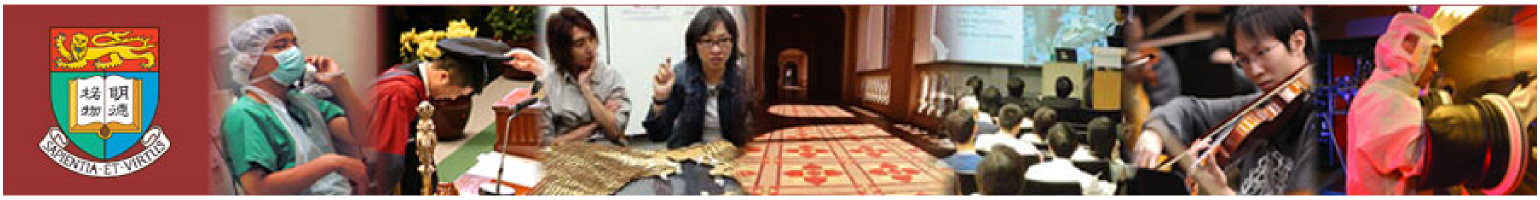

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Alfvén seismic vibrations of crustal solid-state plasma in quaking paramagnetic neutron star

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Magneto-solid-mechanical model of two-component, core-crust, paramagnetic neutron star responding to quake-induced perturbation by differentially rotational, torsional, oscillations of crustal electron-nuclear solid-state plasma about axis of magnetic field frozen in the immobile paramagnetic core is developed. Particular attention is given to the node-free torsional crust-against-core vibrations under combined action of Lorentz magnetic and Hooke's elastic forces; the damping is attributed to Newtonian force of shear viscose stresses in crustal solid-state plasma. The spectral formulas for the frequency and lifetime of this toroidal mode are derived in analytic form and discussed in the context of quasiperiodic oscillations of the x-ray outburst flux from quaking magnetars. The application of obtained theoretical spectra to modal analysis of available data on frequencies of oscillating outburst emission suggests that detected variability is the manifestation of crustal Alfvén's seismic vibrations restored by Lorentz force of magnetic field stresses. © 2010 American Institute of Physics. [doi:10.1063/1.3518758]

I. INTRODUCTION

The investigations of neutron star seismic vibrations offer unique opportunity of studying their internal structure and solid-mechanical and electrodynamical properties of superdense degenerate matter. The most conspicuous feature of these nonconvective solid stars is the capability of accommodating magnetic fields of extremely high intensity[1] that serve as a chief promoter of their observable electromagnetic activity. The absence of nuclear energy sources in these final stage (FS) stars suggests that their magnetic fields are definitely not generated by persistent current-carrying flows in self-exciting dynamo processes, as is the case of liquid main-sequence (MS) stars. It seems quite likely, therefore, that stability to spontaneous decay[2] of fossil magnetic fields of isolated neutron stars[3] is maintained by permanent magnetization of neutron-dominated (poorly conducting) degenerate Fermi-matter. Such an understanding has been laid at the base of paramagnetic neutron star model.[4–7] In this model the degenerate Fermi-matter of nonrelativistic neutrons (whose degeneracy pressure withstands the pressure of self-gravity) is regarded as being in the permanently magnetized state of field-induced Pauli's paramagnetic saturation which is characterized by alignment of spin magnetic moments of neutrons along the axis of frozen-in magnetic field. The most striking dynamical manifestation of spin paramagnetic polarization of nonconducing neutron matter is that such a matter can transmit perturbations by transverse magnetomechanical waves; in such a wave the vector-fields of magnetization and material displacements undergo coupled differentially rotational vibrations traveling along the axis of magnetic field. In a spherical mass of paramagnetic neutron star, this unique feature of field-induced spin magnetic polarization of neutron matter is manifested in that such a star can undergo solely torsional vibrations about axis of its dipole magnetic moment. Based on this finding, it was argued in above works that the model of paramagnetic neutron star executing torsional axisymmetric vibrations, weakly damped by nuclear matter viscosity, is able to explain long periodic ($[5<P<12]$ s—nontypical to young neutron stars) pulsed character of magnetar radiation (both, Soft Gamma Repeaters and Anomalous X-ray Pulsars) in seismically quiescent regime of their emission, as being produced by torsional vibrations, rather than rotation as is the case of radio pulsars.

Recent years have seen a resurgence of interest in torsional vibrations of magnetars, prompted by observations[8–10] of quasiperiodic oscillations (QPOs) during the outburst flare from SGR 0525–66, SGR 1806–20, and SGR 1900+14. The statistics of x-ray burst of SGRs exhibits typical for earthquakes features.[11] It is believed, therefore, that detected QPOs are of seismic origin. Particular attention in this development of magnetar asteroseismology has been paid to the following set of data on QPO frequencies:[12]

$$\text{SGR } 1806-20{:}18,26,29,92,150,625,1840, \tag{1}$$

$$\text{SGR } 1900+14{:}28,54,84,155 \text{ (Hz)}. \tag{2}$$

The corresponding periods are substantially shorter than the above mentioned periods of seismically quiescent pulsed emission. In works[13–17] motivated by this discovery, several models of postquake vibrational relaxation of above magnetars have been investigated. Particular attention has been given to the regime of node-free or nodeless shear axisymmetric vibrations. This regime is interesting in its own right because such vibrations have been and still are poorly inves-

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tigated in theoretical asteroseismology of both solid FS stars and such solid celestial objects as Earth-like planets.[18,19] In particular, in Refs. 13–15, a case of the elastic-force-driven nodeless shear oscillations, both torsional ${}_0t_\ell$ and spheroidal ${}_0s_\ell$, entrapped in the crust of finite-depth ΔR has been studied in some details with remarkable inference that dipole overtones of spheroidal and torsion vibrations of crust against immobile core exhibit features generic to Goldstone soft modes. On the other hand, one can probably cast doubt on arguments of the model presuming the dominant role of solid-mechanical Hooke's force of elastic stresses because such interpretation rests on poorly justifiable assumption about dynamically passive role of an ultrastrong magnetic field, that is, that the field frozen in the star remains unaltered in the process of vibrations. Bearing this in mind and assuming that the presence of charged particles in neutron-dominated stellar matter imparts to it, the properties of electric conductor, in Refs. 16 and 17, the postquake relaxation of above magnetars has been studied in the model of perfectly conducting solid star executing global torsional vibrations restored by joint action of Lorentz force of magnetic field stresses and Hooke's force of solid-mechanical elastic stresses. It was found that such a model provides fairly reasonable account of general trends in QPO frequencies for all data from SGR 1900+14 and for SGR 1900+14 from the range $30 \leq \nu \leq 200$ Hz, but faces serious difficulties in interpreting low-frequency vibrations with $\nu=18$ and $\nu=26$ Hz in data from SGR 1806–20. Also, the model of global torsional vibrations leaves some uncertainties regarding the nature of vibrations with $\nu=625$ and $\nu=1840$ Hz. This last issue has been scrutinized in recent work[17] from the standpoint of a solid star model with nonhomogeneous poloidal magnetic field of well-known Ferraro's form. Moreover, it was found that these high-frequency QPOs can be properly explained as being produced by very high overtones of node-free torsional Alfvén oscillations.

