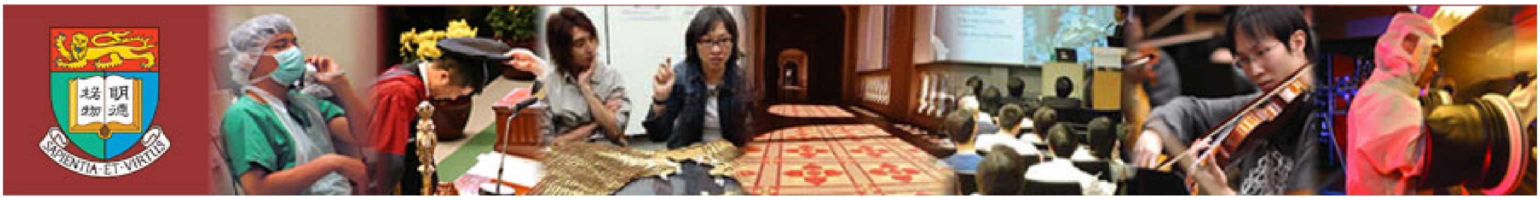

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Expected Stock Returns and the Conditional Skewness

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Motivated by the parsimonious jump-diffusion model of Zhang, Zhao and Chang (2010), we show that the aggregate market returns can be predicted by the conditional skewness of returns and the variance risk premium, a difference between the physical and risk-neutral variance of market returns, even though the variance is supposed to be constant only if jump exists. The magnitude of the predictability is particularly striking at the intermediate quarterly return horizon, even combing other predictor variables, like P/D ratio, the default spread and the consumption-wealth ratio (CAY). We also find that the third central moments are significant in explaining the variance risk premium, which further implies that the potential link between the variance risk premium and the excess market return is the third central moments, not the skewness.

Keywords: Equilibrium asset pricing; Conditional skewness; Return predictability; Variance risk premium; Third central moments.

JEL Classification Code: G12;

11.5

Expected Stock Returns and the Conditional Skewness

Abstract

Motivated by the parsimonious jump-diffusion model of Zhang, Zhao and Chang (2010), we show that the aggregate market returns can be predicted by the conditional skewness of returns and the variance risk premium, a difference between the physical and risk-neutral variance of market returns, even though the variance is supposed to be constant only if jump exists. The magnitude of the predictability is particularly striking at the intermediate quarterly return horizon, even combing other predictor variables, like P/D ratio, the default spread and the consumption-wealth ratio (CAY). We also find that the third central moments are significant in explaining the variance risk premium, which further implies that the potential link between the variance risk premium and the excess market return is the third central moments, not the skewness.

1 Introduction

The predictability of stock returns is still one of the most studied and widely attented issues in economics. Large literature documents the predictability of stock returns at the firm-level cross-sectional analysis. However, time-series predictability of the aggregate market returns is rarely studied. The Capital Asset Pricing Model (CAPM) implies that market risk premium should be rewarded by single factor, but literature, such as Fama and French (1992) and Boudoukh, Richardson and Smith (1993), find that the estimated market risk premium is not different from zero or at times, significantly less than zero. This implies that a single factor asset pricing model is not enough to explain or predict the market returns. Harvey and Siddiue (2000b) present an asset pricing model where skewness is priced which helps explain the negative ex ante market risk premiums.

Actually skewness is being attached more and more importance by researchers in recent years. Brunnermeier and Parker (2005) and Brunnermeier, Gollier and Parker (2007) think that investors overestimate their return and exhibit a preference for skewness in a portfolio choice, so positively skewed assets tend to have lower returns. Mitton and Vorkink (2007) develop an equilibrium model incorporating the skewness in utility function. Boyer, Mitton and Vorkink (2008) empirically investigate the relation between idiosyncratic skewness and expected returns. But these papers focus on the individual stocks and their cross-sectional behavior no caring about the fully diversified portfolio, such as market index return.

Many continuous-time models proposed in financial literature, stochastic volatility and jump diffusion model are the two popular ones to model the skewness. To be the relation intuitive, we hope to have a straightforward and clear model. The existence of jump in market has been largely documented. Todorov (2009) argue that jump is also a source of variance risk and play a very important role in explaining the variance risk premium. Zhang, Zhao and Chang (2010) also develop a general equilibrium model and implies that the expected market returns can be explained by the two factors, variance and skewness of the market returns. Furthermore, we find that the results also suggests expected market returns can also be predicted by the variance risk premium, a difference between the physical and risk-neutral variance of market returns, even though the variance is supposed to be constant only if jump exists. This parsimonious model motivate us to investigate the link between expected returns and variance risk premium, especially the role of the skewness or the third central moments of returns. Bollerslev, Tauchen and Zhou (2009) show that variance risk premium is able to explain a non-trivial fraction of the time-series variation in aggregate stock market which is motivated by a general equilibrium model. Bakshi and Madan (2006) derive a model to connect the volatility spread, the departure between risk-neutral and physical index volatility, to the higher order physical return moments and the parameters of the pricing kernel process. Our parsimonious model also gives the similar relation between the variance risk premium and the third central moments of returns.

Our empirical study focuses on the time-series behavior of the market excess return. Using the S&P 500 index as the proxy of market portfolio, and European out-of-money call and put index options to replicate the risk-neutral moments developed by Bakshi, Kapadia and Madan (2003), we run the regressions of the market returns for predictor variables.

