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Citation	The 2011 IEEE Global Telecommunications Conference (GLOBECOM 2011), Beijing, China, 5-9 December 2011. In Globecom. IEEE Conference and Exhibition, 2011, p. 1-5
Issued Date	2011
URL	http://hdl.handle.net/10722/165231
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Uplink LMMSE Beamforming Design for Cellular Networks with AF MIMO Relaying

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Abstract—In this paper, linear beamforming design for uplink amplify-and-forward relaying cellular networks, in which multiple mobile terminals rely on one relay station to communicate with the base station, is investigated. In particular, the base station, relay station and mobile terminals are all equipped with multiple antennas. Based on linear minimum mean-square-error (LMMSE) criterion and exploiting a hidden convexity in the problem, the precoder matrices at multiple mobile terminals, forwarding matrix at relay station and equalizer matrix at base station are jointly designed. Furthermore, several existing linear beamforming designs for multi-user (MU) MIMO systems and AF MIMO relaying systems can be considered as special cases of the proposed solution. Simulation results are presented to demonstrate the performance advantage of the proposed algorithm.

I. INTRODUCTION

Cooperative communication is a promising technique to improve the quality and reliability of wireless links [1]–[3]. One of the most important application scenarios of cooperative communications is cellular network. Due to shadowing or deep fading of wireless channels, base station may not be able to sufficiently cover all mobile terminals in a cell, especially those on the edge. Deployment of relay stations is an effective and economic way to improve the communication quality in cellular networks.

In cooperative cellular networks, there are two major strategies in relaying. Relay station can either decode the received signal before retransmission [4] or simply amplify-and-forward (AF) the received signal to the corresponding destination without decoding [5]. AF strategy has low complexity and minimal processing delay, and is more secure. These reasons make AF preferable in practical implementation. In fact, deployment of AF relay station with multiple antenna to enlarge coverage of base station is one of the most important components in the future communication standards, e.g., LTE, IMT-Advanced and Winner project [6] [7].

In a cellular network, the base station and relay station are usually allowed to be equipped with multiple antennas. In this paper, we consider a general case where each mobile

This research work was supported in part by Sino-Swedish IMT-Advanced and Beyond Cooperative Program under Grant No.2008DFA11780 and Program for Changjiang Scholars and Innovative Research Team in University.

terminal is also equipped with multiple antennas. In particular, we consider the joint precoder matrices, forwarding matrix, and equalizer matrix design for uplink AF relaying cellular network, under transmit power constraints. In general, for transceiver design under power constraints, there are two different objectives: maximizing the transmission rate or improving the transmission accuracy. How to achieve the first objective for uplink cellular networks with AF MIMO relaying has been discussed in [8]. In this paper, we focus on the second objective.

In terms of accuracy, mean-square-error (MSE) of detected data is a natural performance measure in signal processing [9], [10]. Linear beamforming design problem can be formulated as an optimization problem minimizing the sum MSE of multiple detected data streams. MSE minimization problem in AF MIMO relaying systems has been investigated in [11]. However, the algorithm proposed in [11] is a brute force algorithm with high complexity. In this paper, a novel algorithm exploiting the hidden convexity of the problem is proposed. It is found that the resultant solution has a lower complexity than that in [11] and covers several existing algorithms for multi-user (MU) MIMO or AF MIMO relaying systems as special cases.

The following notations are used throughout this paper. Boldface lowercase letters denote vectors, while boldface uppercase letters denote matrices. The notation \mathbf{Z}^H denotes the Hermitian of the matrix \mathbf{Z} , and $\text{Tr}(\mathbf{Z})$ is the trace of the matrix \mathbf{Z} . The symbol \mathbf{I}_M denotes an $M \times M$ identity matrix, while $\mathbf{0}_{M,N}$ denotes an $M \times N$ all zero matrix. The notation $\mathbf{Z}^{1/2}$ is the Hermitian square root of the positive semidefinite matrix \mathbf{Z} , such that $\mathbf{Z}^{1/2}\mathbf{Z}^{1/2} = \mathbf{Z}$ and $\mathbf{Z}^{1/2}$ is also a Hermitian matrix. The operation $\text{diag}\{[\mathbf{A} \ \mathbf{B}]\}$ is defined as a block diagonal matrix with \mathbf{A} and \mathbf{B} as block diagonal. The symbol $\mathbb{E}\{\cdot\}$ represents the statistical expectation. The operation $\text{vec}(\mathbf{Z})$ stacks the columns of the matrix \mathbf{Z} into a single vector. The symbol \otimes denotes the Kronecker product. For two Hermitian matrices, $\mathbf{C} \succeq \mathbf{D}$ means that $\mathbf{C} - \mathbf{D}$ is a positive semi-definite matrix.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, a dual-hop AF relaying cellular network is considered. As shown in Fig. 1, one multiple-antenna relay

