the Green's function in (58), while in the secondary term, it is transferred to either testing or basis function in (69). This "operator transfer" reduces the singularity of the Green's function; however, it also destroys the continuity of the propagation factor and leads to non-canceled line integral for interactions where the primary term is involved. For the testing line integral (the first line integral in (69)), it needs to be calculated only when the testing RWG function straddles the interface and the basis RWG functions or half-RWGs (either positive or negative triangles) are in the same two layers shown in Fig. 5. The necessity of basis line integral (the second line integral in (69)) can be analyzed similarly. Since we are filling the matrix elements by triangle-triangle pairs other than RWG-RWG pairs, the line integral can be easily judged and added if necessary, and also the number of Sommerfeld integrals can be reduced to minimum.

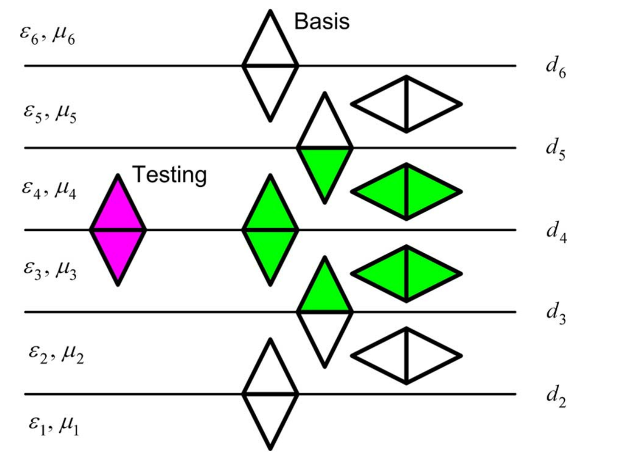
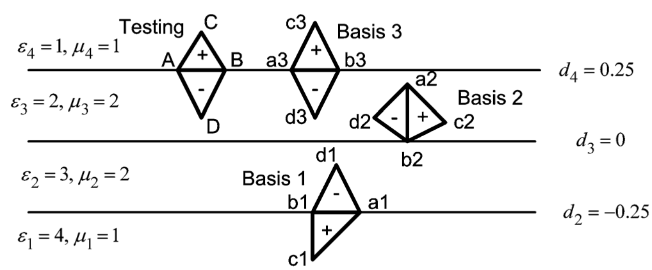


Fig. 5. Cases where testing line integral exists. The testing function is straddling the interface, all radiation from the basis RWGs or half-RWGs (triangles) in color needs invoking testing line integral, while radiation from others does not need this line integral.



A: (-0.27,-0.34,0.25), B: (-0.19,-0.39,0.25), C: (-0.22,-0.32,0.32), D: (-0.20,-0.42,0.19)

a1: (0.19,-0.39,-0.25), b1: (0.10,-0.42,-0.25), c1: (0.11,-0.37,-0.32), d1: (0.18,-0.43,-0.19)

a2: (0.18,-0.43,0.19), b2: (0.09,-0.48,0.08), c2: (0.27,-0.42,0.08), d2: (-0.005,-0.46, 0.19)

a3: (0.00, -0.43,0.25), b3: (0.10,-0.42,0.25), c3: (0.05,-0.39,0.31), d3: (-0.005,-0.46,0.19)

Fig. 6. Line integral test. The testing function is at the top interface. Radiation from: basis 1: no line integral activated; basis 2: testing line integral activated; basis 3: both testing and basis line integrals activated (unit: m).

TABLE I
MATRIX ELEMENT OF $\mathcal{K}_{\mathrm{E}}$: NO LINE INTEGRAL ($\times 10^3$)

	Z_{++}	Z_{+-}	Z_{-+}	Z_{--}	Z
Matrix (real)	9.662	-5.147	1.080	-9.133	-3.538
Matrix (imag)	0.623	0.189	-4.348	0.967	-2.568
Dyadic (real)	-1.027	-0.756	-0.939	-0.816	-3.538
Dyadic (imag)	-1.118	-0.558	-0.577	-0.315	-2.568