We find that ex-post return variance has no predict power for the future excess return which is also found in Bollerslev, Tauchen and Zhou (2009). But the skewness and the third central moments are significant for shorter time horizons, such as one month and three months for individual regressions and the regressions combing other predict variables such as price-earning ratio, price-dividend ratio, the default spread, the term spread, the stochastically daily de-trended risk-free rate and the consumption-wealth ratio (CAY).

At the same time, unsurprisingly, variance risk premium are always significant for all time horizons as documented in Bollerslev, Tauchen and Zhou (2009). To investigate the link between the variance risk premium and skewness, we regress the third central moments and skewness of returns on variance risk premium. We find that the third central moments are significant in explaining the variance risk premium, but the skewness is not. With the significance in the regression of excess return on the third central moments of return, it is reasonable to think that probably potential link between the variance risk premium and the excess return is the third central moments, not the skewness.

The rest of this paper is organized as follows. Section 2 derive the relation between the excess return and the return moments and the variance risk premium. Section 3 describes the measurement of variables. Data resource and description are given in Section 4. Section 5 analyzes our empirical results. Section 6 concludes.

2 Jump Diffusion Model

It has been well documented presence of skewness in the conditional returns distribution. Many continuous-time models except Black-Scholes model have been proposed in the finance literature. The stochastic volatility and jump-diffusion model and model are the most popular ones. In particular, the sources of market variance risk are also from presence of stochastic volatility and occurrence of unanticipated market jumps. So jumps contribute market variance risk and the return skewness at the same time. To clarify the effect of skewness and variance risk premium on the expected market return, we choose the parsimonious jump diffusion model, supposing constant return variance, to identify their relations by closed-form expressions.

Following the jump-diffusion model in a production economy of Zhang, Zhao and Chang (2010), that is, suppose the a single aggregate stock that is understood as a stock index or a market portfolio follows

$$\frac{dSt}{St} = \mu dt + \sigma dBt + (e^x - 1)dNt - \lambda E(e^x - 1)dt; \tag{1}$$

where dB_t denotes the increment of a standard Brownian motion, σ is the volatility of the diffusion and dN_t is the increment of a Poisson process with a constant jump intensity λ and normal-distributed jump size x, that is, $x \sim (\mu_x, \sigma_x^2)$ and $E(dN_t) = \lambda dt$.

A representative investor with the constant relative risk aversion (CRRA) utility function seeks to maximize his expected his utility function of his life time consumption

$$max_{c_t} E_t \int_t^T p(t)U(c_t)dt,$$

where c_t is the rate of consumption at time t, $U(c)$ is a utility function with $U' > 0$, $U'' < 0$, and $p(t) \geq 0$, $0 \leq t \leq T$ is a time preference function. We consider the class of constant relative risk aversion (CRRA) utility function

$$U(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma}, & \gamma > 0, \gamma \neq 1, \\ \ln c, & \gamma = 1, \end{cases} \tag{2}$$

where the constant γ is the relative risk aversion coefficient, $\gamma = -cU''/U'$.

Based on the result of Zhang, Zhao and Chang (2010), in general equilibrium when market is clear, we have the following proposition.

Proposition 1 *In the production economy with jump diffusion and one representative investor with CRRA utility function, the equilibrium equity premium is given* *by*

$$
\begin{aligned}
\phi &= \mu - r \\
&= \gamma\sigma^2 + \lambda E[(1-e^{-\gamma x})(e^x - 1)] \\
&= \frac{\gamma}{\tau}Var_t(Y_\tau) + \frac{1}{2\tau}\gamma(1-\gamma)E_t[Y_\tau - E_t(Y_\tau)]^3 - \frac{1}{2}\lambda\gamma^2 E(x^4) + \lambda E[o(x^5)] \qquad (3) \\
&= \frac{\gamma}{\tau}Var_t(Y_\tau) + \frac{1-\gamma}{2\tau}[Var_t(Y_\tau) - Var^Q(Y_\tau)] - \frac{1}{4}\lambda\gamma^2(\gamma+1)E(x^4) + \lambda E[o(x^5)] \qquad (4)
\end{aligned}
$$

Furthermore, the variance risk premium $VRP(t,\tau)$ and the third central moments of return have the following relation:

$$VRP(t,\tau) = \gamma E_t[Y_\tau - E_t(Y_\tau)]^3 + \lambda\tau E[O(x^4)] \qquad (5)$$

where $Y_\tau = \ln(S_{t+\tau}/S_t)$ is the continuously compounded return within $(t, t+\tau)$, Q denotes the risk-neutral measure and the variance risk premium $VRP(t,\tau)$ is defined as defined as the difference between the physical and risk-neutral variance of stock returns:

$$VRP(t,\tau) = Var_t(Y_\tau) - Var^Q(Y_\tau) \qquad (6)$$

Proof: see Appendix A.

Even though it is a parsimonious model, literature has documented that jump risk premium is significant in the S&P 500 index options, such as Pan (2002) and Bates (2000). The expression in equation 3 and 4 give a closed-form expressions for excess return and the third central moments of return, the variance risk premium and risk aversion parameters. If the risk aversion parameter $\gamma > 1$, it implies that the excess return has a negative relation with the third central moments of returns and variance risk premium and a positive relation with return variance. Mitton and Vorkink (2007) document the portfolio returns of underdiversified investors are substantially more positively skewed than those of diversified investors. The preference for skewness pushes up the price of the assets with high skewness, so that the market portfolio has a lower return and negative skewness due to its well-diversification. But the constraint of this model is that the excess returns are always positive because the jump risk premium is always positive whatever the sign of the jump size. In fact, it is not worrying. Based on the definition of skewness, we know that negative skewness means there is a substantial probability of a big negative return, so in case of downside market, we can expect a big negative skewness.

