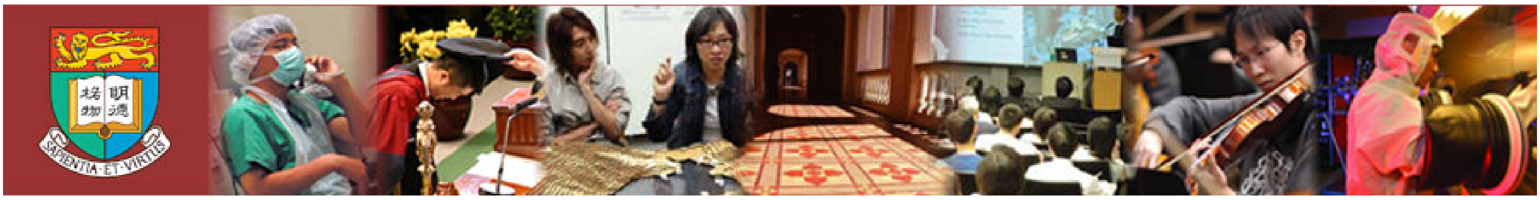

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Controller Synthesis for Positive Systems Under ℓ_1-induced Performance

Xiaoming Chen, James Lam, Ping Li and Zhan Shu

***Abstract*— In this paper, we investigate the problem of controller design for positive systems with the use of linear Lyapunov function. We first present an analytical method to compute the exact value of the ℓ_1-induced norm. Then, we propose a novel characterization under which discrete positive system is asymptotically stable with a prescribed ℓ_1-induced performance. Based on the characterization, a necessary and sufficient condition for the existence of desired controllers is presented, and an iterative linear matrix inequality approach is developed to solve the design condition. Finally, a numerical example is given to illustrate the effectiveness of the proposed theoretical results.**

***Index Terms*— Iterative algorithm, Linear Lyapunov functions, ℓ_1-induced performance, Positive systems**

I. INTRODUCTION

In many practical systems, there is a kind whose state variables naturally take non-negative values. These systems have been studied in different fields of application ranging from biology and chemistry, to economy and sociology [1], [2]. Positive dynamic systems possess many special characteristics, bringing about many new problems to tackle. Consequently, in recent years positive systems have drawn considerable research interest in the control community and a large number of theoretical contributions to this field have appeared [3], [4], [5]. To name a few, Luenberger proposed a system-theoretic approach to positive systems in [6]. Since then, many results have been reported in the literature, see [7], [8], [9] for instance. For example, necessary and sufficient conditions for positive realizability by means of convex analysis were derived in [8]. A positive state-space representation of a given transfer function was characterized by Farina and Benvenuti in [9]. Stability theory for nonnegative and compartmental dynamic systems with time delay was investigated in [10], [11], [12]. As for the results on 2-D positive systems, we refer readers to [4], [13]. Necessary and sufficient conditions are provided to solve the stabilization problem of positive systems in [14], [15]. Some results on the model reduction problem for positive systems can be found in [16], [17].

Moreover, it is noted that many previous results concerning the positive systems were based on quadratic Lyapunov functions and a large number of these results were formulated under the linear matrix inequality (LMI) framework [18]. Recently, some results based on linear Lyapunov functions have emerged [1], [19], [20]. The motivation for using a linear Lyapunov function is that the state of a positive system is nonnegative and hence a linear Lyapunov function serves as a valid candidate. Compared with previous results based on quadratic Lyapunov functions, the results obtained by means of linear Lyapunov functions are easier to analyze. Unfortunately, little attempt has been made to investigate the issue of controller synthesis for positive systems via linear Lyapunov functions, which motivates the present research.

In this paper, we investigate the problem of controller design for positive linear systems by virtue of linear Lyapunov function. More specifically, we present an ℓ_1-induced performance index which fits well into the newly introduced linear Lyapunov function. Based on the performance, desired controllers are derived under which the stability of the closed-loop system and the satisfaction with the proposed performance are fulfilled. It is worth pointing out that the approach developed in this paper has the advantage that the results obtained are simple and under the framework of linear programming, easy to compute, and characterized under the framework of linear programming.