As a logical extension of above line of investigation, in this paper we consider in some details a case of node-free torsional vibrations locked in the crust with focus on toroidal Alfvén mode. In so doing we work from the two-component model of paramagnetic neutron star, pictured in Fig. 1, whose crust and core materials are regarded as endowed with substantially different electrodynamic properties. The immobile massive core, primarily consisting of degenerate neutron matter in the above described permanently magnetized state of Pauli's paramagnetic saturation, is regarded as a main source of magnetic field of the star crust. This implies that the core material is just incapable of sustaining Alfvén vibrations which owe their existence to extremely large (effectively infinite) electrical conductivity of matter.[20–22] The microcomposition of crust, which is dominated by nuclei embedded in degenerate Fermi-gas of relativistic electrons, suggests that its metal-like material possesses properties of perfectly conducting solid-state plasma. Such a view suggests that seismic stability of the star to quake-induced tectonic displacements of crust against core is primarily determined by well-known effect of magnetic (magnet-metal) cohesion mediated by magnetic field lines which operate as a superhard piles endowing the core-crust construction of neutron star with supplementary (to gravity forces) stiffness of magnetic nature. The intermediate layer between core and crust (the inner crust whose density several times less than the core density) is most likely composed of quasiboson matter of paired neutrons. But it is highly unlikely that such quasiboson matter is capable of undergoing phase transition to the Bose–Einstein condensation (BEC) which is characterized by vanishingly small pressure. This suggests that BEC state of paired neutrons, if exist, can only insufficiently contribute to the total mass budget of neutron star—a compact object in which pressure of self-gravity is brought to equilibrium by degeneracy Fermi-pressure of nonrelativistic neutrons in the core and relativistic electrons in the crust. In seismodynamics of the paramagnetic neutron star under consideration, the inner crust is thought of as operating like a lubricant facilitating differentially rotational shear displacements of crust relative to much denser matter of massive core. From the view point of this core-crust model, the starquake is thought of as impulsive release of energy of magnetic core-crust cohesion (by means of disruption of magnetic field lines on the core-crust interface) resulting in the

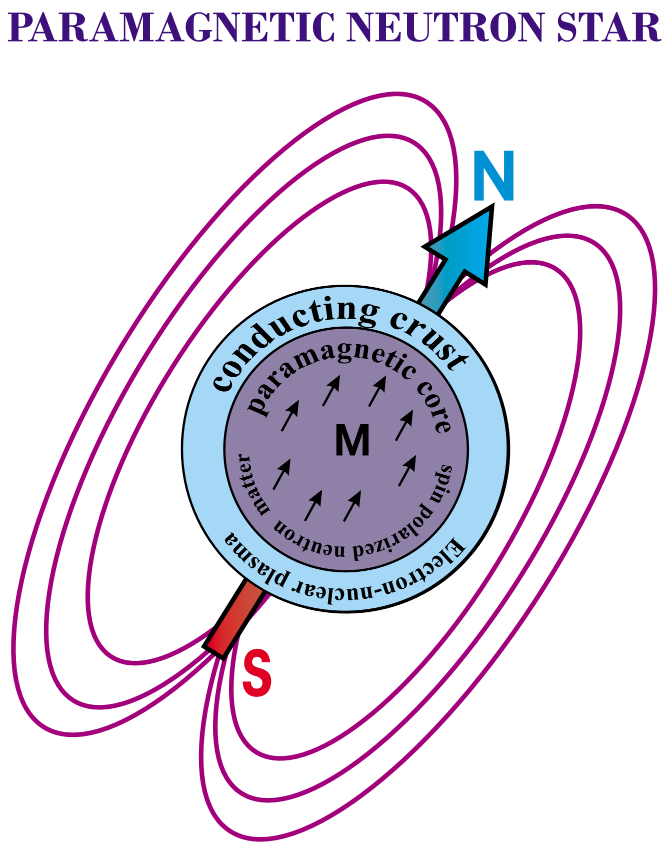

FIG. 1. (Color online) The internal constitution of two-component, core-crust, model of paramagnetic neutron star. The massive core is considered as a poorly conducting permanent magnet composed of degenerate Fermi-gas of nonrelativistic neutrons in the state of Pauli's paramagnetic saturation caused by field-induced alignment of spin magnetic moment of neutrons along the axis of uniform internal and dipolar external magnetic field frozen in the star on the stage of gravitational collapse of its MS progenitor. A highly conducting metal-like material of the neutron star crust, composed of nuclei embedded in the superdense degenerate Fermi-gas of relativistic electrons, is regarded as electron-nuclear solid-state magnetoactive plasma capable of sustaining Alfvén oscillations.

crust fracturing by revealed magnetic stresses. In this paper we focus, however, not on dynamics of quake, but on the postquake vibrational relaxation of the star, namely, on node-free torsional oscillations of crustal solid-state plasma about axis of magnetic field frozen in an immobile paramagnetic core. In Sec. II, a brief outline is given of theory of solid-magnetics appropriate for the perfectly conducting viscoelastic continuous medium pervaded by a magnetic field. In Sec. III, the spectral formulas for the frequency and lifetime of differentially rotational, torsional, nodeless vibrations of the crust restored by combined action of magnetic Lorentz and elastic Hooke's forces are obtained. In Sec. IV, the computed frequency spectra are used for the forward asteroseismic analysis of the fast oscillations of x-ray outburst from above mentioned magnetars. The obtained results are briefly summarized in Sec. V.

II. GOVERNING EQUATIONS OF SOLID-MAGNETICS

It is generally realized today that seismic vibrations of superdense matter of nonconvective FS-stars (white dwarfs, pulsars, and quark stars) can be properly described by equations of solid-mechanical theory of viscoelastic continuous media.[23–28] In what follows we deal with the shear differentially rotational fluctuations of viscoelastic crustal matter of density ρ which are described by quake-induced material displacements u_i (basic variable of solid-mechanics). The noncompressional character of vibrations under consideration implies that $\delta\rho=-\rho\nabla_k u_k=0$. With this in mind, the governing equations of solid-magnetics (solid-mechanical counterpart of equations of magneto-fluid-mechanics) can be written in the form

$$\rho\ddot{u}_i=\nabla_k\tau_{ik}+\nabla_k\sigma_{ik}+\nabla_k\pi_{ik},\quad \nabla_k u_k=0, \tag{3}$$

presuming that Hooke's elastic stresses σ_{ik} and Newton's viscous stresses π_{ik} are described by linear constitutive equations

$$\sigma_{ik}=2\mu u_{ik},\quad u_{ik}=\tfrac{1}{2}[\nabla_i u_k+\nabla_k u_i], \tag{4}$$

$$\pi_{ik}=2\eta\dot{u}_{ik},\quad \dot{u}_{ik}=\tfrac{1}{2}[\nabla_i\dot{u}_k+\nabla_k\dot{u}_i], \tag{5}$$

where μ stands for the shear modulus, η for shear viscosity, and u_{ik} is the tensor of shear strains or deformations. The central to our further discussion is the tensor of fluctuating magnetic field stresses,

$$\tau_{ik}=\frac{1}{4\pi}[B_i\delta B_k+B_k\delta B_i-B_j\delta B_j\delta_{ik}], \tag{6}$$

$$\delta B_i=\nabla_k[u_iB_k-u_kB_i]. \tag{7}$$

As in our previous works,[16,29] we consider model with homogeneous internal magnetic field whose components in spherical polar coordinates read

$$B_r=B\cos\theta,\quad B_\theta=-B\sin\theta,\quad B_\phi=0, \tag{8}$$

and external dipolar magnetic field is described by $\mathbf{B}=\nabla\times\mathbf{A}$, where $\mathbf{A}=[0,0,A_\phi=\mathrm{m}_s/r^2]$ is the vector potential with the standard parametrization of the dipole magnetic moment $\mathrm{m}_s=(1/2)BR^3$ of star of radius R and by B is understood the magnetic field intensity at its magnetic poles, $B=B_p$.

