Title	On Alexander Wylie's Jottings on the Science of the Chinese Arithmetic
Author(s)	Siu, MK; Chan, Y C
Citation	History and Pedagogy of Mathematics (HPM) Satellite Meeting of International Congress on Mathematical Education (ICME), Daejeon, Korea, 16-20 July 2012. In Proceedings of HPM, 2012, p. 229-237
Issued Date	2012
URL	http://hdl.handle.net/10722/177469
Rights	Creative Commons: Attribution 3.0 Hong Kong License

ON ALEXANDER WYLIE'S JOTTINGS ON THE SCIENCE OF THE CHINESE ARITHMETIC

Man-Keung SIU[*], Yip-Cheung CHAN[**]

[*] The University of Hong Kong, Pofulam Road, Hong Kong, CHINA
mathsiu@hkucc.hku.hk
[**] The Chinese University of Hong Kong, Shatin, Hong Kong, CHINA
mathchan@cuhk.edu.hk

ABSTRACT

Starting from August of 1852 the British Protestant missionary and sinologist, Alexander Wylie (1815–1887), published in nine instalments an account *Jottings on the Science of the Chinese Arithmetic* in the newspaper *North China Herald*. He explained clearly the purpose of his account at the beginning:

> "The object of the following desultory notes, made from time to time, in the course of some researches entered upon, with another purpose in view, is to draw attention to the state of the arithmetical science in China, a subject which has not been so fully explored as it might with advantage, and on which some erroneous statements have been current in modern publications."

Alexander Wylie is a well-known figure in the last quarter of the Qing Dynasty for his contribution in transmitting Western science into China during the latter half of the 19th century. In mathematics he was known for translating three treatises in collaboration with the Qing mathematician Li Shanlan (1811–1882) — *Supplementary Elements of Geometry* in 1856 but published in 1865 (believed to be based on the English translation of Book VII to XV of *Elements* by Henry Billingsley in 1570), *Treatise of Algebra* in 1859 (based on *Elements of Algebra* by Agustus De Morgan in 1835) and *Analytical Geometry and Differential and Integral Calculus Step by Step* in 1859 (based on *Elements of Analytical Geometry and of the Differential and Integral Calculus* of Elias Loomis in 1850). He was also the author of *Compendium of Arithmetic* published in 1853.

This presentation will discuss the knowledge of Chinese science and mathematics which most European sinologists of the 18th and 19th centuries possessed and the low regard they held it in, but the viewpoint of which was critically examined by Wylie in his account.

Keywords: Alexander Wylie, Chinese mathematics, arithmetic, algebra

1 Introduction

Alexander Wylie (1815–1887) was a Protestant missionary of the London Missionary Society and later an agent of the British and Foreign Bible Society in China. He was sent to China by the London Missionary Society in 1847. His contribution was not only on spreading Christian faith to China, but

[*] First Author

perhaps more importantly on the intellectual exchange of scientific and mathematical knowledge between China and Western countries. He was well-known in transmitting Western science and mathematics into China by publishing and translating scientific books (in collaboration with Li Shan-lan李善蘭) such as *Compendium of Arithmetic* (Wylie, 1853), *Supplementary Elements of Geometry* (Wylie and Li, 1865), *Treatise of Algebra* (Wylie and Li, 1859a), *Analytical Geometry and Differential and Integral Calculus Step by Step* (Wylie and Li, 1859b). On the other hand, he was also a sinologist who brought Chinese literature, philosophy, science and mathematics to the Western world especially Britain. During his 30 years of stay in China, Wylie collected many Chinese books in different disciplines. He published *Notes on Chinese Literature* (Wylie, 1867) which provides a bibliography with detailed explanatory notes on Chinese books. He had also written a number of articles related to China which were published in newspapers and periodicals. His colleague James Thomas selected some of these articles and edited as *Chinese Researches* (Wylie, 1897). These two books provided valuable sources for Westerners in the 19th century to know about China (in a new light).

