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STEERING VECTOR ESTIMATION AND BEAMFORMING UNDER UNCERTAINTIES

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ABSTRACT

In this paper, we propose a new method for estimating the steering vector under uncertainties, which is utilized for improving the robustness of beamforming. We show that the desired steering vector can be estimated in closed form from a convex optimization problem by making use of the subspace principle. As this method is developed based on an extended version of the orthonormal PAST (OPAST), the steering vector can be recursively estimated with very low complexity and moving sources can be handled. To further improve the performance of beamforming, the uncertainty of the array covariance matrix is taken into account. Numerical results demonstrate that the proposed method performs well in the presence of uncertainties.

***Index Terms*—** steering vector, subspace tracking, robust beamforming, convex optimization.

1. INTRODUCTION

Adaptive beamforming has been widely applied in many fields such as radar, sonar, and wireless communication [1]. It basically aims to enhance the desired signal received while suppressing the noise and interference, with the help of steering vector of the desired signal, which is a function of the source coordinate and geometry of the array. In absence of array uncertainties and with perfect estimate of the direction-of-arrival (DOA) of the desired signal, the steering vector can be determined and used to suppress the interference with any of conventional adaptive beamforming algorithms, such as the Capon beamformer [2].

However, the steering vector in practice may not be determined accurately due to the presence of uncertainties such as DOA mismatch, sensor gain/phase uncertainties, position variations and mutual coupling. It is known that these distortions may significantly degrade the performance of the conventional beamforming methods. As a result, various approaches have been proposed to address these uncertainties. In [3], [4], additional linear constraints have been proposed to better attenuate the interference and broaden the response around the nominal look direction. Unfortunately, these constraints may reduce the degree of freedom for suppressing interference and they are not explicitly related to the uncertainty of the steering vector. In [5], [6], quadratic constraints on the Euclidean norm of the beamformer weight vector or uncertainty of the steering vector have been exploited. This leads to a class of diagonal loading robust beamforming. However, it is somewhat difficult to relate the diagonal loading level with uncertainty bounds of the array steering vector. Thus, some methods have been proposed to address this problem in [7]–[9].

In this paper, instead of relying completely on linear or norm constraints, the problem of robust beamforming is handled by means of recursive steering vector estimation based on the OPAST algorithm [10]. Firstly, a new method which is capable of estimating the deterministic error in steering vector is proposed. A convex problem is formulated based on the subspace principle, and the explicit expression of the steering vector estimate is derived. Then, considering the array covariance matrix uncertainty and incorporating the steering vector estimate into the Capon beamformer, a new robust Capon beamformer is obtained.

2. PROBLEM FORMULATION

An antenna array with N sensor elements impinged by $K+1$ narrow-band uncorrelated signals including one desired signal and K interferences is considered. It is assumed that $K+1<N$. The array output vector $\boldsymbol{x}(t)$ is

$$\boldsymbol{x}(t)=\boldsymbol{s}(t)+\boldsymbol{i}(t)+\boldsymbol{n}(t), \quad (1)$$

where $\boldsymbol{s}(t)=\boldsymbol{a}(\theta_0)s_0(t)$, $\boldsymbol{i}(t)=\sum_{k=1}^{K}\boldsymbol{a}(\theta_k)s_k(t)$, and $\boldsymbol{n}(t)$ are the desired signal, interference and noise components, respectively. $\boldsymbol{a}(\theta_0)$ and $\{\boldsymbol{a}(\theta_k)\}_{k=1}^{K}$ are the steering vectors of the desired signal and interferences, respectively. The noise is considered to be additive white Gaussian noise (AWGN). The sensor outputs are linearly combined by a beamformer weight vector $\boldsymbol{w}$ to form the desired output as $y(t)=\boldsymbol{w}^H\boldsymbol{x}(t)$, and the beamformer weight vector can be obtained from the following problem [2]