A. The energy method

This method of computing frequency of shear vibrations rests on the equation of energy balance,

$$\frac{\partial}{\partial t}\int\frac{\rho\dot{u}^2}{2}d\mathcal{V}=-\int[\tau_{ik}+\sigma_{ik}+\pi_{ik}]\dot{u}_{ik}d\mathcal{V}, \tag{9}$$

which is obtained by scalar multiplication of equation of magneto-solid-mechanics (3) with $\dot{u}_i$ and integration over the volume of seismogenic layer. From the technical point of view, the shear character of material distortions brought about by forces under consideration owes its origin to the symmetric form of stress-tensors in terms of which these forces are expressed. It is this last feature of solid-mechanical elastic stresses and magnetic field stresses that endows the solid-state plasma pervaded by homogeneous magnetic field with the capability of responding to noncompression perturbation by reversal shear vibrations (which are not accompanied by fluctuations in density). The physical significance and practical usefulness of the energy method under consideration are that it can be efficiently utilized not only in the study of nonradial seismic vibrations of neutron stars but also can be applied to the study of more wide class of solid degenerate stars such as white dwarfs stars[30,31] and ultradense quark-matter stars[32] whose material is most likely in the solid aggregate state.[33,34] At this point it seems appropriate to mention here theoretical investigations of vibration properties of atomic nuclei (thought of as ultrafine pieces of continuous nuclear matter) in which it has been found that nuclear giant-resonant excitations (fundamental vibration modes generic to all nuclei of periodic chart) are properly described in terms of spheroidal and torsional elastic vibrations of a solid sphere.[35] This suggests that degenerate nucleon Fermi-matter, regarded as continuous medium, can be thought of as a strained Fermi-solid, rather than flowing Fermi-liquid.

The key idea of this method consists in using of the following separable form of material displacements:

$$u_i(\mathbf{r},t)=a_i(\mathbf{r})\alpha(t), \tag{10}$$

where $a_i(\mathbf{r})$ is the time-independent solenoidal field and amplitude $\alpha(t)$ carries information about temporal evolution of fluctuations. Thanks to this form of u_i, all the above tensors of fluctuating stresses and strains take similar separable form,

$$\tau_{ik}(\mathbf{r},t)=\left[\tilde{\tau}_{ik}(\mathbf{r})-\tfrac{1}{2}\tilde{\tau}_{jj}(\mathbf{r})\delta_{ik}\right]\alpha(t), \tag{11}$$

$$\tilde{\tau}_{ik}(\mathbf{r})=\frac{1}{4\pi}[B_i(\mathbf{r})b_k(\mathbf{r})+B_k(\mathbf{r})b_i(\mathbf{r})], \tag{12}$$

$$b_i(\mathbf{r})=\nabla_k[a_i(\mathbf{r})B_k(\mathbf{r})-a_k(\mathbf{r})B_i(\mathbf{r})], \tag{13}$$

$$\sigma_{ik}(\mathbf{r}) = 2\mu a_{ik}(\mathbf{r})\alpha(t), \quad \pi_{ik}(\mathbf{r}) = 2\eta a_{ik}(\mathbf{r})\dot{\alpha}(t), \tag{14}$$

$$a_{ik}(\mathbf{r}) = \tfrac{1}{2}[\nabla_i a_k(\mathbf{r}) + \nabla_k a_i(\mathbf{r})]. \tag{15}$$

On inserting Eqs. (10)–(15) in the integral equation of energy balance (9), we arrive at equation for $\alpha(t)$ having the well-familiar form,

$$\frac{d\mathcal{E}}{dt} = -2\mathcal{F}, \ \ \mathcal{E} = \frac{\mathcal{M}\dot{\alpha}^2}{2} + \frac{\mathcal{K}\alpha^2}{2}, \ \ \mathcal{F} = \frac{\mathcal{D}\dot{\alpha}^2}{2}, \tag{16}$$

$$\mathcal{M}\ddot{\alpha} + \mathcal{D}\dot{\alpha} + \mathcal{K}\alpha = 0, \tag{17}$$

$$\alpha(t) = \alpha_0 \exp(-t/\tau)\cos(\Omega t), \tag{18}$$

$$\Omega^2 = \omega^2[1-(\omega\tau)^{-2}], \quad \omega^2 = \frac{\mathcal{K}}{\mathcal{M}}, \quad \tau = \frac{2\mathcal{M}}{\mathcal{D}}, \tag{19}$$

where the inertia $\mathcal{M}$, viscous friction $\mathcal{D}$, and stiffness $\mathcal{K}$ of damped oscillator are given by

$$\mathcal{M} = \int \rho(\mathbf{r})a_i(\mathbf{r})a_i(\mathbf{r})d\mathcal{V}, \quad \mathcal{K} = \mathcal{K}_e + \mathcal{K}_m, \tag{20}$$

$$\mathcal{K}_e = 2\int \mu(\mathbf{r})a_{ik}(\mathbf{r})a_{ik}(\mathbf{r})d\mathcal{V}, \tag{21}$$

$$\mathcal{K}_m = \int \tilde{\tau}_{ik}(\mathbf{r})a_{ik}(\mathbf{r})d\mathcal{V}, \tag{22}$$

$$\mathcal{D} = 2\int \eta(\mathbf{r})a_{ik}(\mathbf{r})a_{ik}(\mathbf{r})d\mathcal{V}. \tag{23}$$

All the above equations valid for arbitrary volume occupied by magnetoactive solid-state plasma whose density, shear modulus, and shear viscosity are arbitrary functions of position. As in our previous works, here we confine our computations to the case of uniform profile of these later parameters. Moreover, because the purpose of our present study is the frequency spectrum of node-free torsion vibrations about magnetic axis of the star, several comments should be made regarding the axisymmetric field of material displacements **a** which is taken in one and the same shape in computing parameter of inertia $\mathcal{M}$ and spring constants of both solid-mechanical $\mathcal{K}_e$ and magnetomechanical $\mathcal{K}_m$ stiffness.