The rest of this paper is organized as follows. In Section II, preliminaries are introduced and the ℓ_1-induced performance is developed for positive linear systems. In Section III-A, a method to compute the exact value of ℓ_1-induced norm is proposed. In Section III-B.1, a novel characterization is put forward under which the positive linear system is asymptotically stable and satisfies the performance. Based on the analysis condition, the controller is designed for positive systems in Section III-B.2. In Section IV, an example is proposed to show the effectiveness and applicability of the theoretical results. Conclusions are given in Section V.

II. PROBLEM FORMULATION

In this section, we introduce notations and several results concerning positive linear systems.

Let $\mathbb{R}$ be the set of real numbers; $\mathbb{R}^n$ denotes the *n*-column real vectors; $\mathbb{R}^{n\times m}$ is the set of all real matrices of dimension $n \times m$; $\bar{\mathbb{R}}^n_+$ is the nonnegative orthants of $\mathbb{R}^n$; that is, if $x \in \mathbb{R}^n$, then $x \in \bar{\mathbb{R}}^n_+$ is equivalent to $x \geq\geq 0$. $\mathbb{N}$ is the set of natural numbers. For a matrix $A \in \mathbb{R}^{m\times n}$, a_{ij} denotes the element located at the ith row and the jth column. $A \geq\geq 0$ (respectively, $A >> 0$) means that for all i and j, $a_{i,j} \geq 0$ (respectively, $a_{i,j} > 0$). The notation $A \geq\geq B$ (respectively, $A >> B$) means that the matrix $A - B \geq\geq 0$ (respectively, $A - B >> 0$); The superscript "T" denotes matrix transpose. $\|\cdot\|$ represents the Euclidean norm for vectors. The 1-norm of a vector $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))$ is defined as

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$\|x(k)\|_1 \triangleq \sum_{i=1}^{n} |x_i(k)|$ and the induced 1-norm of a matrix $Q \triangleq [Q_{ij}] \in \mathbb{R}^{p\times q}$ is denoted by $\|Q\|_1 \triangleq \max_{1\leq j\leq q} (\sum_{i=1}^{p} |Q_{ij}|)$. The ℓ_1-norm is defined as $\|x\|_{\ell_1} \triangleq \sum_{k=0}^{\infty} \|x(k)\|_1$. We denote $\mathbf{1} = [1, 1, \ldots, 1]^T$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

Consider a discrete-time linear system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bw(k),\ x(0) = x_0, \\ y(k) &= Cx(k) + Dw(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n, w(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^r$ are the system state, input, and output, respectively; A, B, C and D are constant system matrices.

Definition 1: System (1) is said to be a discrete-time positive linear system if for all $x(0) \geq\geq 0$ and all input $w(k) \geq\geq 0$, we have $x(k) \geq\geq 0$ and $y(k) \geq\geq 0$ for $k \in \mathbb{N}$. The following lemma provides a characterization for positive linear systems.

Lemma 1 ([15]): System (1) is a discrete-time positive linear system if and only if $A \geq\geq 0,\ B \geq\geq 0,\ C \geq\geq 0,\ D \geq\geq 0$.

It should be stressed here that, $x(0) \geq\geq 0$ and $w(k) \geq\geq 0$ are essential for the positivity of the output $y(k)$. When $x(0) \geq\geq 0$ and $w(k) \geq\geq 0$ are not satisfied, $x(k)$ may not stay in the first orthant even if the conditions of Lemma 1 hold. In the real world, this is often guaranteed by the features of practical physical systems. For example, in some population models, the state variables and input represent biomass, the number of species or density.

Now we introduce the ℓ_1-induced performance used in this paper. We say that system (1) has ℓ_1-induced performance at the level γ if, under zero initial conditions, for all nonzero $w \in \ell_1$,

$$\|y\|_{\ell_1} < \gamma \|w\|_{\ell_1}, \tag{2}$$

or equivalently,

$$\sum_{k=0}^{\infty} \|y(k)\|_1 < \gamma \sum_{k=0}^{\infty} \|w(k)\|_1, \tag{3}$$

where $\|y(k)\|_1 = \sum_{i=1}^{r} y_i(k)$ and $\|w(k)\|_1 = \sum_{i=1}^{m} w_i(k)$ are the 1-norm of $y(k)$ and $w(k)$, respectively; $\gamma > 0$ is a given performance level.

The problem to be addressed in this paper is described as follows.