$$\min\ \boldsymbol{w}^H\boldsymbol{R}\boldsymbol{w} \quad \text{s.t. } \boldsymbol{w}^H\boldsymbol{a}_0=1, \quad (2)$$

where $\boldsymbol{R}=E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$ is the array covariance matrix, $\boldsymbol{a}_0$ denotes $\boldsymbol{a}(\theta_0)$ for simplicity. The solution of (2) is given by

$$\boldsymbol{w}_{MVDR}=\frac{\boldsymbol{R}^{-1}\boldsymbol{a}_0}{\boldsymbol{a}_0^H\boldsymbol{R}^{-1}\boldsymbol{a}_0}, \quad (3)$$

which is called minimum variance distortionless response (MVDR) beamformer or Capon beamformer.

It should be noted that this beamformer assumes that the true steering vector is known accurately. Unfortunately, the assumption is usually violated due to various imperfections of the array. Therefore, the true steering vector $\boldsymbol{a}$ of the desired signal should be written as

$$\boldsymbol{a} = \boldsymbol{a}_0 + \upsilon, \tag{4}$$

where υ denotes the uncertainty of the nominal steering vector $\boldsymbol{a}_0$. Generally, υ is unknown to users and the beamforming performance will degrade considerably if it is simply ignored. To tackle this problem, a new approach to estimate υ for correcting $\boldsymbol{a}_0$ is developed.

3. ROBUST BEAMFORMING ALGORITHM

A. Steering Vector Estimation

According to the subspace principle, the true steering vector $\boldsymbol{a}$ is orthogonal to the noise subspace, i.e.,

$$\boldsymbol{U}_n^H \boldsymbol{a} = \boldsymbol{U}_n^H (\boldsymbol{a}_0 + \upsilon) = \boldsymbol{0}, \tag{5}$$

where $\boldsymbol{U}_n$ is the noise subspace which is obtained here by subspace tracking algorithm to reduce the computational complexity and handle scenarios involving moving sources. We known that tracking errors of the noise subspace are inevitable and it will depend on the speed of the moving sources and other stochastic errors such as sensor noises. Hence, the true noise subspace is given by

$$\boldsymbol{U}_n = \hat{\boldsymbol{U}}_n + \Delta, \tag{6}$$

where $\hat{\boldsymbol{U}}_n$ is the noise subspace estimate and Δ is the stochastic error. Generally, it is reasonable to assume that it is zero mean with covariance $\boldsymbol{C}$, i.e.,

$$E[\Delta] = 0,\; E[\Delta\Delta^H] = \boldsymbol{C}. \tag{7}$$

We shall focus on the estimation of υ with the above property. Firstly, substituting (6) to (5) and one gets

$$\hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \upsilon) = -\Delta^H (\boldsymbol{a}_0 + \upsilon), \tag{8}$$

Taking the Euclidean norm and then expectation over Δ on both sides of (8), we have

$$\begin{aligned} \| \hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \upsilon) \|^2 &= E[(\boldsymbol{a}_0 + \upsilon)^H (\Delta\Delta^H)(\boldsymbol{a}_0 + \upsilon)] \\ &= (\boldsymbol{a}_0 + \upsilon)^H \boldsymbol{C} (\boldsymbol{a}_0 + \upsilon). \end{aligned} \tag{9}$$

Since $\boldsymbol{C}$ and υ are typically small, the terms involving the product of $\boldsymbol{C}$ and υ are ignorable, this yields

$$\| \hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \upsilon) \|^2 \approx \boldsymbol{a}_0^H \boldsymbol{C} \boldsymbol{a}_0 = \zeta. \tag{10}$$