From the physical point of view, the main argument justifying the use of one and the same field of material displacements in computing frequency spectra of torsional vibrations driven by forces of elastic and magnetic field stresses rests on general statement of continuum-mechanical theories of magnetoactive perfectly conducting continuous media—magnetic field pervading (both liquid-state and solid-state) plasmas imparts to such a medium a supplementary portion of elasticity which is manifested in its capability of transmitting noncompressional mechanical perturbations by transverse Alfvén waves. Such a view is substantiated by the commonly known fact that transverse wave in incompressible continuous medium is the feature of material oscillatory behavior which is generic to elastic solid, not an incompressible flowing liquid. The transverse hydromagnetic wave propagating, along the lines of constant magnetic field B frozen-in the perfectly conducting medium, with Alfvén speed $v_A=[2P_B/\rho]^{1/2}$ (where $P_B=B^2/8\pi$ is the magnetic field pressure), is characterized by dispersion equation $\omega = v_A k$ which is similar to that for transverse wave of shear mechanical displacements, $\omega = c_t k$, traveling in an elastic solid with the speed $c_t=[\mu/\rho]^{1/2}$, where μ is the shear modulus (which has physical dimension of pressure). The profound discussion of analogy between oscillatory behavior of incompressible perfectly conducting plasmas and elastic solid (similarity of transverse hydromagnetic wave in incompressible perfectly conducting media and transverse wave of shear mechanical displacements in an elastic solid) can be found in monographs of Chandrasekhar[20] (section Alfvén waves), and more extensively this issue is discussed in monograph of Alfvén and Fältammar.[21] Regarding the difference between node-free oscillatory behavior of solid sphere and spherical mass of an incompressible liquid, it is appropriate to note that the liquid sphere is able to sustain solely spheroidal node-free vibrations of fluid velocity. The canonical example is the Kelvin fundamental mode of oscillating fluid velocity in a heavy spherical mass of incompressible homogeneous liquid restored by forces represented as gradient of pressure and gradient of potential of self-gravity.[36] In the meantime, the node-free vibrations of solid sphere restored by elastic force (represented as divergence of shear mechanical stresses) are characterized by two eigenmodes. Namely, the even-parity spheroidal mode of nonrotational vibrations of material displacements and the odd-parity torsional mode of differentially rotational vibrations, the problem in which the very notion of the torsion vibration mode has come into existence.[13] In our studies focus is laid on poorly investigated regime of node-free vibrations in which solenoidal field of material displacements, $\nabla\cdot\mathbf{a}=0$, obeys the vector Laplace equation, $\nabla^2\mathbf{a}=0$. In this regime the instantaneous material displacements are described by the toroidal field of the form $\mathbf{a}=A_\ell\nabla\times[\mathbf{r}r^\ell P_\ell(\cos\theta)]$. Substituting this field in the above given integrals for inertia $\mathcal{M}$ and solid-mechanical stiffness $\mathcal{K}_e$ and integrating over the entire volume of oscillating star, we have found in our previous studies that elastic-force-driven node-free global torsional vibrations, $\mathcal{M}\ddot{\alpha}+\mathcal{K}_e\alpha=0$, are characterized by the frequency spectrum $\omega_e(\ell)=[\mathcal{K}_e/\mathcal{M}]^{1/2}$ of the form

$$\omega_e^2(\ell) = \omega_e^2[(2\ell+3)(\ell-1)], \tag{24}$$

$$\omega_e = \frac{c_t}{R}, \quad c_t = \sqrt{\frac{\bar{\mu}}{\bar{\rho}}}, \tag{25}$$

where ω_e is the natural unit of frequency of shear elastic vibrations and c_t is the speed of transverse wave in the elastic solid characterized by average shear modulus $\bar{\mu}$ and average density $\bar{\rho}$.

The above line of argument about similarity between oscillatory behavior of elastic solid and magnetoactive plasma suggests that perfectly conducting matter of neutron star pervaded by homogeneous magnetic field should be able to sustain the Lorentz-force-driven differentially rotational seismic vibrations (triggered by quake) about magnetic axis in which oscillating field of material displacements has one and the same form as in the above outlined Hooke's-force-driven torsional vibrations. Adhering to this assumption and making use of the above node-free toroidal field **a** as a trial function for computing $\mathcal{M}$ and $\mathcal{K}_m$, we found in Refs. 16 and 17 that torsional vibrations restored by magnetic Lorentz force, $\mathcal{M}\ddot{\alpha}+\mathcal{K}_m\alpha=0$, are characterized by the frequency spectrum,

$$\omega_m^2(\ell)=\omega_A^2\left[(\ell^2-1)\frac{2\ell+3}{2\ell-1}\right], \quad (26)$$

$$\omega_A=\frac{v_A}{R},\quad v_A=\frac{B}{\sqrt{4\pi\rho}}, \quad (27)$$

where ω_A is the natural unit of frequency of Alfvén vibrations and v_A is the speed of Alfvén wave. The practical usefulness of outlined computations with one and the same trial toroidal field of displacements is that allows us to assess the relative role of restoring Hooke's and Lorentz forces in torsional seismic vibrations of neutrons stars which are responsible, as is believed, for the fast oscillations of x-ray flares from quaking magnetars. With all above in mind, one of the main purposes of present paper is to make such an assessment in a mathematically consistent fashion for the torsional node-free vibrations entrapped in the crust, the problem which is considered, to the best of our knowledge, for the first time.

B. Material displacements in torsional mode of crust-against-core nodeless vibrations

The above equations of the energy method show that the main trial function of the frequency spectrum computation is the toroidal field of instantaneous displacements. Because one of the main our purposes here is to assess the relative role of elastic and magnetic forces in quake-induced torsion nodeless vibrations of magnetars, we again adopt all the above arguments regarding the choice of this field in the form of the general solution to vector Laplace equation.

It is convenient to start with the rate of material displacements which is described by general formula of rotational motions,

$$\delta\mathbf{v}(\mathbf{r},t)=\dot{\mathbf{u}}(\mathbf{r},t)=[\mathbf{\Omega}(\mathbf{r},t)\times\mathbf{r}], \quad (28)$$

$$\mathbf{\Omega}(\mathbf{r},t)=[\nabla\times\delta\mathbf{v}(\mathbf{r},t)]=[\nabla\times\dot{\mathbf{u}}(\mathbf{r},t)]=\dot{\mathbf{\Phi}}(\mathbf{r},t). \quad (29)$$

However, unlike a case of rigid-body rotation, in which the angular velocity is a constant vector, in a solid mass undergoing axisymmetric differentially rotational vibrations, the angular velocity $\mathbf{\Omega}(\mathbf{r},t)$ is the vector-function of position which can be represented as

$$\mathbf{\Omega}(\mathbf{r},t)=\dot{\mathbf{\Phi}}(\mathbf{r},t)=\phi(\mathbf{r})\dot{\alpha}(t),\quad \phi(\mathbf{r})=[\nabla\times\mathbf{a}(\mathbf{r})]. \quad (30)$$

In the regime of node-free vibrations in question, $\mathbf{a}(\mathbf{r})$ is described by the divergence-free odd-parity, axial, toroidal field which is one of two harmonic solenoidal fields of fundamental basis[20] obeying the vector Laplace equation $\nabla^2\mathbf{a}=0$. This field can be expressed in terms of general solution of the scalar Laplace equation as follows:

$$\mathbf{a}(\mathbf{r})=\mathbf{a}_t(\mathbf{r})=\nabla\times[\mathbf{r}\chi(\mathbf{r})]=[\nabla\chi(\mathbf{r})\times\mathbf{r}], \quad (31)$$

$$\nabla^2\chi(\mathbf{r})=0, \quad (32)$$

$$\chi(\mathbf{r})=[A_\ell r^\ell+B_\ell r^{-\ell-1}]P_\ell(\zeta),\quad \zeta=\cos\theta, \quad (33)$$

where $P_\ell(\zeta)$ is the Legendre polynomial of multipole degree ℓ. It follows that the angular field $\phi(\mathbf{r})$ is the poloidal vector-field,

$$\phi(\mathbf{r})=[\nabla\times\mathbf{a}_t(\mathbf{r})]=\nabla\times\nabla\times[\mathbf{r}\chi(\mathbf{r})] \quad (34)$$

$$=\nabla[\mathcal{A}_\ell r^\ell+\mathcal{B}_\ell r^{-\ell-1}]P_\ell(\zeta), \quad (35)$$

$$\mathcal{A}_\ell=A_\ell(\ell+1),\quad \mathcal{B}_\ell=B_\ell\ell. \quad (36)$$

Moreover, this field is irrotational, $\nabla\times\phi(\mathbf{r})=0$.