Problem PPL1CD (Positivity-Preserving ℓ_1-induced Controller Design)

Given a positive system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_w w(k), \\ y(k) = Cx(k) + Du(k) + D_w w(k), \end{cases}$$

the control objective is to find a controller $u(k) = Kx(k)$ such that the closed-loop system

$$\begin{cases} x(k+1) = (A + BK)x(k) + B_w w(k), \\ y(k) = (C + DK)x(k) + D_w w(k), \end{cases} \tag{4}$$

is positive, asymptotically stable, and satisfies the ℓ_1-induced performance in (2) under zero initial condition.

Remark 1: It is noted that some frequently used performance measures such as H_∞ norm are based on the ℓ_2 signal space [21]. In some situation, these performance measures induced by ℓ_2 signal are not very natural to describe some of the features of practical physical systems. On the other hand, 1-norm can provide a more useful description for positive systems because 1-norm gives the sum of the values of the components, which is more appropriate, for instance, if the values represent the amount of material or the number of animal in a species.

III. Main Results

In this section, we first propose a method to compute the exact value of ℓ_1-induced norm for system (1). A novel characterization on the stability and the ℓ_1-induced performance of (1) is established. Then, a necessary and sufficient condition for the existence of controller is proposed, and an iterative LMI approach is developed to compute the controller matrices.

A. Exact Computation of ℓ_1-Induced Norm

In this section, we establish an analytical method through which the value of ℓ_1-induced norm of system (1) is computed directly.

Theorem 1: For a stable positive linear system given in (1), the exact value of ℓ_1-induced norm

$$\|G\|_{\ell_1,\ell_1} = \|C(I - A)^{-1}B + D\|_1, \tag{5}$$

where $G : \ell_1 \to \ell_1$ denotes the convolution operator, that is, $y(k) = (G * w)(k)$.

Proof: From system (1), we know that

$$\begin{aligned} y(0) &= Cx(0) + Dw(0), \\ y(1) &= CAx(0) + CBw(0) + Dw(1), \\ y(2) &= CA^2x(0) + CBw(1) \\ &\quad + CABw(0) + Dw(2), \\ &\ \vdots \\ y(k) &= CA^k x(0) + C\sum_{m=1}^{k} A^{k-m} \\ &\quad Bw(m-1) + Dw(k). \end{aligned}$$

Under the assumption that $x(0) = 0$, we have

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(s) \end{bmatrix} = Q \begin{bmatrix} w(0) \\ w(1) \\ w(2) \\ \vdots \\ w(s) \end{bmatrix}, \tag{6}$$

where

$$
\begin{aligned}
Q &= \begin{bmatrix} D & 0 & \cdots & & 0 \\ CB & D & & & \\ CAB & CB & D & \ddots & \\ \vdots & & & & \\ CA^{s-1}B & \cdots & CAB & CB & D \end{bmatrix} \\
&= [Q_{ij}] \in \mathbb{R}^{(s+1)r\times(s+1)m}.
\end{aligned}
$$

Taking the 1-norm of both sides of (6) yields

$$\sum_{k=0}^{s} \|y(k)\|_1 \leq \|Q\|_1 \sum_{k=0}^{s} \|w(k)\|_1, \tag{7}$$

where $\|Q\|_1 = \max_j(\sum_{i=1}^{(s+1)r} Q_{ij})$, that is, $\|Q\|_1 = \|CB + CAB + \cdots + CA^{s-1}B + D\|_1$.

Since A is stable, as $s \to \infty$, $\|Q\|_1 \to \|C(I-A)^{-1}B + D\|_1$, which leads to $\|y\|_{\ell_1} \leq \|C(I-A)^{-1}B + D\|_1\|w\|_{\ell_1}$.

In the following, we investigate the condition under which $\frac{\|y\|_{\ell_1}}{\|w\|_{\ell_1}}$ reaches its supreme value. Suppose all the components of vector $w(k)$ are equal to zero except at time $k = 0$. We have

$$
\begin{aligned}
y(0) &= Dw(0), \\
y(1) &= CBw(0), \\
y(2) &= CABw(0), \\
&\vdots \\
y(s) &= CA^{s-1}Bw(0),
\end{aligned} \tag{8}
$$

which yields $\sum_{k=0}^{s} \|y(k)\|_1 = \mathbf{1}^T(CB + CAB + \cdots + CA^{s-1}B + D)w(0)$.