So we expect that the third central moments of returns and variance risk premium have the predict power for the future returns. Furthermore, from equation 5 we conjecture that part of reason for the significance of variance risk premium for the prediction of the excess return is the positive linear correlation with the third central moments, not the physical skewness.

3 Empirical Measurements

The theoretical model outlined in the previous section suggests that the difference between physical and risk-neutral second central moments may serve as two useful predictor of the excess expected stock returns at the same time by effectively isolating the systematic risk. Let τ period return is given by the log price relative: $R(t,\tau) \equiv$

$\ln(S_{t+\tau}/S_t])$, then we have the results from Bakshi, Kapadia and Madan (2003).

$$
\begin{aligned}
V(t,\tau) &\equiv E^Q[e^{-r\tau}R(t,\tau)^2] \\
&= \int_S^\infty \frac{2[1-\ln(K/S)]}{K^2}C(t,\tau;K)dK + \int_0^S \frac{2[1+\ln(S/K)]}{K^2}P(t,\tau;K)dK \qquad (7) \\
W(t,\tau) &\equiv E^Q[e^{-r\tau}R(t,\tau)^3] \\
&= \int_S^\infty \frac{6\ln(K/S)-3(\ln(K/S)^2}{K^2}C(t,\tau;K)dK \\
&\quad - \int_0^S \frac{6\ln(S/K)+3(\ln(S/K))^2}{K^2}P(t,\tau;K)dK \qquad (8) \\
X(t,\tau) &\equiv E^Q[e^{-r\tau}R(t,\tau)^4] \\
&= \int_S^\infty \frac{12(\ln(K/S))^2-4(\ln(K/S)^3}{K^2}C(t,\tau;K)dK \\
&\quad - \int_0^S \frac{12(\ln(S/K))^2+4(\ln(S/K))^3}{K^2}P(t,\tau;K)dK \qquad (9) \\
\mu(t,\tau) &\equiv E^Q[R(t,\tau)] = e^{r\tau}-1-\frac{e^{r\tau}}{2}V(t,\tau)-\frac{e^{r\tau}}{6}W(t,\tau)-\frac{e^{r\tau}}{24}X(t,\tau) \qquad (10)
\end{aligned}
$$

Where $C(t,\tau,K)$ denote the price of a European call option maturing at time T with strike price K, $P(t,\tau,K)$ denote the price of a European put option maturing at time T with strike price K, S is the spot price at time t and Q is the risk-neutral measure.

Then this means that the risk-neutral second moments or risk-neutral variance of return can be replicated from the out-of-money European options data from the following relation:

$$
\begin{aligned}
E^Q[R(t,\tau)-E(R(t,\tau))]^2 &= E^Q[R(t,\tau)^2]-[E^Q(R(t,\tau))]^2 \\
&= e^{r\tau}V(t,\tau)-\mu(t,\tau)^2 \qquad (11)
\end{aligned}
$$

The variance risk premium is defined the difference between the ex-ante risk neutral expectation of future return variation over the $[t,t+1]$ time interval and the ex-post realized variation over the $[t-1,t]$ time interval. Then the physical variance PhV_t is measured by the summation of the square of daily log return , that is,

$$
PhV_t \equiv E_t[R-E_t[R]]^2 = \sum_{j=1}^{n}[\ln\frac{S_{t-1+\frac{j}{n}}}{S_{t-1+\frac{j-1}{n}}}]^2 \qquad (12)
$$

The physical or realized skewness $PhSk_t$ is from a standard measure, as Mitton and Vorkink (2007),

$$PhSk_t = \frac{\frac{1}{n}\sum_{j=1}^{n}(\ln \frac{S_{t-1+\frac{j}{n}}}{S_{t-1+\frac{j-1}{n}}} - \hat{\mu})^3}{\hat{\sigma}^{3/2}}, \tag{13}$$

where $\hat{\mu}$ is the mean of the daily change of the price.

Then the physical third central moments is calculated by

$$PhT_t = PhSk_t \times (PhV_t)^{\frac{3}{2}} \tag{14}$$

where n is the number of the observation of the corresponding time interval. So the variance risk premium VRP_t and physical skewness/third central moments are directly observable at time t. This is very important for them as the predictors for future excess return. We also note from the expression of the third central moments in the previous section, if the jump size is negative, the third central moments or skewness of the return are also negative.

4 Data Description

Our empirical analysis is based on the aggregate S&P 500 composite index as a proxy for the aggregate market portfolio. Our data is daily and sample spans the period from January 1996 through December 2005.

We retrieve the highly liquid S&P index options from OptionsMatrics along with the "model-free" approach discussed in the previous section. we use the daily call and put European options data to replicate the risk-neutral second central moments of fixed maturities: 1 month, 3months, 6 months and 12 months from January 1996 through December 2005.

Figure 1 plots the daily time series of variance risk premium, physical variance and skewness for 3-month maturity. Variance measure are somewhat higher about the year 2002. Most of points of the variance risk premium are blow zero line. The physical skewness is more negative especially before 2003. This is consistent with the theoretical model developed in the previous section and our earlier empirical conjecture.