As was stated, we study a model of differentially rotational vibrations of peripheral finite-depth crust against immobile core. In this case, the arbitrary constants $\mathcal{A}_\ell$ and $\mathcal{B}_\ell$ can be uniquely eliminated from two boundary conditions: (i) on the core-crust interface $r=R_c$,

$$u_\phi|_{r=R_c}=0, \quad (37)$$

and (ii) on the star surface $r=R$,

$$u_\phi|_{r=R}=[\mathbf{\Phi}\times\mathbf{R}]_\phi|_{r=R}, \quad (38)$$

$$\mathbf{\Phi}=\alpha(t)\nabla_{\hat{n}}P_\ell(\zeta),\quad \mathbf{R}=\mathbf{e}_rR, \quad (39)$$

where

$$\nabla_{\hat{n}}=\frac{1}{R}\nabla_{\mathbf{\Omega}},\quad \nabla_{\mathbf{\Omega}}=\left[\mathbf{e}_\theta\frac{\partial}{\partial\theta}+\mathbf{e}_\phi\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right]. \quad (40)$$

The no-slip condition on the core-crust interface, $r=R_c$, reflects the fact that the amplitude of differentially rotational oscillations gradually decreases down to the star center and turns into zero on the core. The boundary condition on the star surface, $r=R$, is dictated by symmetry of the general toroidal solution of the vector Laplace equation which then is tested to reproduce the moment of inertia of a rigidly rotating solid star.[15,27] The above boundary conditions lead to the coupled algebraic equations

$$\mathcal{A}_\ell R_c^{\ell-1}+\mathcal{B}_\ell R_c^{-\ell-2}=0,\quad \mathcal{A}_\ell R^\ell+\mathcal{B}_\ell R^{-\ell-1}=R, \quad (41)$$

whose solutions are

$$\mathcal{A}_\ell=\mathcal{N}_\ell,\quad \mathcal{B}_\ell=-\mathcal{N}_\ell R_c^{2\ell+1},\quad \mathcal{N}_\ell=\frac{R^{\ell+2}}{R^{2\ell+1}-R_c^{2\ell+1}}. \quad (42)$$

In spherical polar coordinates, the nodeless toroidal field has only one nonzero azimuthal component,

$$a_r=0,\quad a_\theta=0,$$

$$a_\phi=\left[\mathcal{A}_\ell r^\ell+\frac{\mathcal{B}_\ell}{r^{\ell+1}}\right](1-\zeta^2)^{1/2}\frac{dP_\ell(\zeta)}{d\zeta}. \quad (43)$$

The snapshot of material node-free displacements in the crust undergoing torsional oscillations against immobile core

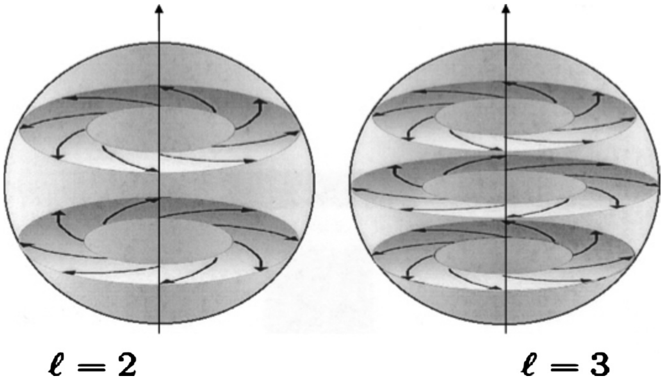

FIG. 2. Material displacements of crustal matter about the dipole magnetic moment axis of paramagnetic neutron star undergoing nodeless differentially rotational, torsional, vibrations in quadrupole and octupole overtones.

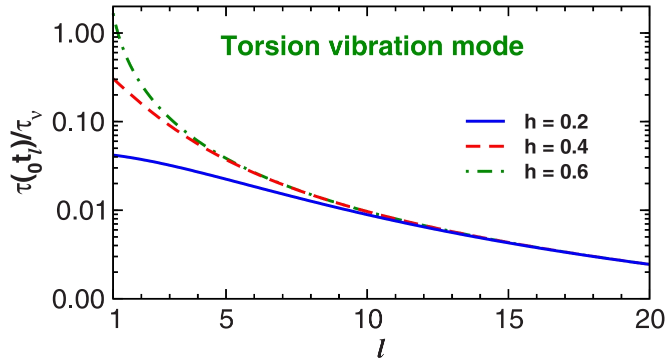

FIG. 3. (Color online) The fractional lifetime of torsion nodeless oscillations of the neutron star crust damped by force of viscous shear stresses as a function of multipole degree ℓ computed at indicated values of the fractional depth h of peripheral seismogenic layer.

of paramagnetic neutron star under consideration is pictured in Fig. 2 for quadrupole, $\ell=2$, and octupole $\ell=3$, overtones of this axial mode. Adopted first boundary condition (37) implying that all stresses (elastic, magnetic, and viscous) vanish on the core-crust interface suggests that quake-induce perturbation sets in the node-free torsional motions only a finite-depth crustal region, whereas central undisturbed region of the star remains at rest. In Sec. III the results of analytic computations are presented in the form showing that spectral formulas for toroidal modes entrapped in the crust are reduced to the above presented ones, Eqs. (24) and (26), for the global oscillations (in the entire volume of the star) when core radius tends to zero; this fact is regarded as a test justifying mathematical correctness of presented computations.

III. SPECTRAL FORMULAS FOR THE FREQUENCY AND LIFETIME

The computation of integrals defining mass parameter $\mathcal{M}$, parameter of vibrational rigidity $\mathcal{K}$, and viscous friction $\mathcal{D}$, which has been presented in some details elsewhere,[13,27] are quite lengthy but straightforward and, therefore, are not presented here. The mass parameter can be conveniently represented in the form

$$\mathcal{M}=4\pi\rho R^5\frac{\ell(\ell+1)}{(2\ell+1)(2\ell+3)}m(\ell), \tag{44}$$

$$m(\ell)=(1-\lambda^{2\ell+1})^{-2}\left[1-(2\ell+3)\lambda^{2\ell+1}\frac{(2\ell+1)^2}{2\ell-1}\lambda^{2\ell+3}-\frac{2\ell+3}{2\ell-1}\lambda^{2(2\ell+1)}\right], \tag{45}$$

$$\lambda=\frac{R_c}{R}=1-h,\quad h=\frac{\Delta R}{R}, \tag{46}$$

$$\Delta R=R-R_c,\quad 0\le\lambda<1. \tag{47}$$

The λ-terms in the above and foregoing equations emerge as result of integration along the radial coordinate from radius of the core-crust interface $r=R_c$ to the star radius, $r=R$. The integral coefficient of viscous friction is given by

$$\mathcal{D}=4\pi\eta R^3\frac{\ell(\ell^2-1)}{2\ell+1}d(\ell), \tag{48}$$

$$d(\ell)=(1-\lambda^{2\ell+1})^{-1}\left[1-\frac{(\ell+2)}{(\ell-1)}\lambda^{2\ell+1}\right]. \tag{49}$$