To validate the matrix representation of $\mathcal{K}_{\mathrm{E}}$, we consider the interaction between one testing function and several basis functions shown in Fig. 6, the representative basis functions are picked from a mesh of a sphere embedded in a four-layer medium. The working frequency is $f = 300$ MHz. The results are validated by applying the dyadic form Green's function. The testing function straddles the interface, and three different basis functions are considered. Table I shows the radiation from basis 1, where no primary field is involved. Hence, no line integral is activated. We can observe that though the interactions between triangle-triangle pairs differ due to the line integral, it can be canceled and the final matrix element is the same. Table II shows the radiation from basis 2, in this case, the testing line

TABLE II
MATRIX ELEMENT OF $\mathcal{K}_\mathrm{E}$: TESTING LINE INTEGRAL ($\times 10^3$)

	Z_{++}	Z_{+-}	Z_{-+}	Z_{--}	Z
Matrix (real)	2.870	−1.521	2.103	−1.436	2.015
Matrix (imag)	3.868	−4.274	−0.712	−0.185	−1.303
Dyadic (real)	0.911	0.437	0.547	0.119	2.015
Dyadic (imag)	−0.054	−0.353	−0.512	−0.385	−1.303

TABLE III
MATRIX ELEMENT OF $\mathcal{K}_\mathrm{E}$: TESTING & BASIS LINE INTEGRALS ($\times 10^4$)

	Z_{++}	Z_{+-}	Z_{-+}	Z_{--}	Z
Matrix (real)	−1.406	0.063	−0.697	1.123	−0.917
Matrix (imag)	0.680	−2.765	3.379	0.131	1.426
Dyadic (real)	−1.406	0.063	−0.697	1.123	−0.917
Dyadic (imag)	0.680	−2.765	3.379	0.131	1.426

integral should be activated, and again the final matrix element agrees. Finally, the radiation from basis 3 is calculated and shown in Table III. Now both testing line integral and basis line integral need to be activated and the matrix representation exactly recovers the dyadic form.

IV. SURFACE INTEGRAL EQUATION

As the layered medium Green's function is formulated properly, we can set up the surface integral equation for general homogeneous objects by applying the extinction theorem [1]. For the object shown in Fig. 1, if we put $\mathbf{r} \in S^-$ and $\mathbf{r}' \in S^+$, we have

$$-\mathbf{E}^o_{\mathrm{inc}}(\mathbf{r}) = \mathcal{L}^o_\mathrm{E}(\mathbf{r},\mathbf{r}') \cdot \mathrm{J}(\mathbf{r}') + \mathcal{K}^o_\mathrm{E}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}(\mathbf{r}'). \quad (79)$$

Here we use superscript "o" to represent that the Green's function is evaluated in the outside region ("i" will be used for inside region). Similarly, if $\mathbf{r} \in S^+, \mathbf{r}' \in S^-$

$$0 = \mathcal{L}^i_\mathrm{E}(\mathbf{r},\mathbf{r}') \cdot [-\mathrm{J}(\mathbf{r}')] + \mathcal{K}^i_\mathrm{E}(\mathbf{r},\mathbf{r}') \cdot [-\mathbf{M}(\mathbf{r}')] \quad (80)$$

where the negative signs of the currents are due to the unique definition of the unit normal vector in one problem ($\mathbf{J} = \hat{n} \times \mathbf{H}$, $\mathbf{M} = \mathbf{E} \times \hat{n}$). One should note that for $\mathcal{L}^i$ and $\mathcal{K}^i$, the choice of the Green's function has some freedom. As long as the material and the boundary condition is maintained, the field can be uniquely determined. The Green's function may even not satisfy the radiation boundary condition (non-physical). However, for simplicity, we usually choose the homogeneous medium Green's function.

In practice, we may have internal resonance problems if we only use the above two E-type equations. To avoid it, the H-type equations are usually applied too. The H-field equations read

$$-\mathbf{H}^o_{\mathrm{inc}}(\mathbf{r}) = \mathcal{L}^o_\mathrm{H}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}(\mathbf{r}') + \mathcal{K}^o_\mathrm{H}(\mathbf{r},\mathbf{r}') \cdot \mathrm{J}(\mathbf{r}') \quad (81)$$

$$0 = \mathcal{L}^i_\mathrm{H}(\mathbf{r},\mathbf{r}') \cdot [-\mathbf{M}(\mathbf{r}')] + \mathcal{K}^i_\mathrm{H}(\mathbf{r},\mathbf{r}') \cdot [-\mathrm{J}(\mathbf{r}')]. \quad (82)$$