Since A is stable, as $s \to \infty$,

$$
\begin{aligned}
\|y\|_{\ell_1} &= \mathbf{1}^T(C(I-A)^{-1}B + D)w(0) \\
&\leq \|C(I-A)^{-1}B + D\|_1\|w\|_{\ell_1}.
\end{aligned}
$$

Now denote $\bar{Q} \triangleq \mathbf{1}^T(C(I-A)^{-1}B + D)$. Without loss of generality, we assume that the norm $\|\bar{Q}\|_1 = \max_j(\bar{Q}_j)$ achieves its supreme value at the j^*_{th} column and all the components of vector $w(0)$ are equal to zero except at the j^*_{th} row, we have that $\frac{\|y\|_{\ell_1}}{\|w\|_{\ell_1}}$ achieves its supremal value $\|C(I-A)^{-1}B + D\|_1$. □

B. Controller Design

In this section, we aim to construct a positive state feedback controller such that the closed-loop system is asymptotically stable and satisfies the performance in (2).

1) Performance Characterization: Before presenting Theorem 2, we first introduce the following lemma in [20].

Proposition 1 ([20]): The positive linear system given by (1) is asymptotically stable if and only if there exist vectors $p \geq\geq 0$, $r >> 0$ satisfy

$$p^T A + r = p^T. \tag{9}$$

When the input w is taken into account, we can derive the following result which provides a fundamental characterization on the stability of system (1) with the performance in (2).

Theorem 2: The positive linear system in (1) is asymptotically stable and satisfies $\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1} (w \neq 0)$ if and only if there exists a vector $p \geq\geq 0$ satisfying

$$
\begin{aligned}
\mathbf{1}^T C + p^T A - p^T &<< 0, & (10)\\
p^T B + \mathbf{1}^T D - \gamma\mathbf{1}^T &<< 0, & (11)
\end{aligned}
$$

Proof: *Sufficiency:* First, we assume that there exists an integer k such that $x(k) \neq 0$. From (10), we can see that (9) holds, and thus the asymptotic stability of system (1) is proved.

Consider the linear Lyapunov function candidate $V(x) = p^T x$, computing the Lyapunov difference yields

$$\Delta V(k) = p^T(Ax(k) + Bw(k)) - p^T x(k).$$

Let

$$
\begin{aligned}
J &= \|y(k)\|_1 - \gamma\|w(k)\|_1 \\
&= \sum_{i=1}^{r} y_i(k) - \gamma \sum_{i=1}^{m} w_i(k) \\
&= \left[\sum_{i=1}^{r} y_i(k) - \gamma \sum_{i=1}^{m} w_i(k) + \Delta V(k)\right] - \Delta V(k) \\
&= \big[\mathbf{1}^T Cx(k) + \mathbf{1}^T Dw(k) - \gamma\mathbf{1}^T w(k) \\
&\quad + p^T(Ax(k) + Bw(k)) - p^T x(k)\big] - \Delta V(k) \\
&= \left[\mathbf{1}^T C + p^T(A-I)\right] x(k) \\
&\quad + (\mathbf{1}^T D + p^T B - \gamma\mathbf{1}^T)w(k) - \Delta V(k) \\
&= \left[\mathbf{1}^T C + p^T(A-I) + \varepsilon\mathbf{1}^T\right] x(k) + (\mathbf{1}^T D \\
&\quad + p^T B - \gamma\mathbf{1}^T)w(k) - \varepsilon\mathbf{1}^T x(k) - \Delta V(k),
\end{aligned} \tag{12}
$$

where $\varepsilon > 0$ is sufficiently small.

From (10) and (11), we have

$$
\begin{aligned}
&\sum_{k=0}^{s}\sum_{i=1}^{r} y_i(k) + \varepsilon \sum_{k=0}^{s}\sum_{i=1}^{n} x_i(k) \\
&< \gamma \sum_{k=0}^{s}\sum_{i=1}^{m} w_i(k) - V(s+1).
\end{aligned} \tag{13}
$$

Since the system is asymptotically stable, we have

$$\sum_{k=0}^{\infty}\sum_{i=1}^{r} y_i(k) + \varepsilon \sum_{k=0}^{\infty}\sum_{i=1}^{n} x_i(k) \leq \gamma \sum_{k=0}^{\infty}\sum_{i=1}^{m} w_i(k), \tag{14}$$

which implies

$$\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1}. \tag{15}$$

Next we consider the case with $x(k) = 0$. From (10), the asymptotic stability of system (1) is proved. It is easy to see that if $x(k) = 0$, we have $y(k) = Dw(k)$ and from (11), $\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1}$ holds. This proves sufficiency.