For the lifetime we obtain

$$\tau({}_0t_\ell)=\frac{2\tau_\nu}{(2\ell+3)(\ell-1)}\frac{m(\ell)}{d(\ell)},\quad \tau_\nu=\frac{R^2}{\nu},\quad \nu=\frac{\eta}{\rho}. \tag{50}$$

In Fig. 3, the fractional lifetime is plotted as a function of multipole degree ℓ with indicated values of fractional depths $h=\Delta R/R$. It shows that the higher ℓ the shorter lifetime. It is easy to see that in the limit, $\lambda=(R_c/R)\to 0$, we regain the spectral formula for lifetime of global torsional nodeless vibrations of solid star,[7,13]

$$\tau({}_0t_\ell)=\frac{2\bar{\tau}_\nu}{(2\ell+3)(\ell-1)},\quad \bar{\tau}_\nu=\frac{R^2}{\bar{\nu}},\quad \bar{\nu}=\frac{\bar{\eta}}{\bar{\rho}}, \tag{51}$$

in which by $\bar{\tau}_\nu$ is understood, in this latter case, the average kinematic viscosity of the star matter as a whole; the extensive discussion of this transport coefficient can be found in Ref. 37. For the node-free torsional oscillations of solid star, the last equation has one and the same physical significance as the well-known Lamb formula does for the time of viscous damping of spheroidal node-free vibrations which in the context of neutron star pulsations has been extensively discussed in Ref. 38. Regarding the problem under consideration, we cannot see, however, how the obtained formulas can be applied to observational data on QPOs in SGRs. Nonetheless, their practical usefulness is that they can be utilized in the study of a more wide class of solid celestial objects such as Earth-like planets[18,19] and white dwarf stars.[39]

From above it is clear that the integral coefficient of elastic rigidity $\mathcal{K}_e$ of torsional vibrations has analytic form similar to that for coefficient of viscous friction $\mathcal{D}$, namely,

$$\mathcal{K}_e = 4\pi\mu R^3 \frac{\ell(\ell^2-1)}{2\ell+1} k_e(\ell), \quad (52)$$

$$k_e(\ell) = (1-\lambda^{2\ell+1})^{-1}\left[1 - \frac{(\ell+2)}{(\ell-1)}\lambda^{2\ell+1}\right]. \quad (53)$$

The frequency as a function of multipole degree ℓ of node-free elastic vibrations in question $\nu_e(\ell)$ [measured in hertz and related to angular frequency as $\omega_e(\ell)=2\pi\nu_e(\ell)=\mathcal{K}_e/\mathcal{M}$] is given by

$$\nu_e^2(\ell) = \nu_e^2[(2\ell+3)(\ell-1)]\frac{k_e(\ell)}{m(\ell)}, \quad (54)$$

$$\omega_e = 2\pi\nu_e = \frac{c_t}{R}, \quad c_t = \sqrt{\frac{\mu}{\rho}}, \quad \lambda = 1-h, \quad h = \frac{\Delta R}{R}. \quad (55)$$

It is easy to see that in the limit $\lambda \to 0$, we regain spectral formula for the frequency of global torsional oscillations, having the form of Eq. (54) with $k_e(\ell)=m(\ell)=1$. Understandably that in this latter case, all material characteristics belong to the star as a whole.

The magnetomechanical stiffness of Alfvén vibrations $\mathcal{K}_m$ can conveniently be written as

$$K_m = B^2 R^3 \frac{\ell(\ell^2-1)(\ell+1)}{(2\ell+1)(2\ell-1)} k_m(\ell), \quad (56)$$

$$k_m(\ell) = (1-\lambda^{2\ell+1})^{-2}\left\{1 + \frac{3\lambda^{2\ell+1}}{(\ell^2-1)(2\ell+3)} \times\left[1 - \frac{1}{3}\ell(\ell+2)(2\ell-1)\lambda^{2\ell+1}\right]\right\}. \quad (57)$$

This leads to the following two-parametric spectral formula:

$$\nu_m^2(\ell) = \nu_A^2\left[(\ell^2-1)\frac{2\ell+3}{2\ell-1}\right]\frac{k_m(\ell)}{m(\ell)}, \quad (58)$$

$$\omega_A = 2\pi\nu_A = \frac{v_A}{R}, \quad v_A = \frac{B}{\sqrt{4\pi\rho}}. \quad (59)$$

In the forward asteroseismic analysis of QPO data relying on this latter spectral formula, the Alfvén frequency ν_A and the fractional depth of seismogenic zone h are regarded as free parameters which are adjusted so as to reproduce general trends in the observed QPO frequencies. In Fig. 4, the fractional frequencies and periods of this toroidal Alfvén mode as functions of multipole degree ℓ are plotted with indicated values of fractional depth of the seismogenic layer h. Remarkably, the lowest overtone of global oscillations is of quadrupole degree, $\ell=2$, whereas for vibrations locked in the crust, the lowest overtone is of dipole degree, $\ell=1$, as is clearly seen in Fig. 5. This suggests that dipole vibration can be thought of as Goldstone's soft mode whose most conspicuous property is that the mode disappears (the frequency tends to zero) when key parameter regulating the depth of seismogenic zone $\lambda \to 0$.

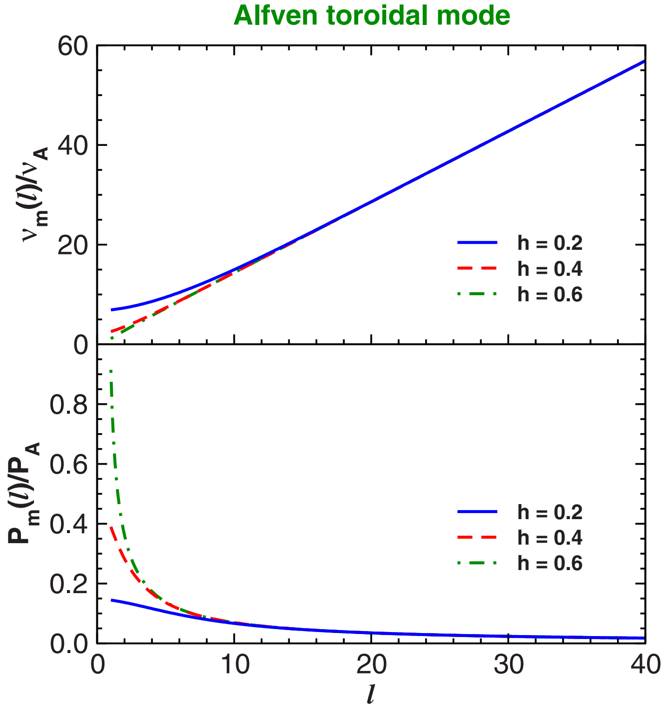

FIG. 4. (Color online) Fractional frequency and period of nodeless torsional magneto-solid-mechanical oscillations, toroidal Alfvén mode ${}_0a^t_\ell$, entrapped in the neutron star crust as functions of multipole degree ℓ computed at indicated values of the fractional depth h of peripheral seismogenic layer. The value $h=1$ corresponds to global torsional oscillations excited in the entire volume of the star. Here $\nu_A=\omega_A/2\pi$, where $\omega_A=v_A/R$ with $v_A=B/\sqrt{4\pi\rho}$ being the velocity of Alfvén wave in crustal matter of density ρ and $P_A=2\pi/\omega_A$.