As a result, we have $\| \hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \upsilon) \|^2 \approx \zeta$, which is then relaxed to a quadratic inequality $\| \hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \upsilon) \|^2 \le \zeta$, since typically only the bounds on the uncertainties are required. Note that, though $\boldsymbol{a}$ is not known exactly, it often lies within a small region around the nominal steering vector $\boldsymbol{a}_0$. Therefore, it is natural to choose the smallest υ such that (10) is satisfied. Consequently, the problem at hand is to minimize the Euclidean norm of υ subject to a quadratic inequality:

$$\min \; \| \upsilon \|^2 \quad \text{s.t.} \; \| \hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \upsilon) \|^2 \le \zeta. \tag{11}$$

It can be seen that the problem in (11) is convex and hence an optimal solution does exist. We now employ the Lagrange multiplier method to solve this problem. The *Lagrangian L* associated with (11) is given by

$$L(\upsilon, \lambda) = \| \upsilon \|^2 + \lambda \left(\| \hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \upsilon) \|^2 - \zeta \right), \tag{12}$$

where λ is the Lagrange multiplier. By setting the partial derivative of (12) with respect to υ to zero, one gets the first order necessary condition for optimality as follows

$$\upsilon + \lambda \hat{\boldsymbol{U}}_n \hat{\boldsymbol{U}}_n^H \upsilon + \lambda \hat{\boldsymbol{U}}_n \hat{\boldsymbol{U}}_n^H \boldsymbol{a}_0 = \boldsymbol{0}. \tag{13}$$

On the other hand, since the problem is convex and the objective function is differentiable, any stationary point is also the global solution. Hence, solving (13), we get the optimal solution to (11) as

$$\hat{\upsilon} = -\lambda (\boldsymbol{I} + \lambda \hat{\boldsymbol{U}}_n \hat{\boldsymbol{U}}_n^H)^{-1} \hat{\boldsymbol{U}}_n \hat{\boldsymbol{U}}_n^H \boldsymbol{a}_0. \tag{14}$$

To determine λ, a common way is to substitute (14) back to the quadratic constraint and it will in general give rise to a nonlinear equation. Fortunately, we shall show that a closed form solution can be derived.

To begin with, we assume that the noise subspace estimate can be factorized as $\hat{\boldsymbol{U}}_n = \widetilde{\boldsymbol{U}}_n \widetilde{\boldsymbol{U}}_n^H$, where $\widetilde{\boldsymbol{U}}_n$ denotes a orthonormal basis of the noise subspace, and it is satisfied by $\widetilde{\boldsymbol{U}}_n^H \widetilde{\boldsymbol{U}}_n = \boldsymbol{I}$. Hence, we have $\hat{\boldsymbol{U}}_n = \hat{\boldsymbol{U}}_n^H$ and $\hat{\boldsymbol{U}}_n \hat{\boldsymbol{U}}_n^H = \hat{\boldsymbol{U}}_n$. Using these properties and the matrix inverse lemma, the inverse term in (14) can be simplified as

$$\begin{aligned} (\boldsymbol{I} + \lambda \hat{\boldsymbol{U}}_n \hat{\boldsymbol{U}}_n^H)^{-1} &= (\boldsymbol{I} + \lambda \widetilde{\boldsymbol{U}}_n \widetilde{\boldsymbol{U}}_n^H)^{-1} \\ &= \boldsymbol{I} - \lambda \widetilde{\boldsymbol{U}}_n (\boldsymbol{I} + \lambda \widetilde{\boldsymbol{U}}_n^H \widetilde{\boldsymbol{U}}_n)^{-1} \widetilde{\boldsymbol{U}}_n^H \\ &= \boldsymbol{I} - \lambda (1+\lambda)^{-1} \widetilde{\boldsymbol{U}}_n \widetilde{\boldsymbol{U}}_n^H \\ &= \boldsymbol{I} - \lambda (1+\lambda)^{-1} \hat{\boldsymbol{U}}_n. \end{aligned} \tag{15}$$