In an accompanying workshop by the same authors, titled " *Chinese Arithmetic in the Eyes of a British Missionary and Calculus in the Eyes of a Chinese Mathematician*", we will focus on Wylie's introduction of (Western) algebra and calculus into China. The present paper supplements and complements this workshop. We will focus on how Wylie introduced Chinese mathematics to his own country. In particular, we will discuss a series of newspaper articles titled *Jottings on the Science of the Chinese Arithmetic*[1], which were first published in nine instalments from August to November of 1852 in *North China Herald* and later reprinted in *Chinese Researches* (Wylie, 1897, pp. 159–194). This series of articles played a pioneering role in the study of the history of Chinese mathematics in the Western world. It may be the only reliable (Western) source on the history of Chinese mathematics before the publication of Yoshio Mikami's *The Development of Mathematics in China and Japan* in 1913 (Wang, 1999). Dauben (2000) gives the following comment on *Jottings:*

> "This article is the first in English to give a reliable account, for the most part, of Chinese mathematics ⋯. Given the pioneering nature of this work, it is not surprising that it contains various errors and inaccuracies ⋯. Nevertheless, the "Jottings" is an important work for the history of Chinese mathematics, and was to have a significant influence upon such prominent historians of mathematics as Moritz Cantor, Florian Cajori, and David E. Smith." (Dauben, 2000, p. 781–782)

According to Wylie, the objective of this series of articles is to clarify some erroneous statements about the status of mathematics in China that were found in (Western) publications at his time. He explained clearly this purpose at the beginning:

> "The object of the following desultory notes, made from time to time, in the course of some researches entered upon, with another purpose in view, is to draw attention to the state of the arithmetical science in China, a subject which has not been so fully explored as it might with advantage, and on which some erroneous statements have been current in modern publications."

In this presentation, we will first outline Westerners' common views at Wylie's time on the status of Chinese mathematics. Then, we will discuss how Wylie responded to these views in his *Jottings* and

[1] In subsequent discussion, we will use "*Jottings*" as the abbreviation for the article *Jottings on the Science of the Chinese Arithmetic*.

evaluate his viewpoints in the light of contemporary literature on history of Chinese mathematics. Finally, we will discuss the implication of this historical document to current mathematics education. Since this paper is to be submitted five months before the actual presentation so that the authors lack the opportunity of benefitting from comments and views of colleagues in the audience, the present record will focus mainly on Wylie's response to the erroneous statements, while its pedagogical implication will be discussed in more detailed during the actual presentation.

The Chinese terms in the text are written in a system adopted by Wylie in his writings, which is not exactly the (older) Wade-Giles system nor the (more modern) Pinyin system.

2 Common views of Westerners in the 19th Century about Chinese mathematics

In this section, we will give a brief account (with support from excerpts of source materials) on the common views of Westerners in the 19th Century about Chinese mathematics. It will provide background information to the subsequent discussions on Wylie's *Jottings*. We are indebted to Wang (2004) in directing us to some of these source materials.

Generally speaking, Westerners in the 19th Century thought that Chinese possessed only very limited mathematical knowledge which was far behind them (the Europeans). They also thought that mathematics was a neglected discipline to the Chinese. These views are evidenced from the following quotations:

- "The knowledge of mathematics even among learned men is very small, and the common people study it only as far as their business requires." (William, 1848, p. 147)
- "For their [Chinese] acquaintance with the exact sciences cannot for a moment bear comparison with that of Europeans." (Murray et al., 1836, p. 224)
- "It happens that men of genius neglect that kind of knowledge [knowledge of mathematics and astronomy], and pursue the more popular branches which lead to honour and emolument." (Murray et al., 1836, p. 225)

2.1 Contribution of Jesuit missionaries to Chinese mathematics

The prevalent Western view at the time was that Chinese books on mathematics were based on contribution from the Jesuit missionaries. For instance, William (1848) pointed out that the *Swan-fah Tung Tsung* (*General Comprehensive Arithmetic*) and the *Tsuimí-shan Fang Sho Hioh* (*Mathematics of the Lagerstraemia Hill Institution*) contained a lot of material from the mathematical writings of the Jesuit missionaries. Similarly, Davis claimed that:

> "In the science of numbers, and in geometry, the Chinese have, as usual, nothing to teach us; being, on the contrary, indebted for a good deal to Europe, as may be seen from the logarithmic tables and other works prepared for the Emperor Kâng-hy by the Jesuits." (Davis, 1851, p. 282)

However, the impact of Western mathematics transmitted by the Jesuits was small. For instance, Murray et al claimed that:

> "the progress which it had made in that country [China], when compared to the time it had been cultivated before the Jesuit missionaries obtained a footing among them, was extremely small.

> ⋯ it may be inferred, that there existed in that country no mathematics by which it could be improved." (Murray et al, 1836, p. 231)

2.2 Chinese numeric notation, arithmetic and algebra

According to the understanding of Westerners, "the [numeric] notation of the Chinese is based on the decimal principle, but their figures are not changed in value by position, and it is difficult therefore to write out clearly the solution of a question." (William, 1848, p. 146). William continued to explain that this was overcome, in arithmetical calculations, by the assistance of an abacus. However, he pointed out that its disadvantage is that "if an error be made, the whole must be performed again, since the result only appears when the sum is finished" (William, 1848, p. 146). Therefore, he concluded that: "This mode of notation ⋯ falls far behind the Arabic system now in general use in the west" (William, 1848, p. 146).

Other literature of Westerners in this period shared similar opinion. For instance, Murray et al. (1836) gave the following comments on the abacus and Chinese numeric notation and arithmetic:

> "It must, however, be admitted, that although this machine [the abacus] be well adapted for explaining the principles of arithmetic, it would be a very inadequate substitute for our Arabic numerals, more especially in those laborious calculations which the progress of European science has rendered indispensable. Sir George Staunton says, that the Chinese have no characters, except those in their common language, to express sums in an abbreviated form, after the manner of the Arabic figures used by Europeans. When, however, they have occasion to introduce numbers in their writings, they have recourse to their ordinary terms, each of which denotes a numerical value, independently of its relative position,—a method less tedious indeed than the expression of the same numbers by the method of alphabetical writing, but which by no means equals the conciseness of the same process in the Arabic notation. The universal multiplication and subdivision of all quantities by decimal proportions, facilitates their calculations, and prevents the necessity of methods to abridge them." (Murray et al., 1836, p. 228–229)

Davis (1851) not only repeated the above opinions, he even claimed that: "No algebraic knowledge is to be found in China" (Davis, 1851, p. 282). Unfortunately, this erroneous statement was rather popular among Westerners in that period. Indeed, in his *Jottings*, Wylie put quite a lot of effort to correct this misunderstanding.

2.3 Summary

Based on the source materials above, common views of Westerners at Wylie's time can be listed below:

1. Chinese mathematics was far behind Western mathematics.
2. Nothing about Chinese Mathematics was worth learning by Westerners. On the contrary, Chinese mathematics benefited wholly from Western mathematics (for example, logarithm) which was transmitted by the (Jesuit) missionaries.
3. Chinese numeric notation was cumbersome. It fell far behind the Arabic numeric system used by Westerners. Although the notation was based on decimal principle, it did not have local value (that is, the numeric figures were not changed in value by positions).

4. Abacus was an apparatus which assisted the Chinese to do arithmetic calculation, but it was not very useful—at least, it was inadequate as a substitute for the Arabic numeric system.
5. There was no algebra in Chinese mathematics.

In the next section, we will describe how Wylie responded to these views in his *Jottings*.

3 Wylie's response in his *Jottings*

Wang (1998) gave a detailed analysis on the structure and content of Wylie's *Jottings*, with selected passages translated into Chinese. In this presentation, we analyse *Jottings* from another perspective, namely, how Wylie responded to Westerners' common (erroneous) views about Chinese mathematics. In the following discussion, the page numbers of *Jottings* refer to those in the *Chinese Researches* reprint edition.