IV. APPLICATION TO QPOS IN THE OUTBURST X-RAY FLUX FROM SGR 1806–20 AND SGR 1900+14

The basic physics underlying current understanding of interconnection between quasiperiodic oscillations of detected electromagnetic flux and vibrations of neutron star has

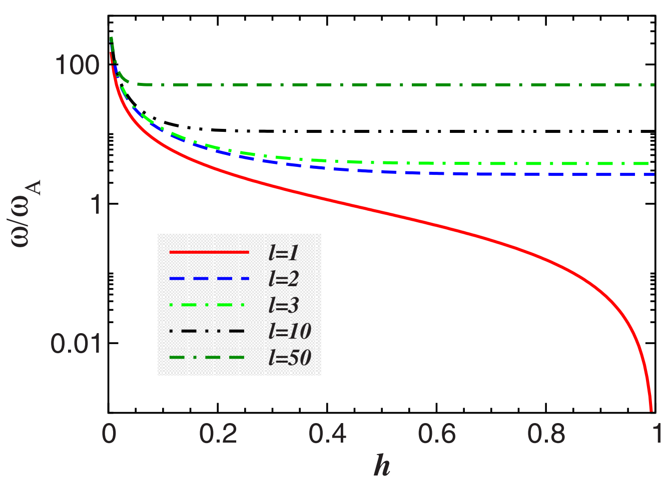

FIG. 5. (Color online) Fractional frequency of nodeless torsional Alfvén oscillations of indicated overtones ℓ as a function of the fractional depth h of peripheral seismogenic layer. The vanishing of dipole overtone in the limit of $h\to 1$, the case when entire mass of neutron star sets in torsional oscillations, suggests that dipole vibration possesses property typical to Goldstone's soft modes.

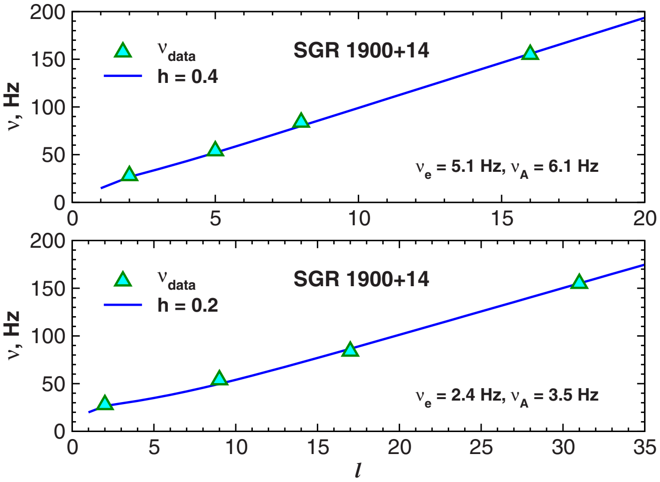

FIG. 6. (Color online) Theoretical fit of the QPOs frequency in the x-ray flux from SGR 1900+14 on the basis of three-parametric theoretical spectrum of frequency of torsional seismic vibrations in the crustal region of indicated fractional depth.

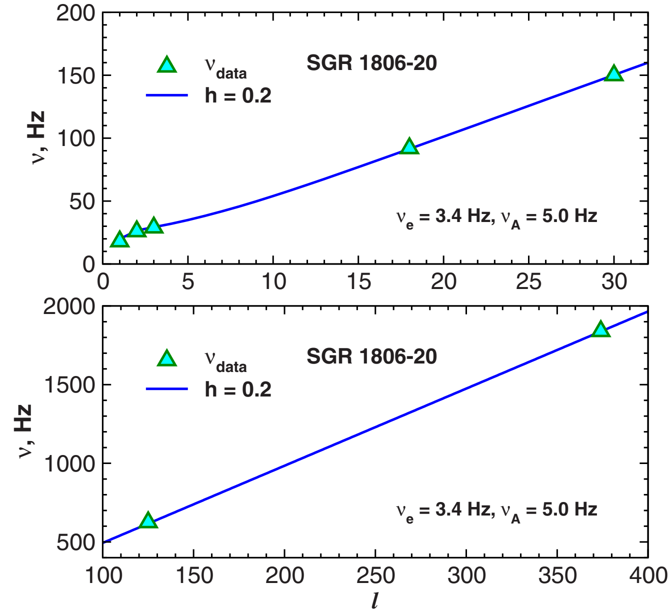

FIG. 7. (Color online) The same as Fig. 6, but for SGRs 1806–20 with h=0.2.

been recognized long ago.[23,25] Owing to the effect of strong flow-field coupling, which is central to the propagation of Alfvén waves, the quake-induced perturbation excites coupled vibrations of perfectly conducting solid-state plasma of the crust (as well as gaseous plasma of magnetar corona expelled from the surface by outburst) and frozen-in lines of magnetic field. Outside the star the vibrations of magnetic field lines are coupled with oscillations of gas-dust plasma expelled from the star surface by quake. Additionally, it is these fluctuations of outer lines of magnetic field, operating like transmitters of beams of charged particles producing coherent (curvature and/or synchrotron) high-energy radiation, are detected as QPOs of light curves of the SGRs giant flares.

In applying the obtained spectral formulas to the frequencies of detected QPOs, we examine two scenarios, namely, when quake-induced torsional vibrations are restored by joint action of Lorentz magnetic and Hooke's elastic forces and when oscillations are of pure Alfvén's nature, that is, produced by torsional seismic vibrations of crust against core under the action of solely one Lorentz force of magnetic field stresses.

A. Crust vibrations driven by combined action of Lorentz magnetic and Hooke's elastic forces

In this case, the asteroseismic analysis of detected QPOs rests on the three-parametric spectral formula,

$$\nu^2(\ell)[\nu_A,\nu_e,h] = \nu_m^2(\ell)[\nu_A,h] + \nu_e^2(\ell)[\nu_e,h]. \qquad (60)$$

The suggested theoretical ℓ-pole specification of the detected frequencies is presented in Fig. 6 for SGR 1900+14 and in Figs. 7 and 8, exhibiting remarkable correlation between depth of seismogenic zone and fundamental frequencies of magnetic and elastic oscillations—the larger ΔR, the higher basic frequencies of Alfvénic ν_A and elastic ν_e vibrations. It is seen from computations for SGR 1806–20 that reasonable fit of data can be attained with $h=0.2$ (for the star model with radius of 20 km, $\Delta R=2$ km) and with $h=0.4$ ($\Delta R=5$ km).

It is worth emphasizing that at above values of h, the obtained here tree-parametric spectral formula much better matches the data as compared to that for global, in the entire volume, vibrations studied in Ref. 16. On this ground we conclude that if the detected QPOs are produced by seismic vibrations of peripheral region of the star under coherent action of Lorentz and Hooke's forces, then the depth of seismogenic layer ΔR should be quite large, somewhere in the range $0.2R < \Delta R < 0.4R$.