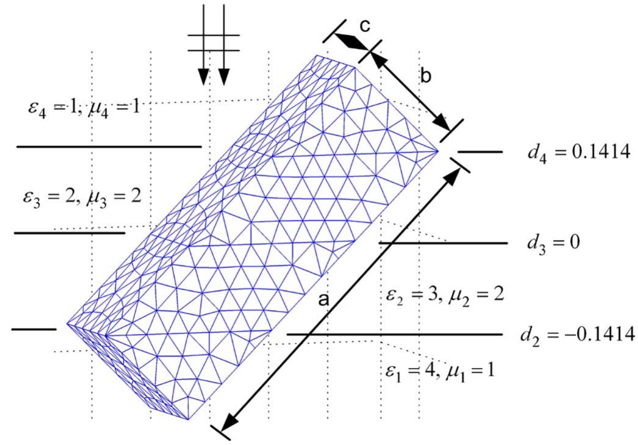


Fig. 7. A PEC cuboid embedded in a 4-layer medium, where $a = 0.6$, $b = c = 0.2$ (unit: m). It is penetrating different layers and is excited by a plane wave.

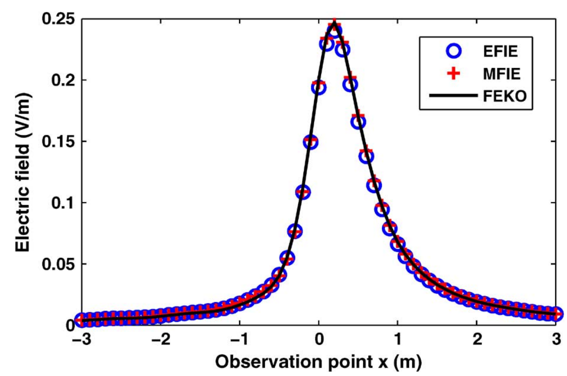


Fig. 8. The scattered field of the PEC cuboid, calculated from EFIE, MFIE, and FEKO.

The popular Poggio–Miller–Chang–Harrington–Wu–Tsai (PMCHWT) formula [32]–[34] is applied here though other alternatives are available in homogeneous medium [33]. Finally, the surface integral equation in matrix notation reads

$$\begin{bmatrix} -\mathbf{E}^o_{\mathrm{inc}} \\ -\mathbf{H}^o_{\mathrm{inc}} \end{bmatrix}\Bigg|_{\tan} = \begin{bmatrix} (\mathcal{L}^o_\mathrm{E} + \mathcal{L}^i_\mathrm{E}) & (\mathcal{K}^o_\mathrm{E} + \mathcal{K}^i_\mathrm{E}) \\ (\mathcal{K}^o_\mathrm{H} + \mathcal{K}^i_\mathrm{H}) & (\mathcal{L}^o_\mathrm{H} + \mathcal{L}^i_\mathrm{H}) \end{bmatrix} \cdot \begin{bmatrix} \mathrm{J} \\ \mathbf{M} \end{bmatrix}\Bigg|_{\tan}. \quad (83)$$

V. NUMERICAL RESULTS

Several numerical results are presented in this section. In order to validate the operators, a PEC cuboid penetrating different layers shown in Fig. 7 is first simulated. The cuboid is tilted by $\theta = 45^\circ$ and the dimension is $a = 0.6$ m, $b = c = 0.2$ m. It is illuminated by a TE plane wave with normal incidence ($f = 300$ MHz). The observation line is at ($-3 \le x \le 3$, $y = 0$, $z = 0.5$) m. Both EFIE and MFIE (magnetic field integral equation) are implemented to test the $\mathcal{L}_\mathrm{E}$ operator and $\mathcal{K}_\mathrm{H}$ operator. The scattered co-polarization field is computed and validated by FEKO, as is shown in Fig. 8. Next, a dielectric capsule structure ($\epsilon_r = 4$ and $\mu_r = 1$) which also penetrates the layered medium is simulated, as is shown in Fig. 9. The dimension of the object is $h = 0.25$ m, $r = 0.5$ m. It is excited by a Hertzian dipole at ($x = 0.5, y = 0, z = 0.9$) m