In addition to the variance and skewness risk premium, we also consider other more traditional predictor variances (see, Bollerslev, Tauchen and Zhou (2009)). Specially, we obtain daily P/E rations and dividend yields for the S%P 500 from Standard & Poor's. Daily Data on the 3-month T-bill, the default spread (between Moody's BAA and AAA corporate bond spreads), the daily term spread (between the 10-year T-bond and the 3-month T-bill yields), and the stochastically daily de-trended risk-free rate (the 1-month T-bill rate minus its backward 12-month moving average) are taken from the public website of the Federal Reserve Bank of St. Louis. The consumption-wealth ratio (CAY), as defined in Lettau and Ludvigson (2001), is downloaded from Lettau and Ludvigson's website.

Anticipating the empirical results, we find that the predictability afforded by the physical variance, variance risk premium, the third central moments and skewness respectively. Table 1 reports the corresponding summary statistics and predictor variables based on the annualized daily return basis. The mean monthly excess return of S&P 500 index over the sample equals 3.49 percent annually. The sample means for the variance risk premium, physical skewness, physical variance and third central moments are -0.89%, -3.18%, 29.53 and -0.05 square percent respectively. Average negative variance risk premium and skewness are primarily consistent with large literature, for example, Zhang, Zhao and Chang (2010), Bollerslev, Tauchen and Zhou (2009), Bali and Hovaimian (2009).

5 Empirical Results

The main purpose of our empirical study is to investigate whether the past skewness/third central moments can forecast the future return. So all of our simple linear regressions of the S&P 500 excess returns are based on different sets of lagged predictor variables. All of the reported t-statistics are Newey-West adjusted values taking account of the overlap in the regressions. We focus on our discussion on the estimated slope coefficients and their statistical significance as determined by the robust t-statistics. At the same time, we also report the corresponding adjusted R^2s.

5.1 Intertemporal Relation between the Conditional Skewness/Third Central Moments and Market Returns

5.1.1 Ex-post Variance and Market Returns

Based on the model in section 2, we run the regressions of excess market excess returns on physical variance, physical skewness/third central moments, variance risk premium and the control variables mentioned in the previous section:

$$ExcessRe_{t+1} = \alpha + \gamma PhV_t + \beta PhSk_t/PhT_t + \phi VRP_t + \eta ControlV_t + \epsilon_{t+1} \quad (15)$$

The results are reported in Table 2, 3, 4, 5 respectively for monthly, quarterly, semi-annual and annual returns. The "Simple" columns report the regression coefficients and Newey-West adjusted t-statistics on the excess returns for one single factor only. The "Multiple" columns give the regression results combing other control variables. We find that none of the coefficients on physical variance is significant for the simple regression, even when combing other predictor variables, it is still not significant. R^2 is also very low, especially in quarterly return regressions, it only has -0.04%. This means that the ex-post variance has no predict power for the future returns.

This finding can not be explained by the CAPM model, which also implies that the expected risk premium is positive, but we all know from market that excess market return may not be positive, especially during the period of downside market. This contradiction is also noticed by other researches, for example Bollerslev, Tauchen and Zhou (2009) and Diacogiannis and Feldman (2010) which give their explanations. This violation of CAPM results in more more model incorporating higher order risks, such as Harvey and Siddique (2000b) develop a single factor asset pricing model incorporating conditional skewness. In their three-moment CAPM, the market return is also a function of the conditional skewness and the price of skewness. This motives us to investigate the relation between the skewness and the market return by market data.

5.1.2 Ex-post Conditional Skewness and Market Returns

Mitton and Vorkink (2007) document the portfolio returns of underdiversified investors are substantially more positively skewed than those of diversified investors. The preference for skewness pushes up the price of the assets with high skewness, so that the market portfolio has a lower return and negative skewness due to its well-diversification. Harvey and Siddique (2000a) show that conditional skewness helps explain the cross-sectional variation of expected returns across assets, but they do not care about the aggregate market.

Using the S&P 500 index as the proxy of market portfolio, we find that the coefficients on the third central moments are all negatively significant for quarterly regressions in Table 3. The coefficients of physical third central moments and skewness range from -0.12 to -0.13 and from -0.23 to -0.26 respectively ,which implies a good economic significance. Moreover, the statistical significance parameters are high in absolute value ranging from -1.78 to -3.34. Hence, we observe not only a robust economic significance, but also highly statistically significant parameter estimates.

Atilgan, Bali and Demirtas (2010) also investigate the intemporal relation between the implied volatility spread and expected returns on the aggregate stock market. They argue that this relation is not driven by information flow from option markets to stick market rather than volatility spreads acting as a proxy for skewness. The main reason for this argument is that the physical skewness as a control variable in the regressions of excess return is not significant. But they put all of the control variables together in the regressions, which may decrease or increase the significance of the some factors if the variables are linear correlated. For example, the past variance is highly significant in all of the regressions in their Table 3. But it is inconsistent with literature, such as Bollerslev, Tauchen and Zhou (2009) and Diacogiannis and Feldman (2010) we have mentioned above. We run the regressions for physical skewness not only as an individual factor but also combining with other predictors. All of the adjusted t-statistics imply the higher the ex-post skewness, the lower the future excess returns.

On the another hand, based on our theoretical results on section 2, variance risk premium is also an important factor for forecasting the future return. In fact, R^2 on variance risk premium, the third central moments and skewness individual regressions are highest, 9.08%, 7.67% and 10.40% for quarterly return regression. Combing the the third central moments with other predictors, variance risk premium, price-dividend ratio, the default spread ($DFSP$) and the relative risk free rate ($RREL$), R^2 increases and the the third central moments remains statistically significant. It is unsurprisingly find that the variance risk premium are significant in the individual regression and the coefficients are also remain statistically significant in the joint regressions combining other predictors. This is highly consistent with Bollerslev, Tauchen and Zhou (2009) which document that high (low) variance risk premia predicting high (low) future returns (aggregate stock market returns). We also note that the conditional physical skewness is more significant than the third central moments for monthly and quarterly regressions even combing other predictors.