Necessity: Assume that system (1) is asymptotically stable and satisfies the performance $\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1}$. Now it follows that (14) holds, that is, under zero initial conditions,

$$\sum_{k=0}^{\infty}\left(\sum_{i=1}^{r} y_i(k) - \gamma \sum_{i=1}^{m} w_i(k)\right) < 0, \tag{16}$$

which is equal to

$$(\mathbf{1}^T D + \mathbf{1}^T C(I-A)^{-1}B - \gamma \mathbf{1}^T)\sum_{k=0}^{\infty} w(k) < 0. \tag{17}$$

As $0 \neq w \in \ell_1$ is arbitrary, inequality (17) implies

$$\mathbf{1}^T D + \mathbf{1}^T C(I-A)^{-1}B - \gamma \mathbf{1}^T << 0. \tag{18}$$

Define $\tilde{p} \triangleq (\mathbf{1}^T C(I-A)^{-1})^T \geq\geq 0$ and $p \triangleq \tilde{p} + \epsilon\alpha >> 0$, where $\alpha >> 0$ satisfies $\alpha^T(I-A) >> 0$, and $\epsilon > 0$ is sufficiently small. We have

$$\begin{aligned} \mathbf{1}^T C + p^T A - p^T &= \mathbf{1}^T C - (\tilde{p}^T + \epsilon\alpha^T)(I-A) \\ &= \mathbf{1}^T C - \mathbf{1}^T C - \epsilon\alpha^T(I-A) \\ &= -\epsilon\alpha^T(I-A) \\ &<< 0. \end{aligned} \tag{19}$$

On the other hand,

$$\begin{aligned} &\mathbf{1}^T D + p^T B - \gamma\mathbf{1}^T \\ &= \mathbf{1}^T D + \mathbf{1}^T C(I-A)^{-1}B + \epsilon\alpha^T B - \gamma\mathbf{1}^T. \end{aligned} \tag{20}$$

From (18) and ϵ is sufficiently small, we have that (11) holds. □

Remark 2: The condition obtained in Theorem 2 is necessary and sufficient in terms of linear programming which can be verified and solved efficiently. Hence, we can easily obtain γ and feasible solution p with the help of convex optimization techniques.

2) Controller Synthesis: This subsection is devoted to the synthesis of the state-feedback controller. Based on the analysis in subsection III-B.1, a necessary and sufficient condition for the existence of a solution to Problem PPL1CD is obtained. Then, an iterative LMI approach is developed to compute the controller matrices accordingly.

Theorem 3: The closed-loop system (4) is positive, asymptotically stable and satisfies $\|y\|_{\ell_1} < \gamma\|w\|_{\ell_1}$ if and only if there exist a matrix K and a vector $p \geq\geq 0$ satisfying that

$$A + BK \geq\geq 0, \tag{21}$$

$$C + DK \geq\geq 0, \tag{22}$$

$$\mathbf{1}^T(C + DK) + p^T(A + BK) - p^T << 0, \tag{23}$$

$$p^T B_w + \mathbf{1}^T D_w - \gamma\mathbf{1}^T << 0. \tag{24}$$

Although the existence of a controller can be characterized according to Theorem 3, it is still difficult to obtain K due to the presence of the term $p^T BK$. In the following, our aim is to derive a numerically tractable mean to synthesize the required controllers with the help of convex optimization.

It is noted that when matrix K is fixed, (23) turns out to be linear with respect to the other variables. Therefore, a natural way is to fix K, and solve (23)–(24) by linear programming. Thus, the following iterative algorithm can be proposed to solve the problem (see [22]).

Algorithm PPL1CD:

1) Set $i = 1$. Select an initial matrix K_1 such that system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_w w(k), \\ y(k) = Cx(k) + Du(k) + D_w w(k), \end{cases}$$

with

$$u(k) = K_1 x(k) \tag{25}$$

is asymptotically stable and $A + BK_1 \geq\geq 0$ and $C + DK_1 \geq\geq 0$.