Substituting (15) into (14), one gets

$$\begin{aligned} \hat{\upsilon} &= -\lambda \left(\boldsymbol{I} - \lambda (1+\lambda)^{-1} \hat{\boldsymbol{U}}_n \right) \hat{\boldsymbol{U}}_n \hat{\boldsymbol{U}}_n^H \boldsymbol{a}_0 \\ &= -\lambda \left(\hat{\boldsymbol{U}}_n - \lambda (1+\lambda)^{-1} \hat{\boldsymbol{U}}_n \right) \boldsymbol{a}_0 \\ &= -\lambda (1+\lambda)^{-1} \hat{\boldsymbol{U}}_n \boldsymbol{a}_0. \end{aligned} \tag{16}$$

Recall (10), one gets the following equation on λ:

$$\begin{aligned} \| \hat{\boldsymbol{U}}_n^H (\boldsymbol{a}_0 + \hat{\upsilon}) \|^2 &= \left\| \hat{\boldsymbol{U}}_n^H \left(\boldsymbol{a}_0 - \lambda (1+\lambda)^{-1} \hat{\boldsymbol{U}}_n \boldsymbol{a}_0 \right) \right\|^2 \\ &= \left\| \hat{\boldsymbol{U}}_n^H \boldsymbol{a}_0 - \lambda (1+\lambda)^{-1} \hat{\boldsymbol{U}}_n^H \boldsymbol{a}_0 \right\|^2 \\ &= \left\| (1+\lambda)^{-1} \hat{\boldsymbol{U}}_n^H \boldsymbol{a}_0 \right\|^2 = \zeta \end{aligned} \tag{17}$$

Solving the above equality, one gets

$$\lambda = \alpha^{-1} - 1 \text{, and } \alpha = \left(\zeta^{-1}\boldsymbol{a}_0^H \hat{\boldsymbol{U}}_n \boldsymbol{a}_0\right)^{-1/2}. \qquad (18)$$

Finally, by substituting (18) into (16), we have

$$\hat{\upsilon} = (\alpha - 1)\hat{\boldsymbol{U}}_n \boldsymbol{a}_0 . \qquad (19)$$

We notice that υ is estimated in closed form. Moreover, if the noise subspace is exactly estimated and $\zeta = 0$, we have $\alpha = 0$ and $\hat{\boldsymbol{a}} = \boldsymbol{a}_0 + \hat{\upsilon} = (\boldsymbol{I} - \hat{\boldsymbol{U}}_n)\boldsymbol{a}_0 = (\boldsymbol{I} - \tilde{\boldsymbol{U}}_n \tilde{\boldsymbol{U}}_n^H)\boldsymbol{a}_0$, which is the solution of the conventional projection approach [11]. Therefore, the proposed approach generalizes the projection approach to include possible uncertainties arising from tracking and/or other stochastic errors. Also, the proposed method provides an analytical solution, which simplifies the implementation.

B. Subspace Tracking and Robust Beamforming

We now proceed to introduce the method for recursively tracking the subspace $\hat{\boldsymbol{U}}_n(t)$ and covariance $\boldsymbol{C}(t)$ for robust beamforming. From the extended OPAST algorithm shown in Table I, the signal subspace $\tilde{\boldsymbol{U}}_s(t)$ is updated as

$$\tilde{\boldsymbol{U}}_s(t) = \tilde{\boldsymbol{U}}_s(t-1) + \tilde{\boldsymbol{e}}(t)\boldsymbol{g}^H(t), \qquad (20)$$

where $\tilde{\boldsymbol{e}}(t)$ and $\boldsymbol{g}(t)$ are defined in Table I. Since $\tilde{\boldsymbol{U}}_s(t)$ satisfies $\tilde{\boldsymbol{U}}_s^H(t)\tilde{\boldsymbol{U}}_s(t) = \boldsymbol{I}$, then $\hat{\boldsymbol{U}}_n(t)$ can be calculated as

$$\hat{\boldsymbol{U}}_n(t) = \tilde{\boldsymbol{U}}_n(t)\tilde{\boldsymbol{U}}_n^H(t) = \boldsymbol{I} - \tilde{\boldsymbol{U}}_s(t)\tilde{\boldsymbol{U}}_s^H(t). \qquad (21)$$