We also note that for semi-annual return regressions, the coefficients of the third central moments, the skewness and the variance risk premium are very different from our conjecture. Model in section 2 implies that the excess returns are always positive. But in fact, the average of semi-annual excess return for our sample is -10.44%. This means that our model can not explain the situation when the market excess return is negative. However, the positive coefficients of variance risk premium and skewness/third central moments are also reasonable, because these two factors are negative during this period at the same time. So during the period of downside market, or the market excess return is negative, data analysis implies that there is a positive relation between the conditional skewness and the future returns. Recall the definition of skewness, a big negative skewness means a higher possibility to have a negative return. For example, the market crash means the occurrence of a big negative jump which implies a negative skewness from our model, and at the same time, the investors averagely earn a negative return.

5.2 Intertemporal Relation between Variance Risk Premium and the Third Central Moments of Market Return

From the previous analysis, we know that variance risk premium and the third central moments or skewness of returns are the significant factors for forecasting market returns. Especially, variance risk premium as a predict factor has been well documented by literature, such as Bakshi and Kapadia (2003), carr and Wu (2009) which indicate that market variance risk is indeed priced. But what is the fundamental explanations behind this relation has been paid much attention. Recently conditional skewness as a higher order moments become popular to examine the link. Bakshi and Madan (2006) suppose that the aggregate investor behavior is modeled through a power utility class of pricing kernels m(R), then the difference between physical and risk-neutral variance is a combination of physical third, fourth central moments and risk aversion parameters. Our jump diffusion model is built a product economy, but the general equilibrium also implies a similar result variance risk premium and the third central moments have a linear relation because of the existence of jump in the market.

From the previous analysis of subsection, we know that both of them have predict power for the excess market return. We hope to test whether the third central moments or skewness of returns have the explanation power for variance risk premium for the aggregate market. Table 6 presents results from the time-series regressions of the variance risk premium on the third central moments of market returns. To differentiate the role of the third central moments and the skewness of market returns, we also run the regressions on the skewness:

$$VRP_t = \alpha + \beta PhT_t/PhSk_t + \epsilon_t \tag{16}$$

We find that the third central moments are all significant for different time horizons. The coefficients of it range from 0.0121 to 0.0299. Almost all of absolute value of the Newey-West adjusted t-statistics are more than 2. The highest estimate and t-value appear for annual regression, 0.0299 and 3.85 respectively. At the same time, R^2 for annal regression is also highest, 33.98%. But neither of the physical skewness is significant. This means that variance risk premium include the information from the third central moments, not from the skewness. This implies that the physical variance has no direct effect on the difference between the physical and risk-neutral variance. Therefore, we think that one of explanation behind a potential expected returns and volatility spreads may be the third central moments, not skewness, see, Atilgan, Bali and Demirtas (2010).

6 Conclusion

Recent year, many empirical studies imply that Capital Asset Pricing Model (CAPM) is violated and can not explain the expected stock return, so that researchers pay more attention to the skewness. But one hand, most of work focus on the individual stocks, not the aggregate market. On the other hand, the stochastic volatility and jump-diffusion model and model are the most popular ones proposed. In particular, the sources of market variance risk are also from presence of stochastic volatility and occurrence of unanticipated market jumps. So jumps contribute market variance risk and the return skewness at the same time. To clarify the effect of skewness and variance risk premium on the expected market return, we choose the parsimonious jump diffusion model, supposing constant return variance, to identify their relations by closed-form expressions. In this paper, we derive a relation between the expected aggregate market return and the return variance and skewness from a parsimonious jump-diffusion model of Zhang, Zhao and Chang (2010).

From the model, the existence of jumps induce the relation between variance risk premium, conditional skewness and the excess market returns. To test this relation, we regress the excess S&P 500 index returns for ten years on the variance and skewness firstly and found that ex-post variance has no any significant predict power for the future market returns, but skewness and the third central moments are significant especially for quarterly returns.

At the same time, we use the out-of-money call and put European options data to replicate the risk-neutral moments based on Bakshi, Kapadia and Madan (2003). We also find the consistent result with literature that variance risk premium has significant explanation power for the future stock return. These empirical findings motives us to investigate investigate whether the potential explanation on the link between the market return and the variance risk premium is skewness or not. We find that it should be the third central moments, not the skewness, link the variance risk premium and expected stock returns. This finding is also consistent with the result of Bakshi and Madan (2006).

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Table 1: Summary Statistics

This table reports the summary statistics and correlation matrix for the variables from January 4, 1996 to December 30, 2005. $R_{mt} - R_{ft}$ denotes the logarithmic returns on the S&P 500 in excess of the 3-month T-bill rate. $PhV_t - RnV_t$ denotes the difference between the model-free realized variance and implied variance or risk-neutral variance for 3-month maturity. $PhT_t - RnT_t$ denotes the difference between the realized third central moments and risk-neutral third central moments. The predictor variables include the price-dividend ratio $log(P_t/D_t)$, the default spread $DFSP_t$ defined as the difference between Moody's BAA and AAA bond yield indices, the stochastically de-trended risk free rate $RREL_t$ defined as the 1-month T-bill rate minus its trailing twelve month moving average and the term spread $TMSP_t$ defined as the difference between the 10-year and 3-month Treasury yields. All the variables are based on daily basis.