station helps multiple mobile terminals to transmit signals to the base station (BS). Furthermore, there are L_k data streams to be transmitted from the k^{th} mobile terminal to the BS, and the signal from the k^{th} mobile terminal is denoted as \mathbf{s}_k . Without loss of generality, it is assumed that the transmitted data streams are independent: $\mathbb{E}\{\mathbf{s}_k\mathbf{s}_j^H\} = \mathbf{0}_{L_k, L_j}$ when $k \neq j$ and $\mathbb{E}\{\mathbf{s}_k\mathbf{s}_k^H\} = \mathbf{I}_{L_k}$. At the k^{th} mobile terminal, the transmit signal \mathbf{s}_k is multiplied by a precoder matrix \mathbf{P}_k under a power constraint $\text{Tr}(\mathbf{P}_k\mathbf{P}_k^H) \leq P_{s,k}$, where $P_{s,k}$ is the maximum transmit power at the k^{th} mobile terminal. The received signal \mathbf{x} at the relay station is the superposition of signals from different terminals through different channels and is given by

$$\mathbf{x} = \sum_{k=1}^K \{\mathbf{H}_{MR,k} \mathbf{P}_k \mathbf{s}_k\} + \mathbf{n}. \quad (1)$$

where $\mathbf{H}_{MR,k}$ is the $N_R \times N_{M,k}$ channel matrix between the k^{th} mobile terminal and relay station, and \mathbf{n} is the additive Gaussian noise at the relay station with zero mean and covariance matrix \mathbf{R}_n .

Defining

$$\begin{aligned} \mathbf{H}_{MR} &\triangleq [\mathbf{H}_{MR,1} \cdots \mathbf{H}_{MR,K}] \quad \mathbf{P} \triangleq \text{diag}\{[\mathbf{P}_1, \dots, \mathbf{P}_K]\} \\ \mathbf{s} &\triangleq [\mathbf{s}_1^T \cdots \mathbf{s}_K^T]^T, \end{aligned} \quad (2)$$

the received signal at relay station (1) is rewritten as

$$\mathbf{x} = \mathbf{H}_{MR} \mathbf{P} \mathbf{s} + \mathbf{n}. \quad (3)$$

Since the data transmitted from different mobile terminals are independent, the correlation matrix of \mathbf{x} equals to

$$\mathbf{R}_x = \mathbf{H}_{MR} \mathbf{P} \mathbf{P}^H \mathbf{H}_{MR}^H + \mathbf{R}_n. \quad (4)$$

At the relay station, the received signal \mathbf{x} is multiplied by a linear forwarding matrix \mathbf{F} , with a power constraint $\text{Tr}(\mathbf{F}\mathbf{F}^H) \leq P_r$, where P_r is the maximum transmit power at the relay station. Finally, the received signal at the BS is

$$\mathbf{y} = \mathbf{H}_{RB} \mathbf{F} \mathbf{H}_{MR} \mathbf{P} \mathbf{s} + \mathbf{H}_{RB} \mathbf{F} \mathbf{n} + \boldsymbol{\xi}, \quad (5)$$

where \mathbf{H}_{RB} is the $N_B \times N_R$ channel matrix between the relay station and BS, and $\boldsymbol{\xi}$ is the additive zero mean Gaussian noise with covariance \mathbf{R}_{ξ} .