Furthermore, substituting (20) into (21), one gets

$$\hat{\boldsymbol{U}}_n(t) = \hat{\boldsymbol{U}}_n(t-1) + \Delta_n(t), \qquad (22)$$

where

$$\Delta_n(t) = -\tilde{\boldsymbol{U}}_s(t-1)\boldsymbol{g}(t)\tilde{\boldsymbol{e}}^H(t) - \tilde{\boldsymbol{e}}(t)\boldsymbol{g}^H(t)\tilde{\boldsymbol{U}}_s^H(t) - \|\boldsymbol{g}(t)\|^2\, \tilde{\boldsymbol{e}}(t)\tilde{\boldsymbol{e}}^H(t). \qquad (23)$$

Hence, we notice that the noise subspace can be estimated recursively from the signal subspace with a low arithmetic complexity. On the other hand, (23) provides us with the instantaneous perturbation of the noise subspace from which its covariance can be effectively estimated. More precisely, we propose to estimate the covariance $\boldsymbol{C}(t)$ recursively as

$$\boldsymbol{C}(t) = \beta\boldsymbol{C}(t-1) + (1-\beta)\Delta_n(t)\Delta_n^H(t), \qquad (24)$$

where β is a forgetting factor.

As a result, we have $\alpha(t) = \left(\zeta^{-1}(t)\boldsymbol{a}_0^H(t)\tilde{\boldsymbol{U}}_n(t)\boldsymbol{a}_0(t)\right)^{-1/2}$, $\zeta(t) = \boldsymbol{a}_0^H(t)\boldsymbol{C}(t)\boldsymbol{a}_0(t)$, and

$$\hat{\upsilon}(t) = \left(\alpha(t) - 1\right)\hat{\boldsymbol{U}}_n(t)\boldsymbol{a}_0(t). \qquad (25)$$

Accordingly, the steering vector is estimated as

$$\hat{\boldsymbol{a}}(t) = \boldsymbol{a}_0(t) + \hat{\upsilon}(t). \qquad (26)$$

The traditional MVDR beamformer can thus be invoked to obtain a new robust beamformer by replacing $\boldsymbol{a}_0(t)$ in (3) with $\hat{\boldsymbol{a}}(t)$. Hence, the following robust MVDR (R-MVDR) beamformer is proposed as:

TABLE I

THE EXTENDED OPAST ALGORITHM

Initialize $\boldsymbol{P}(0)$, $\boldsymbol{U}_s(0)$, $\boldsymbol{U}_n(0)$ and $\boldsymbol{C}_{\delta U}(t)$
For $t = 1,2,\ldots, do$
$\boldsymbol{y}(t) = \tilde{\boldsymbol{U}}_s^H(t-1)\boldsymbol{x}(t)$; $\boldsymbol{h}(t) = \boldsymbol{P}(t-1)\boldsymbol{y}(t)$
$\boldsymbol{g}(t) = \boldsymbol{h}(t)/[\beta + \boldsymbol{y}^H(t)\boldsymbol{h}(t)]$; $\boldsymbol{P}(t) = \beta^{-1}Tri\{\boldsymbol{P}(t-1) - \boldsymbol{g}(t)\boldsymbol{h}^H(t)\}$
$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \tilde{\boldsymbol{U}}_s(t-1)\boldsymbol{y}(t)$; $\tau(t) = \|\boldsymbol{g}(t)\|^{-2}\left(\left(1 + \|\boldsymbol{e}(t)\|^2\|\boldsymbol{g}(t)\|^2\right)^{-2} - 1\right)$
$\tilde{\boldsymbol{e}}(t) = \tau(t)\tilde{\boldsymbol{U}}_s(t-1)\boldsymbol{g}(t) + \left(1 + \tau(t)\|\boldsymbol{g}(t)\|^2\right)\boldsymbol{e}(t)$
$\tilde{\boldsymbol{U}}_s(t) = \tilde{\boldsymbol{U}}_s(t-1) + \tilde{\boldsymbol{e}}(t)\boldsymbol{g}^H(t)$
$\Delta_n(t) = -\tilde{\boldsymbol{U}}_s(t-1)\boldsymbol{g}(t)\tilde{\boldsymbol{e}}^H(t) - \boldsymbol{e}(t)\boldsymbol{g}^H(t)\tilde{\boldsymbol{U}}_s^H(t) - \|\boldsymbol{g}(t)\|^2\, \tilde{\boldsymbol{e}}(t)\tilde{\boldsymbol{e}}^H(t)$
$\hat{\boldsymbol{U}}_n(t) = \hat{\boldsymbol{U}}_n(t-1) + \Delta_n(t)$
$\boldsymbol{C}(t) = \beta\boldsymbol{C}(t-1) + (1-\beta)\Delta_n(t)\Delta_n^H(t)$
End t