	$R_{mt} - R_{ft}$	$PhV_t - RnV_t$	Var_t	PhT_t	$PhSk_t$	$\ln(P_t/D_t)$	$DFSP_t$	$RREL_t$	$TMSP_t$	$\ln(P_t/E_t)$	CAY_t
	Summary Statistics										
Mean	3.19	-8.90	29.53	-5.26	-3.18	6.54	0.84	-0.11	1.61	3.42	0.30
Std Dev	57.00	22.84	27.31	2.19	51.00	0.47	0.22	0.72	1.11	0.211.59	
Skewness	-0.48	0.45	2.36	1.07	-0.25	-0.05	0.94	-0.82	-0.25	0.14	-0.37
Kurtosis	4.11	11.40	9.62	2.78	3.82	2.49	3.02	3.80	2.05	1.89	2.03
	Correlation Matrix										
$R_{mt} - R_{ft}$	1.00	-0.12	0.07	-0.06	-0.12	-0.04	-0.12	-0.04	-0.13	0.05	-0.07
$PhV_t - RnV_t$		1.00	0.32	0.14	0.03	0.11	-0.02	0.18	-0.028	-0.018	-0.04
Var_t			1.00	0.12	0.08	-0.01	0.14	-0.21	-0.09	0.24	0.08
PhT_t				1.00	0.59	0.03	0.29	-0.04	0.13	-0.13	0.01
$PhSk_t$					1.00	0.16	0.20	-0.03	0.04	0.01	-0.04
$\ln(P_t/D_t)$						1.00	0.05	0.05	-0.29	0.72	-0.48
$DFSP_t$							1.00	0.35	0.54	-0.39	-0.20
$RREL_t$								1.00	-0.32	0.08	-0.41
$TMSP_t$									1.00	-0.641	-0.181
$\ln(P_t/E_t)$										1.00	-0.04
CAY_t											1.00

Table 2: Monthly Return Regressions

his Table reports the regression results for annualized 1-month daily excess returns on different predict variables. The sample period extends from January 1996 to December 2005. Variables calculated are based on 1-month basis. Newey-West adjusted t-statistics are reported in parentheses. All variables definitions are identical to Table 1.

	Simple									Multiple						
Constant	-0.01	0.04	0.03	0.01	1.76	0.17	0.04	0.05	0.92	-0.14	-0.02	0.01	0.05	0.01	0.09	0.02
	(-0.18)	(0.88)	(0.75)	(0.20)	(3.46)	(0.96)	(0.64)	(0.99)	(1.45)	(-0.26)	(-0.37)	(0.25)	(0.13)	(1.63)	(1.44)	(2.06)
PhV_t	0.13									0.17	0.17			0.19	0.19	0.21
	(0.93)									(1.06)	(1.05)			(1.14)	(1.11)	(1.13)
PhT_t		-0.01								-0.17		-0.11		-0.02		
		(-0.98)								(-1.29)		(-0.67)		(-1.67)		
$PhSk_t$			-0.13								-0.14		-0.12		-0.14	-0.13
			(-1.91)								(-1.99)		(-1.88)		(-2.04)	(-2.00)
$PhV_t - RnV_t$				-0.29								-0.28	-0.28			
				(-2.51)								(-2.34)	(-2.39)			
$\ln(P_t/D_t)$					-0.26											
					(-3.37)											
$DFSP_t$						-0.16										
						(-0.76)										
$RREL_t$							3.87									0.43
							(0.64)									(0.63)
$TMSP_t$								-0.89								-0.48
								(-0.22)								(-0.95)
$\ln(P_t/E_t)$									-0.26					-0.31	-0.28	-0.46
									(-1.86)					(-1.62)	(-1.44)	(-2.10)
$Adj.R^2(\%)$	0.49	0.29	1.31	0.29	4.58	0.38	0.20	-0.01	0.84	0.97	2.03	1.51	2.59	2.20	3.03	4.09

Table 3: Quarterly Return Regressions

This Table reports the regression results for annualized 3-month daily excess returns on different predict variables. The sample period extends from January 1996 to December 2005. Variables calculated are based on 3-month basis. Newey-West adjusted t-statistics are reported in parentheses. All variables definitions are identical to Table 1.

	Simple										Multiple					
Constant	0.03	0.03	0.10	-0.05	1.85	0.24	0.04	0.05	1.05	-1.34	-0.09	-0.02	-0.04	-0.07	4.96	2.01
	(0.62)	(0.91)	(0.24)	(-1.04)	(4.10)	(1.49)	(1.07)	(0.60)	(1.96)	(-2.06)	(-0.02)	(-0.43)	(-0.84)	(2.97)	(3.89)	(2.06)
PhV_t	0.24										0.40	0.37				
	(0.05)										(0.84)	(0.81)				
PhT_t		-0.12									-0.13		-0.12		-0.12	-0.12
		(-2.14)									(-2.78)		(-1.78)		(-2.94)	(-2.80)
$PhSk_t$			-0.25									-0.26		-0.23		
			(-1.91)									(-3.34)		(-3.11)		
$PhV_t - RnV_t$				-0.19									-0.16	-0.17		
				(-3.97)									(-3.66)	(-3.83)		
$\ln(P_t/D_t)$					-0.28										-0.33	-0.30
					(-3.99)										(-4.88)	(-3.96)
$DFSP_t$						-0.25										
						(-1.23)										
$RREL_t$							5.97									0.57
							(1.20)									(1.55)
$TMSP_t$								-0.96								-0.17
								(-0.21)								(-0.45)
$\ln(P_t/E_t)$									-0.30							
									(-1.89)							
CAY_t										1.43					-0.29	
										(2.10)					(-1.86)	
$Adj.R^2(\%)$	-0.04	7.67	10.40	9.08	17.08	3.08	1.87	0.07	3.83	0.48	8.27	10.92	14.34	17.94	26.79	27.64