When a linear equalizer \mathbf{B} is adopted at the BS, the total MSE of the detected data is

$$\begin{aligned} &\text{MSE}_U(\mathbf{B}, \mathbf{F}, \mathbf{P}) \\ &= \mathbb{E}\{\|\mathbf{By} - \mathbf{s}\|^2\} \\ &= \text{Tr}(\mathbf{B}(\mathbf{H}_{RB} \mathbf{F} \mathbf{R}_x \mathbf{F}^H \mathbf{H}_{RB}^H + \mathbf{R}_{\xi}) \mathbf{B}^H) - \text{Tr}(\mathbf{B} \mathbf{H}_{RB} \mathbf{F} \mathbf{H}_{MR} \mathbf{P}) \\ &\quad - \text{Tr}((\mathbf{B} \mathbf{H}_{RB} \mathbf{F} \mathbf{H}_{MR} \mathbf{P})^H) + \text{Tr}(\mathbf{I}_L), \end{aligned} \quad (6)$$

where $L = \sum_{k=1}^K L_k$ is the total number of data streams. Finally, the optimization problem for beamforming matrices design in the uplink case is formulated as

$$\begin{aligned} \min_{\mathbf{B}, \mathbf{F}, \mathbf{P}} &\quad \text{MSE}_U(\mathbf{B}, \mathbf{F}, \mathbf{P}) \\ \text{s.t.} &\quad \text{Tr}(\mathbf{P}_k \mathbf{P}_k^H) \leq P_{s,k}, \quad k = 1, \dots, K \\ &\quad \text{Tr}(\mathbf{F} \mathbf{F}^H) \leq P_r \quad \mathbf{P} = \text{diag}\{[\mathbf{P}_1, \dots, \mathbf{P}_K]\}. \end{aligned} \quad (7)$$

In general, the optimization problem (7) can be solved using an iterative algorithm alternating the three variables \mathbf{B} , \mathbf{F} and \mathbf{P} , which has been proposed in [11]. Unfortunately, it suffers two main weaknesses:

- (1) As the iterative algorithm needs to compute the equalizer \mathbf{B} at each iteration, its convergence speed is slow at high SNRs. It means that the algorithm has high complexity at high SNRs. This is a common problem for iterative transceiver design for MU MIMO systems [12].
- (2) That algorithm is a brute force method and provides little insight into the nature of the design problem.

In order to overcome these weaknesses, in the following, a novel algorithm is proposed, which is found to be insightful and covers several existing algorithms for conventional AF MIMO relaying systems and MU MIMO systems as its special cases. Furthermore, it has a much lower complexity compared to the existing algorithm given in [11].

III. THE PROPOSED ALGORITHM

First, we reduce the number of variables of the optimization problem (7). Noticing that there is no constraint on \mathbf{B} , the optimal \mathbf{B} satisfies $\partial \text{MSE}_U(\mathbf{B}, \mathbf{F}, \mathbf{P}) / \partial \mathbf{B}^* = \mathbf{0}_{L, N_B}$, and the optimal equalizer at the BS can be written as a function of forwarding matrix and precoder matrices, that is, $\mathbf{B} = (\mathbf{H}_{RB} \mathbf{F} \mathbf{H}_{MR} \mathbf{P})^H (\mathbf{H}_{RB} \mathbf{F} \mathbf{R}_x \mathbf{F}^H \mathbf{H}_{RB}^H + \mathbf{R}_{\xi})^{-1}$. Substituting this result into (6), the uplink MSE is simplified as

$$\begin{aligned} &\text{MSE}_U(\mathbf{F}, \mathbf{P}) \\ &= \text{Tr}(\mathbf{I}_L) - \text{Tr}((\mathbf{H}_{RB} \mathbf{F} \mathbf{H}_{MR} \mathbf{P})^H \\ &\quad \times (\mathbf{H}_{RB} \mathbf{F} \mathbf{R}_x \mathbf{F}^H \mathbf{H}_{RB}^H + \mathbf{R}_{\xi})^{-1} (\mathbf{H}_{RB} \mathbf{F} \mathbf{H}_{MR} \mathbf{P})). \end{aligned} \quad (8)$$