$$\boldsymbol{w}_{R-MVDR}(t) = \frac{\boldsymbol{R}^{-1}(t)\hat{\boldsymbol{a}}(t)}{\hat{\boldsymbol{a}}^H(t)\boldsymbol{R}^{-1}(t)\hat{\boldsymbol{a}}(t)}. \qquad (27)$$

The array covariance matrix $\boldsymbol{R}(t)$ is recursively estimated by the popular formula $\boldsymbol{R}(t) = \beta\boldsymbol{R}(t-1) + (1-\beta)\boldsymbol{x}(t)\boldsymbol{x}^H(t)$. Since the performance of the R-MVDR beamformer (27) may be influenced by the perturbation of array covariance matrix, a diagonal loaded R-MVDR beamformer is given as

$$\boldsymbol{w}_{R-MVDR-DL}(t) = \frac{(\boldsymbol{R}(t) + \gamma(t)\boldsymbol{I})^{-1}\hat{\boldsymbol{a}}(t)}{\hat{\boldsymbol{a}}^H(t)(\boldsymbol{R}(t) + \gamma(t)\boldsymbol{I})^{-1}\hat{\boldsymbol{a}}(t)}, \qquad (28)$$

where the value of $\gamma(t)$ is related to the perturbation bound of the array covariance matrix. In real systems, it may be selected with some prior information. In this paper, the instantaneous variation of the array covariance matrix will be adopted to estimate $\gamma(t)$ as $\gamma(t) = k\|\boldsymbol{R}(t) - \boldsymbol{R}(t-1)\|$. It is experimentally found that k can be chosen from a wide range without affecting significantly the performance. Hence, the choice of k is not a crucial problem. Here, we choose $k = 10\%$ for illustration. Finally, it can be seen that the complexity of the proposed method is $O(N^3)$, which is of the same order of MVDR beamformer.

4. SIMULATION RESULTS

A ULA with $N = 10$ sensors separated by half wavelength is considered. The noise is assumed to be AWGN with a power of 0dB. One desired signal and two interferences are assumed to impinge on the array from far-field. The DOA of the desired signal is considered to be time-varying and given by $10^\circ \times 10^{-3} t$, $0 \le t \le 1000$, the two interferences are fixed to be at 40° and 60°. The powers of the interferences are fixed to be 30dB. The forgetting factor is $\beta = 0.99$. Each sensor is suffered from a gain/phase uncertainty as $\rho_i e^{j\phi_i}$, $1 \le i \le N$, which is assumed to be $\{\rho_i\}_{i=1}^N$ = $\{1, 1.0369, 0.9695, 1.0033, 1.0176, 1.0560, 1.0309, 0.9665, 1.0718, 1.0690\}$,and $\{\phi_i\}_{i=1}^N = \{0, -0.2916, -0.2947, 0.2547, 0.4015, 0.1534, 0.3667, -0.3193, -0.4652, -0.1343\}$. For comparison, conventional algorithms are also tested: 1) the diagonal loading (DL) beamformer with loading level being 10 times of the noise power; 2) robust Capon beamformer (RCB) [7] with the error bound equal to $\varepsilon = 3.2460$, and 3) the worst-case beamformer [9]. In the simulations, the DOA of the desired signal is firstly estimated using ESPRIT with the tracked signal subspace. Then, beamforming algorithms are invoked based on the estimated DOA.