3.1 The history of abacus

Wylie wrote: "It has been erroneously stated by some authors that the Chinese have used the 算盤 *Swan-pan* or abacus from time immemorial." (p. 168). It seems that this erroneous statement was rather common among Westerners at that time (see for instances, Murray, 1836, pp. 227–228; Davis, 1851, p. 283–284). Wylie pointed out that the abacus was indeed introduced in "comparatively recent date". He continued to introduce the *Show* or tallies which is a predecessor of abacus. "In ancient time calculations were carried on by means of 籌 *Show* or tallies made of bamboo" (p. 168). We remark that the history of abacus and tallies mentioned by Wylie is basically correct. Martzloff (1997, Chapter 13) pointed out that the counting rods (tallies) can be traced as early as Former Han Dynasty (1st century, B.C.E.) and kept on playing an important role in Chinese mathematics until the Yuan Dynasty (13–14th century). It was also pointed out that "the abacus only entered into common use in China from the second half of the 16th Century [Ming Dynasty]" (p. 215).

The most interesting thing about the tallies which Wylie correctly pointed out is that "the written character is evidently a rude representation of these [the tallies] " (p. 168). He made an analogy of this kind of written representation with the Roman numerals and pointed out that both systems have a new symbol for the increment of 5. It provided evidence suggesting that the Chinese numeric notation depended on the theory of local values at a time much earlier than the European understood this theory.

3.2 Local values in Chinese written numbers

On p. 169 of his *Jottings*, Wylie quoted several books of his time, including *Penny Cyclopaedia of the Society for the Diffusion of Useful Knowledge* (edited by Charles Knight, 1833) and also Sir John Davis's works, which claimed that the Chinese written numeration does not have local value. Wylie disagreed and pointed out that "an example from any native work will be a sufficient reply to the above statements" (p. 169). Then, he quoted a question from Chapter 8 of *Soo-shoo-kew-chang (Nine Sections of the Art of Numbers)*[2] 數書九章by Tsin kew-chaon (Song Dynasty, 13th century) as a "random" example.

[2]Nowadays, this book is known as *Mathematical Treatise in Nine Sections*.

Wylie used this example as an illustration that the arithmetical work in (ancient) China was essentially the same as what the English did, except perhaps with different meanings in some terms. In particular, Wylie argued that "the author [of *Nine Sections of the Art of Numbers*] had the same view with regard to local value,⋯, as that universally adopted by modern civilized nations" (p. 169).

It is interesting to note that in later part of *Jottings* when Wylie introduced the method of *Tien-yuen-yih* 天元一 (Chinese algebra of polynomials) found in the Yuan Dynasty, he has the following comments:

> "In the *Tien-yuen-yih*, unity is employed as the representative of an unknown number; this being combined with an extension of the theory of local value, in order to represent the successive powers of the Monad or unknown number" (p. 182).
>
> "It is not a little remarkable, that while it has been gravely asserted by most respectable authorities in Europe, that the Chinese are ignorant of the meaning of local value, we find here on the contrary, that they have pushed the principle to a degree of refinement unpracticed in the west" (p. 182).

In other words, Wylie pointed out that the polynomial representation in the method of *Tien-yuen-yih* is indeed a generalization of the theory of local value.

3.3 Algebra in ancient China

In order to respond to the claim that there was no algebra in China, Wylie provided some concrete algebraic methods found in ancient China. It is interesting to note Wylie's comments on the dates for the origin of these methods:

> "In examining the productions of the Chinese one finds considerable difficulty in assigning the precise date for the origin of any mathematical process; for on almost every point, where we consult a native author, we find references to some still earlier work on the subject" (p. 175).

Nevertheless, this quotation suggests that Wylie believed that algebraic knowledge did indeed exist in China long time ago.