Based on the definition of $\mathbf{R}_x = \mathbf{H}_{MR} \mathbf{P} \mathbf{P}^H \mathbf{H}_{MR}^H + \mathbf{R}_n$, it can be expressed as

$$\mathbf{R}_x = \mathbf{R}_n^{1/2} \underbrace{(\mathbf{R}_n^{-1/2} \mathbf{H}_{MR} \mathbf{P} \mathbf{P}^H \mathbf{H}_{MR}^H \mathbf{R}_n^{-1/2} + \mathbf{I}) \mathbf{R}_n^{1/2}}_{\triangleq \mathbf{\Xi}}. \quad (9)$$

Now introducing $\tilde{\mathbf{F}} = \mathbf{F} \mathbf{R}_n^{1/2} \mathbf{\Xi}^{1/2}$, the MSE (8) becomes

$$\begin{aligned} &\overline{\text{MSE}}_U(\tilde{\mathbf{F}}, \mathbf{P}) \\ &= \text{Tr}(\mathbf{I}_L) - \text{Tr}((\mathbf{H}_{RB} \tilde{\mathbf{F}} \mathbf{\Xi}^{-1/2} \mathbf{R}_n^{-1/2} \mathbf{H}_{MR} \mathbf{P})^H \\ &\quad \times (\mathbf{H}_{RB} \tilde{\mathbf{F}} \tilde{\mathbf{F}}^H \mathbf{H}_{RB}^H + \mathbf{R}_{\xi})^{-1} (\mathbf{H}_{RB} \tilde{\mathbf{F}} \mathbf{\Xi}^{-1/2} \mathbf{R}_n^{-1/2} \mathbf{H}_{MR} \mathbf{P})). \end{aligned} \quad (10)$$

Thus the uplink beamforming design optimization problem (7) is rewritten as

$$\begin{aligned} \min_{\tilde{\mathbf{F}}, \mathbf{P}} &\quad \overline{\text{MSE}}_U(\tilde{\mathbf{F}}, \mathbf{P}) \\ \text{s.t.} &\quad \text{Tr}(\mathbf{P}_k \mathbf{P}_k^H) \leq P_{s,k}, \quad k = 1, \dots, K \\ &\quad \text{Tr}(\tilde{\mathbf{F}} \tilde{\mathbf{F}}^H) \leq P_r \quad \mathbf{P} = \text{diag}\{[\mathbf{P}_1, \dots, \mathbf{P}_K]\}. \end{aligned} \quad (11)$$

Unfortunately, the optimization problem (11) is still nonconvex for $\tilde{\mathbf{F}}$ and \mathbf{P} , and thus there is no closed-form solution. However, notice that if either $\tilde{\mathbf{F}}$ or \mathbf{P} is fixed, the optimization problem is convex with respect to the remaining variable. Therefore, an iterative algorithm which designs $\tilde{\mathbf{F}}$ and \mathbf{P} alternatively, is proposed as follows.

(1) Design $\tilde{\mathbf{F}}$ when \mathbf{P} is fixed

From (10), it is noticed that $\tilde{\mathbf{F}}$ appears both inside and outside of the inverse operation. In order to simplify the objective function, we use the following variant of matrix inversion lemma

$$\mathbf{C}^H(\mathbf{CC}^H + \mathbf{D})^{-1}\mathbf{C} = \mathbf{I} - (\mathbf{C}^H\mathbf{D}^{-1}\mathbf{C} + \mathbf{I})^{-1}. \quad (12)$$

Taking $\mathbf{C} = \mathbf{H}_{RB}\tilde{\mathbf{F}}$ and $\mathbf{D} = \mathbf{R}_\xi$, the MSE (10) can be reformulated as [10]

$$\begin{aligned} & \text{MSE}_U(\tilde{\mathbf{F}}, \mathbf{P}) \\ &= \text{Tr}((\Xi^{-1/2}\mathbf{R}_n^{-1/2}\mathbf{H}_{MR}\mathbf{P})(\Xi^{-1/2}\mathbf{R}_n^{-1/2}\mathbf{H}_{MR}\mathbf{P})^H \\ & \quad \times (\tilde{\mathbf{F}}^H\mathbf{H}_{RB}^H\mathbf{R}_\xi^{-1}\mathbf{H}_{RB}\tilde{\mathbf{F}} + \mathbf{I})^{-1}) \\ & \quad + \text{Tr}((\mathbf{P}^H\mathbf{H}_{MR}^H\mathbf{R}_n^{-1}\mathbf{H}_{MR}\mathbf{P} + \mathbf{I})^{-1}). \end{aligned} \quad (13)$$