2) For fixed K_i, solve the following optimization problem for p_i and γ_i.
OP: Minimize γ_i subject to the following constraints:

$$\begin{aligned} \mathbf{1}^T(C + DK) + p_i^T(A + BK_i) - p_i^T &<< 0, \\ p_i^T B_w + \mathbf{1}^T D_w - \gamma_i \mathbf{1}^T &<< 0, \\ p_i &\geq\geq 0. \end{aligned}$$

Denote γ_i^* as the minimum value of γ_i.

3) For fixed p_i, solve the following feasibility problem for K_i.
FP: Find K_i subject to the following constraints:

$$\begin{aligned} A + BK_i &\geq\geq 0, \\ C + DK_i &\geq\geq 0, \\ \mathbf{1}^T(C + DK) + p_i^T(A + BK_i) - p_i^T &<< 0. \end{aligned}$$

4) If $\left|\left(\gamma_i^* - \gamma_{i-1}^*\right)/\gamma_i^*\right| < \varepsilon_1$, where ε_1 is a prescribed tolerance, then a solution to *Problem PPL1CD* may not exist. STOP.
else set $i = i + 1$ and $K_i = K_{i-1}$, then go to Step 2.

Remark 3: The selection of K_1 in Step 1 can be made easily. In fact, from [23], we know that system (4) with (25) is positive and asymptotically stable if and only if there exist a diagonal matrix $P \triangleq \mathrm{diag}(p_1, p_2, \ldots, p_n)$ and $Q \triangleq [q_{ij}] \in \mathbb{R}^{l\times n}$ such that

$$\begin{bmatrix} -P & AP + BQ \\ * & -P \end{bmatrix} < 0. \tag{26}$$

$$a_{ij}p_j + \sum_{z=1}^{l} b_{iz}q_{zj} \geq 0. \tag{27}$$

Under this condition, an initial choice of K can be given by $K_1 = QP^{-1}$.

Remark 4: The parameter γ can be optimized iteratively. Notice that $\gamma_{i+1}^* \leq \gamma_i^*$ since the corresponding parameters obtained in Step 4 will be utilized as the initial conditions to derive a smaller γ. Therefore, the convergence of the iterative process is naturally guaranteed. Moreover, it follows from Step 1 that if one cannot find such a matrix K_1, then it can be concluded immediately that there does not exist a solution to *Problem PPL1CD*. In fact, the initial matrix K_1 can be viewed as a state-feedback controller matrix, and be constructed by existing convex optimization approaches.

IV. Illustrative Example

In this section, we present an illustrative example to demonstrate the applicability of the proposed results.

It is well known that positive systems are widely studied in many areas. One of these areas where the discrete-time positive systems arise is population models [20]. Among different age-structured population models, the Leslie model is the most classical and widely used in population dynamics and control In this model, individuals are supposed to be subject to some given specific rates of fertility and mortality [2]. In this example, we investigate the structured population dynamics of a certain pest described by a Lesile model. An external disturbance is brought into consideration and our aim is to annihilate the pests in a certain area. We consider the following Leslie model:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_w w(k), \\ y(k) &= Cx(k) + Du(k), \end{aligned} \tag{28}$$

where

$$A = \begin{bmatrix} 0.2 & 0.3 & 2 \\ 0.8 & 0 & 0 \\ 0 & 0.7 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}, \tag{29}$$

$$B_w = \begin{bmatrix} 0.1 \\ 0.05 \\ 0.1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \; D = 0.15. \tag{30}$$

In this model, $x(k) = \begin{bmatrix} x_1(k) & x_2(k) & x_3(k) \end{bmatrix}^T$ where $x_i(k)$ represents the number of individuals of age i in year k before the reproduction season. The external input, denoted as $w(k)$, is regarded as a measure of the population of the pests from other regions that flows into the area of interest. The output $y(k)$ denotes the sum of the number of pests in the area. In what follows, we shall use the method proposed in this paper to design a required controller.