Ta-yen ("Great Extension") 大衍 (known as "Chinese Remainder Theorem" nowadays) may be the most well-known algebraic method introduced by Wylie in his *Jottings*. As a result of a German translation of *Jottings* (translated by K.L. Biernatzki), this method had drawn the attention of Western historians. Unfortunately, because of some misinterpretation in this German translation, some of these historians thought that this method was mathematically incorrect. After a long process of investigation (thanks to the work of L. Matthiessen in 1881) Westerners realized that *Ta-yen* method was indeed equivalent to the method devised by Gauss. Finally, this method was recognized as the Chinese Remainder Theorem. Readers who are interested in the details of this story may refer to Wang (2004). We now come back to the discussion on how Wylie introduced the method of *Ta-yen* in his *Jottings*. First, Wylie quoted the well-known problem of *Wuh-puh-chi-soo* ("Unknown Numerical Quantities") 物不知數appeared in *Sun-tsze Swan-king* (*Sun Tsze's Arithmetical Classic*)[3](Chin Dynasty, 1st Century):

[3]Nowadays, this book is known as *Master Sun's Arithmetical Manual.*

> "Given an unknown number, which when divided by 3, leaves a remainder of 2; when divided by 5, it leaves 3; and when divided by 7, leaves 2; what is the number?" (p. 175)

After giving a brief discussion of the method of solution, Wylie proceeded to describe the general method given in Chapter 1 (*Ta-yen*) in *Nine Sections of the Art of Numbers*. It is interesting (but may not be so appropriate[4]) that he selected Problem 1 in the Chapter of *Ta-yen* as an illustration of this method. Despite the fact that this principle is not very clearly explained (for instance, the precise procedure of "finding unity"), Wylie's work played a pioneering role in introducing this method to Westerners.

Another method introduced by Wylie is the *Tien-yuen-yih*(unity)[5] as "the representative of an unknown number" (p. 182). This was an ancient Chinese method of representing a polynomial of one variable. More precisely, ancient Chinese used different terms (such as *Yuen*元, *Tai*太, *Tai-kieh*太極) to represent the coefficients of different powers of an unknown quantity, that is, variable x (in today's terminology). As mentioned in a previous section, Wylie regarded it as "an extension of the theory of local value". Furthermore, he also pointed out that "the method invented by Hariot, of placing all the significant terms on one side, is precisely that used by the Chinese [as demonstrated by *Tien-yuen-yih*] some five centuries earlier; and although in itself but a variation in algebraic language, yet it is said by De Morgan to have been the foundation of most important branches of the science" (p. 182).

Next, Wylie pointed out that Horner's method of "solving equations of all orders" which was first published in 1819 (some 30 years before the publication of *Jottings*) could be found in *Nine Sections of the Art of Numbers* (Song Dynasty, 13th century which was 6 century earlier). Again, this gives another example that many algebraic methods known by Westerners were already known by Chinese many centuries earlier. This serves as a refutation of the Westerners' usual claim that "no algebraic knowledge is to be found in China".

3.4 Chinese mathematics versus Western mathematics

The overall purpose of Wylie's *Jottings* is to respond to the common Westerners' view (at his time) that Chinese mathematics was far behind their Western mathematics and nothing in Chinese Mathematics was worth learning. As discussed above, Wylie provided some examples to support an opposite view, namely, quite an amount of mathematical knowledge known to Westerners at his time was actually discovered by ancient Chinese much earlier (some even several centuries earlier). The theory of local values in numeric representations, the concept of negative numbers, *Ta-yen* (Chinese Remainder Theorem), *Tien-yuen-yih* (the method of representing a polynomial), and solving polynomials of any degrees are some examples. Furthermore, detailed introduction of some classical Chinese mathematics books such as *Kew-chang-swan-shun* (*Arithmetical Rules of the Nine Sections*)[6] 九章算術 and *Soo-shoo-kew-chang* (*Nine Sections of the Art of Numbers*) 數書九章 are included in *Jottings*. Despite the fact that it contains some erroneous descriptions on these books (see for instance, Wang 1998), it has opened up a new window for Westerners to know about Chinese mathematics.

[4] See Wang (2004).