Now, $\tilde{\mathbf{F}}$ only appears inside the matrix inverse. If \mathbf{P} is fixed, the last term of (13) is independent of $\tilde{\mathbf{F}}$, and the optimization problem (11) becomes

$$\begin{aligned} \min_{\mathbf{F}} \quad & \text{Tr}(\underbrace{(\Xi^{-1/2}\mathbf{R}_n^{-1/2}\mathbf{H}_{MR}\mathbf{P})(\Xi^{-1/2}\mathbf{R}_n^{-1/2}\mathbf{H}_{MR}\mathbf{P})^H}_{\triangleq \Theta} \\ & \quad \times (\tilde{\mathbf{F}}^H\mathbf{H}_{RB}^H\mathbf{R}_\xi^{-1}\mathbf{H}_{RB}\tilde{\mathbf{F}} + \mathbf{I})^{-1}) \\ \text{s.t.} \quad & \text{Tr}(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H) \leq P_r. \end{aligned} \quad (14)$$

Based on eigen-decomposition, $\Theta = \mathbf{U}_\Theta \Lambda_\Theta \mathbf{U}_\Theta^H$ and $\mathbf{M} = \mathbf{U}_M \Lambda_M \mathbf{U}_M^H$, and defining

$$\Lambda_{\tilde{\mathbf{F}}} \triangleq \mathbf{U}_M^H \tilde{\mathbf{F}} \mathbf{U}_\Theta, \quad (15)$$

the optimization problem (14) can be simplified as

$$\begin{aligned} \min_{\Lambda_{\tilde{\mathbf{F}}}} \quad & \text{Tr}(\Lambda_\Theta(\Lambda_{\tilde{\mathbf{F}}}^H \Lambda_M \Lambda_{\tilde{\mathbf{F}}} + \mathbf{I})^{-1}) \\ \text{s.t.} \quad & \text{Tr}(\Lambda_{\tilde{\mathbf{F}}} \Lambda_{\tilde{\mathbf{F}}}^H) \leq P_r. \end{aligned} \quad (16)$$

Without loss of generality, the diagonal elements of Λ_Θ and Λ_M are assumed to be arranged in decreasing order. The closed-form solution of (16) can be shown to be [10]

$$\Lambda_{\tilde{\mathbf{F}}} = \begin{bmatrix} \left[\left(\frac{1}{\sqrt{\mu_f}} \tilde{\Lambda}_M^{-1/2} \tilde{\Lambda}_\Theta^{1/2} - \tilde{\Lambda}_M^{-1} \right)^+ \right]^{1/2} & \mathbf{0}_{L, N_R - L} \\ \mathbf{0}_{N_R - L, L} & \mathbf{0}_{N_R - L, N_R - L} \end{bmatrix}, \quad (17)$$

where $\tilde{\Lambda}_\Theta$ and $\tilde{\Lambda}_M$ are the $L \times L$ principal submatrices of Λ_Θ and Λ_M , respectively. The scalar μ_f is the Lagrange multiplier which makes $\text{Tr}(\Lambda_{\tilde{\mathbf{F}}} \Lambda_{\tilde{\mathbf{F}}}^H) = P_r$ hold. Based on (15) and (17), the optimal $\tilde{\mathbf{F}}$ can be recovered as

$$\tilde{\mathbf{F}} = \mathbf{U}_{M,L} \left[\left(\frac{1}{\sqrt{\mu_f}} \tilde{\Lambda}_M^{-1/2} \tilde{\Lambda}_\Theta^{1/2} - \tilde{\Lambda}_M^{-1} \right)^+ \right]^{1/2} \mathbf{U}_{\Theta,L}^H, \quad (18)$$

where $\mathbf{U}_{M,L}$ and $\mathbf{U}_{\Theta,L}$ are the first L columns of \mathbf{U}_M and \mathbf{U}_Θ , respectively. Finally, the optimal \mathbf{F} is given by $\mathbf{F} = \tilde{\mathbf{F}} \Xi^{-1/2} \mathbf{R}_n^{-1/2}$.