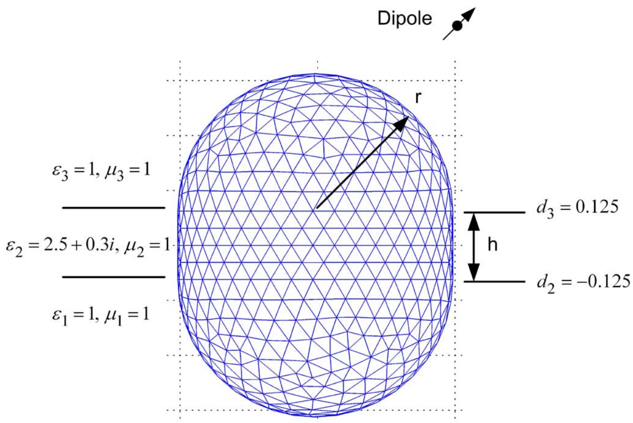


Fig. 9. A dielectric capsule structure with $\epsilon_r = 4$ and $\mu_r = 1$ embedded in a 3-layer medium, where $h = 0.25$, $r = 0.5$ (unit: m). It is penetrating different layers and is excited by a Hertzian dipole.

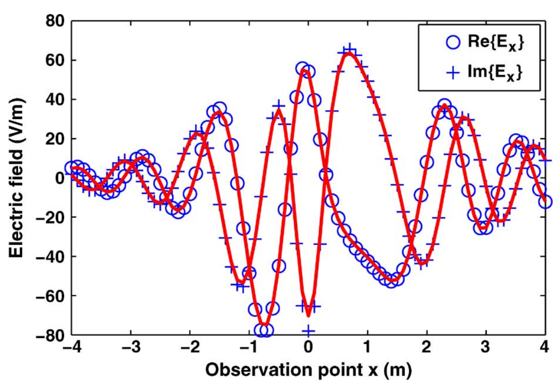


Fig. 10. The x component of the near electric field. The result is validated by FEKO with $10\lambda_0 \times 10\lambda_0$ truncated layered medium (solid line).

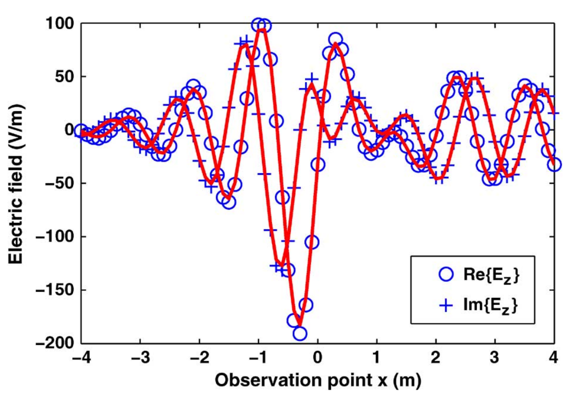


Fig. 11. The z component of the near electric field. The result is validated by FEKO with $10\lambda_0 \times 10\lambda_0$ truncated layered medium (solid line).

with polarization of ($\theta = 45°$, $\phi = 0°$). The working frequency is $f = 300$ MHz and the observation line is set to be at $(-4 \leq x \leq 4, y = 0, z = -0.9)$ m. The simulation is also validated by FEKO where the layered medium is truncated into a finite domain by a dimension of $10\lambda_0 \times 10\lambda_0$. The results are shown in Figs. 10 and 11, where for the imaginary part, a negative sign is adjusted manually due to different time convention.

VI. Conclusion

A new Green's function formulation for modeling homogeneous objects in layered medium is systematically developed in this paper. The dyadic form of the Green's function is developed by applying the pilot vector potential approach, and all components are derived in terms of 0-order and 1-order Sommerfeld integrals. The matrix representation is further derived by applying integration by parts and vector identities. The line integral issue for interactions between straddling basis functions is discussed in details for different situations, based on the continuity property of the propagation factor and the consistency analysis of the primary term and the secondary term. The extinction theorem is revisited in inhomogeneous environment and the relevant integral equations are set up. Several numerical results are presented to validate this formulation. The duality principle of this Green's function is further discussed to make the formulation succinct.