By resorting to Theorem 3, we have that $\gamma^* = 0.2900$ approximately after 20 iterations and a feasible solution is achieved with

$$p = \begin{bmatrix} 1.8994 & 1.1936 & 0.4009 \end{bmatrix}^T,$$

which yields the controller gain matrix as

$$K = \begin{bmatrix} -0.3979 & -0.5987 & -3.9996 \end{bmatrix}.$$

The performance of the open-loop and the closed-loop system is evaluated via simulation. The initial condition used in the simulation is

$$x(0) = \begin{bmatrix} 50 & 10 & 30 \end{bmatrix}^T.$$

Figure 1 shows the response of open-loop system and Figure 2 shows the state response of the closed-loop system when the external disturbance $w(k) \equiv 0$, from which we can see that the state converges to zero. To illustrate the disturbance attenuation performance, the external disturbance $w(k)$ is assumed to be

$$w(k) = \begin{cases} 50, & 5 \le k \le 10, \\ 0, & \text{otherwise.} \end{cases} \tag{31}$$

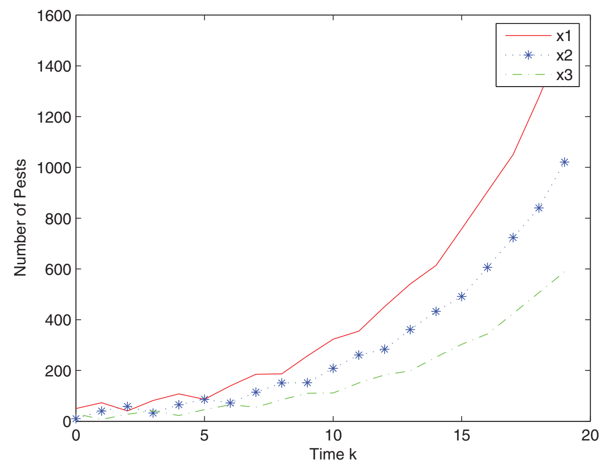

Fig. 1. Open-loop unforced response

Figure 3 shows the response of state variables with $w(k)$. From this example, we see that even under the influx of pests from other regions, the pests in this region can be annihilated finally by applying the control. In practice, the spraying of pesticide is usually one of the pest control methods. Here, the number of insects of all ages can be reduced by spraying pesticide, which corresponds to the introduction of state-feedback controller $u(k) = Kx(k)$ where K represents the amount of pesticide.

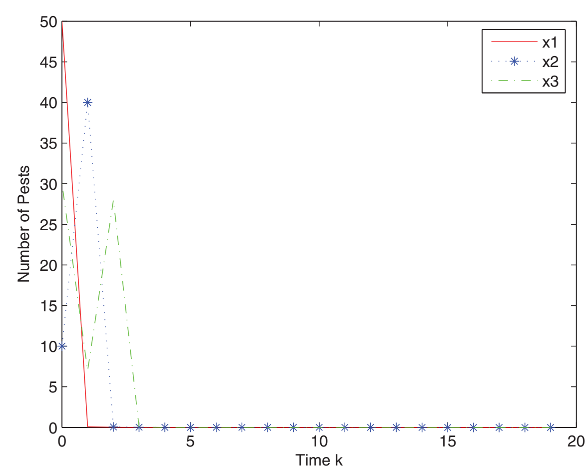

Fig. 2. Closed-loop unforced response

V. Conclusion

The problem of state-feedback controller design for positive system with the use of linear Lyapunov function has been studied. A method has been established to compute the exact value of ℓ_1-induced norm for positive system. A characterization has been proposed to ensure the asymptotic stability of the controlled system with a prescribed ℓ_1-induced performance level. The necessary and sufficient

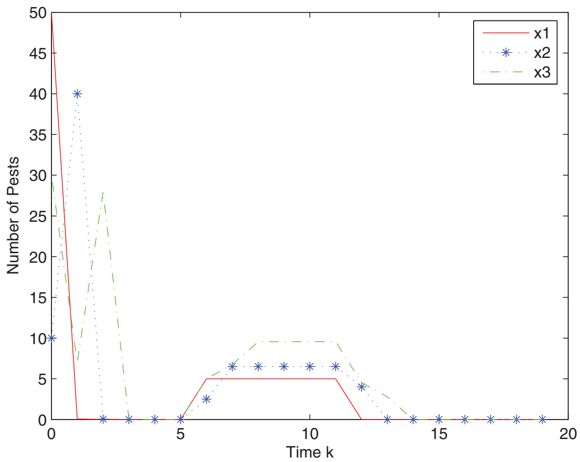

Fig. 3. Closed-loop forced response

condition for the existence of a desired controller has been established accordingly. Then, an iterative LMI algorithm has been developed to solve the design condition. Finally, an example has been presented to illustrate the effectiveness of the theoretical results.

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