Table 4: Semiannual Return Regressions

This Table reports the regression results for annualized 6-month excess returns on different predict variables. The sample period extends from January 1996 to December 2005. Variables calculated are based on 6-month basis. Newey-West adjusted t-statistics are reported in parentheses. All variables definitions are identical to Table 1.

	Simple										Multiple					
Constant	-0.15	-0.10	-0.08	0.02	-1.68	-0.40	-0.11	-0.14	-0.75	3.19	-0.14	-0.10	0.01	-0.03	-1.44	-1.98
	(-2.95)	(-2.75)	(-2.10)	(0.22)	(-4.27)	(-2.74)	(-2.98)	(-1.76)	(-1.56)	(2.05)	(-2.56)	(-2.03)	(0.14)	(0.56)	(-1.21)	(-4.29)
PhV_t	0.30										0.21	0.11				
	(1.04)										(0.65)	(0.38)				
PhT_t		0.22									0.16		0.16		0.27	0.15
		(1.10)									(0.67)		(0.95)		(1.56)	(0.80)
$PhSk_t$			0.16									0.15		0.16		
			(2.29)									(2.09)		(2.26)		
$PhV_t - RnV_t$				0.34									0.33	0.33		
				(2.78)									(2.74)	(3.58)		
$\ln(P_t/D_t)$					0.24										0.24	0.28
					(3.92)										(3.98)	(4.09)
$DFSP_t$						0.35										
						(1.91)										
$RREL_t$							-7.67									6.30
							(-1.62)									(-1.93)
$TMSP_t$								2.51								3.89
								(0.55)								(1.13)
$\ln(P_t/E_t)$									0.19							
									(1.33)							
CAY_t										-3.44					-0.26	
										(-2.11)					(-0.24)	
$Adj.R^2(\%)$	2.04	2.16	8.30	8.59	25.85	13.05	6.32	1.52	3.10	6.02	3.04	8.49	9.73	16.32	29.19	38.66

Table 5: Annual Return Regressions

This Table reports the regression results for annual excess returns on different predict variables. The sample period extends from January 1996 to December 2005. Variables calculated are based on 12-month basis. Newey-West adjusted t-statistics are reported in parentheses. All variables definitions are identical to Table 1.

	Simple						Multiple					
Constant	0.11	0.03	0.03	-0.02	1.31	-2.86	0.12	0.11	-0.03	2.03	-0.01	-0.60
	(1.55)	(0.79)	(1.02)	(-0.40)	(2.55)	(-1.90)	(2.32)	(2.02)	(-0.64)	(-0.27)	(4.10)	(-0.61)
PhV_t	-0.24						-0.27	-0.24				
	(-0.96)						(-1.43)	(-1.14)				
PhT_t		-0.11					0.05		0.10		-0.24	-0.23
		(-0.11)					(0.37)		(0.95)		(-1.92)	(-1.88)
$PhSk_t$			0.07					0.07		0.08		
			(1.44)					(1.21)		(1.58)		
$PhV_t - RnV_t$				-0.25					-0.36	-0.27		
				(-1.96)					(-2.00)	(-1.87)		
$\ln(P_t/E_t)$					-0.38						-0.59	-0.58
					(-2.41)						(-3.99)	(-3.93)
CAY_t						3.02						2.71
						(1.91)						(3.37)
$Adj.R^2(\%)$	6.13	0.00	3.08	5.36	20.36	7.53	6.81	9.12	7.34	9.50	32.76	38.82

Table 6: Variance Risk Premium Regressions

This Table reports the regression results for variance risk premium on physical skewness and third central moments based on 1-month, 3-month, 6-month and 12-month basis respectively. The regression is: $PhV_t - RnV_t = a + bPhT_t/PhSk_t + e_t$. The sample period extends from January 1996 to December 2005. Newey-West adjusted t-statistics are reported in parentheses. Variables definitions are identical to Table 1.

Constant	PhT_t	$PhSk_t$	$Adj.R^2(\%)$
Panel A: Regression for 1-month			
-0.0009		0.0155	0.08
(-6.91)		(0.65)	
-0.0009	0.0141		1.79
(-14.75)	(2.22)		
Panel B: Regression for 3-month			
-0.0043		0.0107	0.69
(-15.05)		(1.40)	
-0.0044	0.0121		2.88
(-14.75)	(2.17)		
Panel C: Regression for 6-month			
-0.0346		0.0149	0.05
(-12.67)		(0.17)	
-0.0348	0.0173		1.86
(-22.00)	(1.97)		
Panel D: Regression for 12-month			
-0.0169		0.0418	1.25
(-4.59)		(0.72)	
-0.0163	0.0299		33.98
(-7.09)	(3.85)		

A Proof of proposition 1

Based on the equilibrium model of Zhang, Zhao and Chang (2010), which give the results about equity premium (ϕ) and first, second and third physical and risk-neutral central moments.