Appendix
Duality Principle of Layered Medium Green's Function

The magnetic type Green's function $\bar{\mathbf{G}}_m$ and $\nabla \times \bar{\mathbf{G}}_m$ in (5) and (6) can be obtained from the duality principle. One form of the duality principle reads [1],

$$\begin{aligned} &\mathbf{E} \rightarrow \mathbf{H}, \mathbf{H} \rightarrow \mathbf{E}, \mu \rightarrow -\epsilon, \epsilon \rightarrow -\mu \\ &\mathbf{M} \rightarrow -\mathrm{J}, \mathrm{J} \rightarrow -\mathbf{M}, \rho_m \rightarrow -\rho, \rho \rightarrow -\rho_m. \end{aligned} \tag{84}$$

A. Primary Term of the Green's Function

Since the scalar homogeneous medium Green's function in (8) satisfies the scalar Helmholtz equation

$$(\nabla^2 + k_m^2) g(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r}, \mathbf{r}') \tag{85}$$

we can easily show that

$$g_m(\mathbf{r}, \mathbf{r}') = g_e(\mathbf{r}, \mathbf{r}') = g(\mathbf{r}, \mathbf{r}') \tag{86}$$

hence

$$\bar{\mathbf{G}}_m(\mathbf{r}, \mathbf{r}') = \bar{\mathbf{G}}_e(\mathbf{r}, \mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k_m^2} \right) g(\mathbf{r}, \mathbf{r}'). \tag{87}$$

B. Secondary Term of the Green's Function

The propagation factor $F^\alpha(z, z')$ of the secondary term satisfies the following differential equation [19]

$$\left[\frac{d}{dz} \frac{1}{p(z)} \frac{d}{dz} + \frac{1}{p(z)} k_z^2(z) \right] F^\alpha(z, z') = 0. \tag{88}$$

It is obvious that

$$F^{\mathrm{TE}}(z, z') \rightarrow F^{\mathrm{TM}}(z, z'), F^{\mathrm{TM}}(z, z') \rightarrow F^{\mathrm{TE}}(z, z') \tag{89}$$

hence,

$$g^{\mathrm{TE}}(\mathbf{r}, \mathbf{r}') \rightarrow g^{\mathrm{TM}}(\mathbf{r}, \mathbf{r}'), g^{\mathrm{TM}}(\mathbf{r}, \mathbf{r}') \rightarrow g^{\mathrm{TE}}(\mathbf{r}, \mathbf{r}'). \tag{90}$$

Finally, we have

$$\bar{\mathbf{G}}_m(\mathbf{r},\mathbf{r}') = \bar{\mathbf{G}}_m^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') + \frac{1}{k_{mn}^2}\bar{\mathbf{G}}_m^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') \tag{91}$$

where

$$\bar{\mathbf{G}}_m^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') = (\nabla\times\hat{z})(\nabla'\times\hat{z})g^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') \tag{92}$$

$$\bar{\mathbf{G}}_m^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') = (\nabla\times\nabla\times\hat{z})(\nabla'\times\nabla'\times\hat{z})\,g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}'). \tag{93}$$

Hence,

$$\bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') \to \bar{\mathbf{G}}_m(\mathbf{r},\mathbf{r}'), \bar{\mathbf{G}}_m(\mathbf{r},\mathbf{r}') \to \bar{\mathbf{G}}_e(\mathbf{r},\mathbf{r}') \tag{94}$$

and similar duality holds between $\nabla\times\bar{\mathbf{G}}_e$ and $\nabla\times\bar{\mathbf{G}}_m$.

With these properties, we can now easily verify the correctness of definition of the $\mathcal{L}_{\mathrm{H}}$ and $\mathcal{K}_{\mathrm{E}}$ in (5) and (6).

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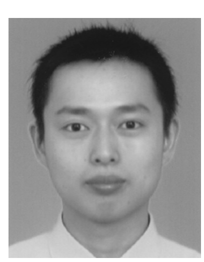

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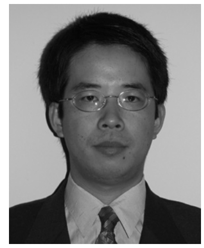

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