The DOA tracking results and output SINRs at −5dB and 5dB SNRs are shown respectively in Fig. 1 and Fig. 2. Obviously, we notice that, the DOA cannot be tracked well due to the presence of gain/phase uncertainties. This will introduce a DOA mismatch for beamforming. It can be seen from Fig. 1(b) and Fig. 2(b) that the conventional methods are significantly influenced by gain/phase uncertainties and DOA mismatch, especially at higher SNRs. On the contrary, the proposed method can achieve better performance.

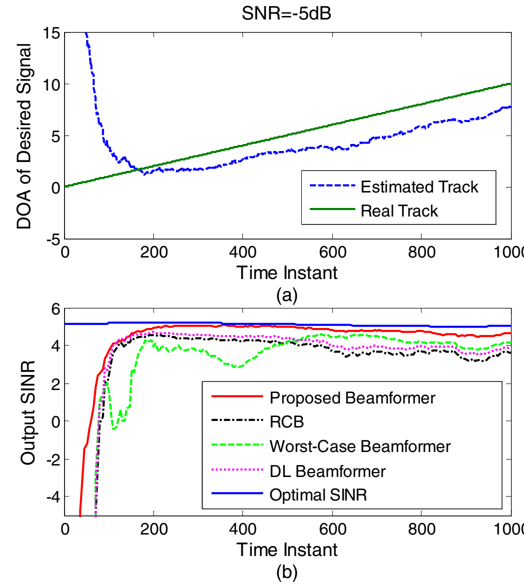

Fig.1. Resultant DOA tracking and output SINR when SNR= −5dB.

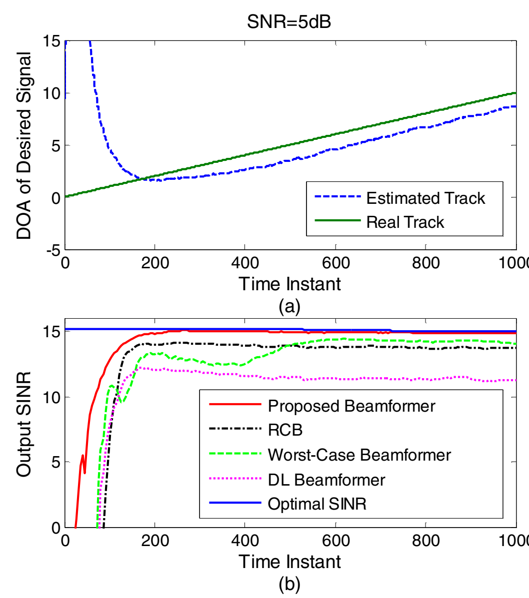

Fig.2. Resultant DOA tracking and output SINR when SNR=5dB.

5. CONCLUSIONS

A new method for correcting possible deterministic errors in the steering vector due to array uncertainties is presented. It uses the subspace principle and the resulting problem can be formulated as a convex problem and solved in closed form. Using an extended OPAST algorithm, the proposed method can be employed to handle scenarios involving moving sources while requiring a low complexity. The resultant robust beamformer resembles the diagonally loaded MVDR beamformer with a recursively estimated steering vector and the loading level given by the perturbation bound of the array covariance matrix. Simulation results demonstrate the effectiveness of the proposed method.

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