[5] Authors such as Wang (2004) pointed out that Wylie has (mistakenly) mixed up *Ta-yen and Tien-yuen-yih*. Indeed, they are actually not related.

[6] Nowadays, this book is known as *Nine Chapters on the Mathematical Art*.

Wylie held a balanced view on Chinese mathematics and Western mathematics. On the one hand, he did not underestimate Chinese mathematics; on the other hand, he recognized the contribution of Western mathematics transmitted by the missionaries to the progress of mathematics in China. In the last part of *Jottings* (p. 188 and onwards), he gave a brief account on mathematics in Qing Dynasty and pointed out how Western mathematics influenced the development of mathematical ideas in that period. For instance, the work of Li Shan-lan, who became one of Wylie's close co-workers, on logarithm (a mathematical idea transmitted by the Jesuit missionaries) was introduced. As revealed in the following comment, Wylie paid rather high regard to Li's work:

> "This small indication of self-satisfaction may be very well overlooked, as quite pardonable in one who has had no better aid than that afforded by the *Leuh-lih-yuen-yuen*, and who has here given us, as the result of four years' thought, a theorem, which in the days of Briggs and Napier, would have been sufficient to raise him to distinction." (p. 194)

The following closing remark in *Jottings* suffices to describe Wylie's view on Chinese mathematics and Western mathematics, which was indeed rather innovative in his time!

> "It is true the Celestials are disposed to look with a feeling akin to contempt on the mushroom antiquity of our Western lore; yet it is equally true that a spirit of inquiry still germinates among them, which if fostered by a greater freedom of intercourse, will doubtless tend much to smooth the asperities which now exist, and this prove mutually advantageous." (p. 194)

REFERENCES

- Dauben, J. W. (ed.), 1985/2000, *The History of Mathematics from Antiquity to the Present: A Selective Annotated Bibliography* (*Revised Edition*). Garland (revised CD_ROM edition published by American Mathematical Society).
- Davis, J. F., 1851, *The Chinese: A General Description of China & its Inhabitants*. London: C. Cox.
- Martzloff, J.C., 1997, *A History of Chinese Mathematics*. Berlin, Heidelberg: Springer-Verlag.
- Murray, H., Wallace, W., Burnett, G., 1836, *A Historical and Descriptive Account of China* (*Vol. III*), Edinburg: Oliver & Boyd.
- Wang, X., 1998, "A. Wylie's evaluation of Chinese mathematics" [in Chinese], *Zhongguo Keji Shiliao* (China Historical Materials of Science and Technology) **19** (2), 10–23.
- Wang, X., 1999, "Alexander Wylie and the history of Chinese mathematics" [in Chinese], Daiziran Tansuo (*Exploration on Nature*) **18**, 113–117.
- Wang, X., 2004, "Traditional Chinese Mathematics in the 19th Century Western World" [in Chinese], *Qinghua Xuebao* (Tsing Hua Journal of Chinese Studies) **33**, 73–97.
- William, S.W., 1848, *The Middle Kingdom: A survey of the Geography, Government, Education & Social Life, Art, Religion of the Chinese Empire & its inhabitants* (*Vol. II*), New York & London: Wiley & Putnam.
- Wylie, A., 1853, *Compendium of Arithmetic*數學啟蒙, Shanghai: London Missionary Society Press.
- Wylie, A., Li, S. (trans.), 1865, *Supplementary Elements of Geometry*續幾何原本, Shanghai: London Missionary Society Press.
- Wylie, A., Li, S. (trans.), 1859a, *Treatise of Algebra*代數學, Shanghai: London Missionary Society Press.

– Wylie, A., Li, S. (trans.), 1859b, *Analytical Geometry and Differential and Integral Calculus Step by Step*代微積拾級, Shanghai: London Missionary Society Press.

– Wylie, A., 1867, *Notes on Chinese Literature*, Shanghai: American Presbyterian Mission Press.

– Wylie, A., 1897/1966, *Chinese Researches, Taipei*: Ch'eng-Wen Publishing (originally published in 1897 in Shanghai).