To simplify and clarify the relation, here we assume that the jump size follows a normal distribution, that is $x \sim (\mu_x, \sigma_x)$. According to the setting of model and the relation between the physical and risk-neutral jump sizes, we firstly need to obtain the relation between the physical and risk-neutral central moments of jump size.

Suppose Q is the risk-neutral measure, and recall the results of Zhang, Zhao and Chang (2010), we have the Q measure central moments about x:

$$
\begin{aligned}
E(x) &= \mu_x \\
E^Q(x) &= \mu_x^Q = \frac{E(e^{-\gamma x}x)}{E(e^{-\gamma x})} = \mu_x - \gamma\sigma_x^2 \\
E(x-\mu_x)^2 &= \sigma_x^2 \\
E^Q(x-\mu_x^Q)^2 &= (\sigma_x^Q)^2 = \frac{E(e^{-\gamma x}(x-\mu_x^Q)^2)}{E(e^{-\gamma x})} \\
&= \frac{E(e^{-\gamma x}x^2)}{E(e^{-\gamma x})} - (\mu_x^Q)^2 \\
&= \sigma_x^2 \\
E(x-\mu_x)^3 &= E(x^3) - 3\mu_x\sigma_x^2 - \mu_x^3 \\
E^Q(x-\mu_x^Q)^3 &= E^Q(x^3) - 3\mu_x^Q\sigma_x^2 - (\mu_x^Q)^3
\end{aligned}
$$

Furthermore, let log return be denoted as $Y_\tau = \ln(S_{t+\tau}/S_t)$, equilibrium jump-diffusion model also give the following cumulants:

$$
\begin{aligned}
Var_t(Y_\tau) &= \sigma^2\tau + \lambda\tau[\mu_x^2 + \sigma_x^2] \\
Var_t^Q(Y_\tau) &= \sigma^2\tau + \lambda^Q\tau[(\mu_x^Q)^2 + (\sigma_x^Q)^2] \\
E_t[Y_\tau - E_t(Y_\tau)]^3 &= \lambda\tau[\mu_x^3 + 3\mu_x\sigma_x^2 + E(x-\mu_x)^3]
\end{aligned}
$$

Because of the normality of jump size x and the properties obtained above, no loss of generality, we take Taylor expansion for jump size x in order to examine their relationship between them, then we obtain the variance risk premium which is the difference between the physical and risk-neutral variance of return:

$$\begin{aligned}
Var_t(Y_\tau) - Var_t^Q(Y_\tau) &= \lambda\tau[\mu_x^2 + \sigma_x^2] - \lambda E(e^{-\gamma x})\tau[(\mu_x^Q)^2 + \sigma_x^2] \\
&= \lambda\tau[E(x^2) - E(x^2 e^{-\gamma x})] \\
&= \lambda\tau[E(x^2) - E(x^2(1 - \gamma x + \frac{1}{2}\gamma^2 x^2 + o(x^3))] \\
&= \lambda\gamma\tau E(x^3) - \frac{1}{2}\lambda\tau\gamma^2 E(x^4) + \lambda\tau E[O(x^5)]
\end{aligned}$$

Similarly, the third central moments of returns can also be expressed by jump intensity and jump size:

$$\begin{aligned}
E_t[Y_\tau - E_t(Y_\tau)]^3 &= \lambda\tau[\mu_x^3 + 3\mu_x\sigma_x^2 + E(x-\mu_x)^3] \\
&= \lambda\tau E(x^3)
\end{aligned}$$

Then it is easy to have a relation between the variance risk premium and the third central moments of return by the jump parameters:

$$Var_t(Y_\tau) - Var^Q(Y_\tau) = \gamma E_t[Y_\tau - E_t(Y_\tau)]^3 - \frac{1}{2}\lambda\tau\gamma^2 E(x^4) + \lambda\tau E[O(x^5)]$$

Recall the result of equity premium ϕ:

$$\phi \equiv \phi_\sigma + \phi_J = \gamma\sigma^2 + \lambda E[(1 - e^{-\gamma x})(e^x - 1)]$$

Taking Taylor expansion for jump size x, we have

$$
\begin{aligned}
\phi &= \gamma\sigma^2 + \lambda E[(1-e^{-\gamma x})(e^x-1)] \\
&= \gamma\sigma^2 + \lambda E[(\gamma x - \frac{1}{2}\gamma^2 x^2 + o(x^3))(1 + x + \frac{1}{2}x^2 + o(x^3) - 1)] \\
&= \gamma\sigma^2 + \lambda\gamma E(x^2) + \frac{1}{2}\gamma(1-\gamma)E(x^3) - \frac{1}{2}\lambda\gamma^2 E(x^4) + \lambda E[o(x^5)] \\
&= \frac{\gamma}{\tau}Var_t(Y_\tau) + \frac{1}{2\tau}\gamma(1-\gamma)E_t[Y_\tau - E_t(Y_\tau)]^3 - \frac{1}{2}\lambda\gamma^2 E(x^4) + \lambda E[o(x^5)] \\
&= \frac{\gamma}{\tau}Var_t(Y_\tau) + \frac{1-\gamma}{2\tau}[Var_t(Y_\tau) - Var^Q(Y_\tau)] - \frac{1}{4}\lambda\gamma^2(\gamma+1)E(x^4) + \lambda E[o(x^5)]
\end{aligned}
$$

Figure 1: Variance risk premium and physical variance and skewness. This figure plots the variance risk premium, physical variance and skewness of 3-month maturity for the S&P 500 index from January 1996 to December 2005