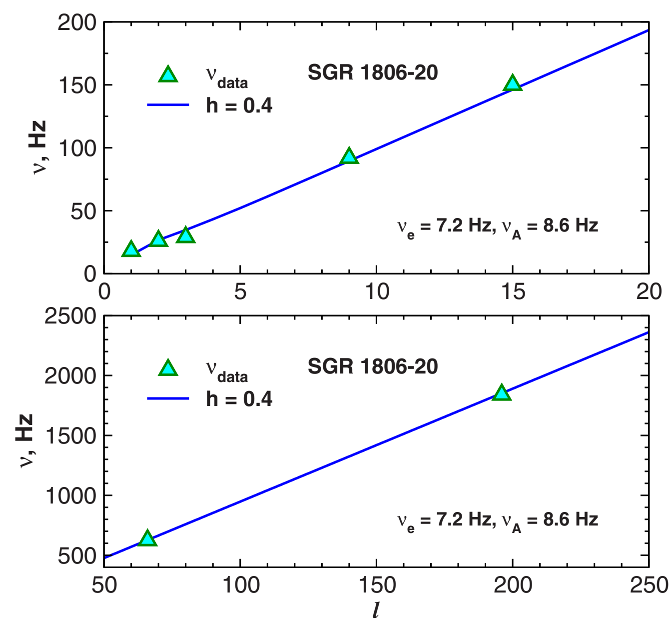

FIG. 8. (Color online) The same as Fig. 6, but for SGRs 1806–20 with h=0.4.

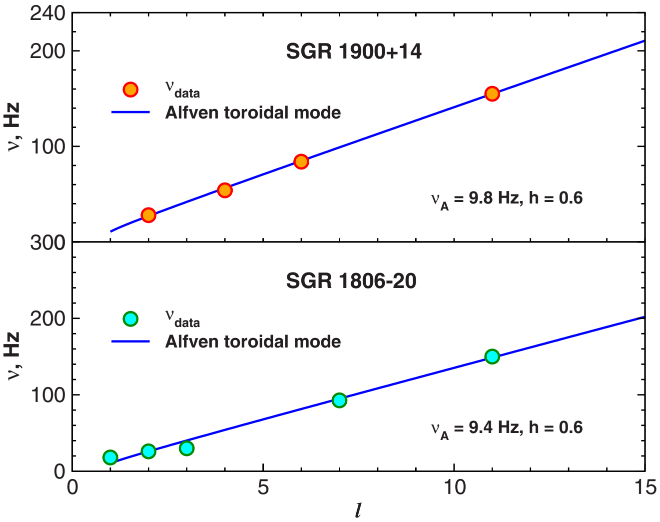

FIG. 9. (Color online) Theoretical description (lines) of detected QPO frequencies (symbols) in the x-ray flux during the flare of SGRs 1806–20 and SGR 1900+14 as overtones of pure Alfvén torsional nodeless oscillations of crustal magnetoactive plasma under the action of solely Lorentz restoring force.

B. Lorentz-force-driven vibrations of crustal solid-state plasma

It seems appropriate to note that pure Alfvén oscillations of crustal electron-nuclear solid-state plasma about axis of magnetic field frozen in the immobile core have been studied some time ago[29] in the context of searching for fingerprints of postglitch vibrational behavior of radio pulsars. In the problem under consideration, one can use one and the same spectral formula for the ℓ-pole specification of detected QPOs which in above notations is written as

$$\nu^2(\ell) = \nu_m^2(\ell)[v_A, h]. \tag{61}$$

The results presented in Figs. 9 and 10 show that at indicated input parameters, i.e., the Alfvén frequency ν_A and the fractional depth of seismogenic layer $h=\Delta R/R$, the model too adequately reproduces general trends in the data with fairly reasonable ℓ-pole specification of overtones pointed out by integer numbers along x-axis. It is seen that the low-frequency QPOs in data for SGR 1806–20 are interpreted as dipole and quadrupole overtones: $\nu({}_0a_1^t)=18$ and $\ell({}_0a_2^t)=26$ Hz. Moreover, the high-frequency kilohertz vibrations with 627 and 1870 Hz are unambiguously specified as high-multipole overtones: $\nu({}_0a_{\ell=42}^t)=627$ Hz and $\nu({}_0a_{\ell=122}^t)=1870$ Hz. However, in this latter scenario of Lorentz-force-dominated vibrations, the best fit of data is attained at fairly large value of fractional depth, $h=0.6$, which is much larger than the expected depth of the crust. In our opinion, this result may be regarded as indication to that the detected QPOs are formed by coherent vibrations of crustal solid-state plasma and plasma of magnetar corona.

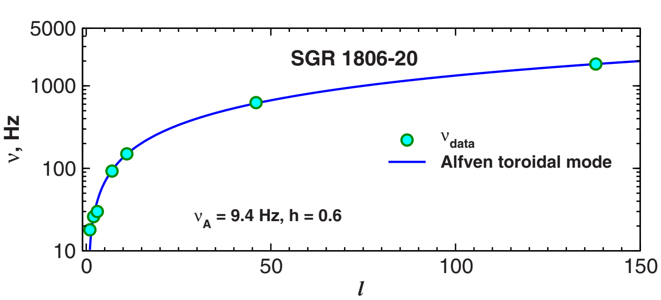

FIG. 10. (Color online) Same as Fig. 9 but for SGRs 1806–20.

V. CONCLUDING REMARKS

Ever since identification of pulsars with rapidly rotating neutron stars, it has been argued[40,41] that two key properties of these compact objects—(i) the degeneracy of neutron (nonconducting) Fermi-matter whose pressure opposes the pressure of self-gravity and (ii) a highly stable to decay superstrong magnetic fields—can be reconciled, if poorly conducting neutron-dominated stellar matter, constituting the neutron star cores, has been brought to gravitational equilibrium in the permanently magnetized state. The most plausible is the state of Pauli's paramagnetic saturation with spin magnetic moments of neutrons polarized along the axis of fossil field inherited from massive progenitor and amplified in magnetic-flux-conserving core-collapse supernova.[6] This idea is central to the considered two-component, core-crust, model of paramagnetic neutron star whose less dense and highly conducting, metal-like, material of the crust is considered as a solid-state, electron-nuclear, plasma pervaded by frozen in the core magnetic field. This difference between electrodynamic properties of core and crust matter (permanently magnetized nonconducting core and perfectly conducting nonmagnetic crust) suggests that magnetic cohesion between massive core (permanent magnet) and crust (metal-like material) should play central role in seismic activity of the star. Working from such an understanding, we have computed frequency spectra of node-free torsional oscillations of crust against immobile core under the action Lorentz and Hooke restoring forces and damped by Newtonian viscous force. As a trial function of oscillating material displacements, we have used the node-free toroidal field computed from vector Laplace equation. The obtained spectral formulas are of some interest in their own right because they can be applied to more wide class of celestial objects. In this work we applied the obtained analytic frequency spectra to magnetars, highly magnetized quaking neutron stars whose bursting seismic activity is commonly associated with release of magnetic field stresses. Focus was laid on forward asteroseismic analysis of fast x-ray flux oscillations during the giant flare of SGR 1900+14 and SGR 1806–20 and, thus, assuming that these oscillations are produced by torsion vibrations of crustal solid-state plasma about axis of dipole magnetic field frozen in the immobile permanently magnetized core. In so doing we have investigated two cases of postquake vibrational relaxation of the star, depending on restoring forces. In first case, the analysis of data has been based on assumption that detected QPOs owe their existence to node-free torsional vibrations of crust against core restored by joint action of Lorentz magnetic and Hooke's elastic forces. Additionally, we found that obtained three-parametric spectral formula provides much better fit of data than two-parametric frequency spectrum of global vibrations (Bastrukov *et al.*). The considered second scenario presumes that vibrations are dominated by solely Lorentz restoring force of magnetic field stresses. We found that obtained two-parametric frequency spectrum can too be fairly reasonably reconciled with detected QPO frequencies. All the above lead us to conclude that Lorentz restoring force of magnetic field stresses plays decisive part in quake-induced torsional vibrations of crustal solid-state plasma of magnetars.

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