(2) Design \mathbf{P} when $\tilde{\mathbf{F}}$ is fixed

Since Ξ in (10) depends on \mathbf{P} , the MSE expression in (10) is a complicated function of \mathbf{P} , direct optimization of \mathbf{P} seems

intractable. However, based on the property of trace operator $\text{Tr}(\mathbf{DC}) = \text{Tr}(\mathbf{CD})$, the total MSE (10) can be reformulated as

$$\begin{aligned} & \overline{\text{MSE}}_U(\tilde{\mathbf{F}}, \mathbf{P}) \\ &= \text{Tr}(\mathbf{I}_L) - \text{Tr}((\mathbf{H}_{RB}\tilde{\mathbf{F}})^H(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\mathbf{H}_{RB}^H + \mathbf{R}_\xi)^{-1} \\ & \quad \times \mathbf{H}_{RB}\tilde{\mathbf{F}})(\Xi^{-1/2}\underbrace{\mathbf{R}_n^{-1/2}\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^H\mathbf{H}_{MR}^H\mathbf{R}_n^{-1/2}}_{\triangleq \Xi^{-1}}\Xi^{-1/2}) \\ &= \text{Tr}(\mathbf{I}_L) - \text{Tr}((\mathbf{H}_{RB}\tilde{\mathbf{F}})^H(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\mathbf{H}_{RB}^H + \mathbf{R}_\xi)^{-1}\mathbf{H}_{RB}\tilde{\mathbf{F}}) \\ & \quad \times (\mathbf{I}_{N_R} - \Xi^{-1}). \end{aligned} \quad (19)$$

Substituting the definition of Ξ into (19), the MSE can be further rewritten as

$$\begin{aligned} & \overline{\text{MSE}}_U(\tilde{\mathbf{F}}, \mathbf{P}) \\ &= \text{Tr}(\underbrace{(\mathbf{H}_{RB}\tilde{\mathbf{F}})^H(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\mathbf{H}_{RB}^H + \mathbf{R}_\xi)^{-1}(\mathbf{H}_{RB}\tilde{\mathbf{F}})}_{\triangleq \Pi} \\ & \quad \times (\mathbf{R}_n^{-1/2}\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^H\mathbf{H}_{MR}^H\mathbf{R}_n^{-1/2} + \mathbf{I}_{N_R})^{-1}) + \text{Tr}(\mathbf{I}_L) \\ & \quad - \text{Tr}((\mathbf{H}_{RB}\tilde{\mathbf{F}})^H(\mathbf{H}_{RB}\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\mathbf{H}_{RB}^H + \mathbf{R}_\xi)^{-1}(\mathbf{H}_{RB}\tilde{\mathbf{F}})), \end{aligned} \quad (20)$$

where \mathbf{P} only appears inside of the inverse operation. As the last two terms of (20) are independent of \mathbf{P} , the optimization problem for \mathbf{P} is

$$\begin{aligned} \min_{\mathbf{P}} \quad & \text{Tr}(\Pi(\mathbf{R}_n^{-1/2}\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^H\mathbf{H}_{MR}^H\mathbf{R}_n^{-1/2} + \mathbf{I}_{N_R})^{-1}) \\ \text{s.t.} \quad & \text{Tr}(\mathbf{P}_k \mathbf{P}_k^H) \leq P_{s,k} \quad k = 1, \dots, K, \\ & \mathbf{P} = \text{diag}\{[\mathbf{P}_1, \dots, \mathbf{P}_K]\}. \end{aligned} \quad (21)$$

Notice that in the objective function, the matrix inversion is a convex function over positive semi-definite matrices [14]. In order to exploit this convexity, we will transform the optimization problem (21) into another problem with positive semi-definite covariance matrices as variables. More specifically, with the definitions of \mathbf{H}_{MR} and \mathbf{P} , we have

$$\mathbf{H}_{MR}\mathbf{P}\mathbf{P}^H\mathbf{H}_{MR}^H = \sum_{k=1}^K \{ \mathbf{H}_{MR,k} \underbrace{\mathbf{P}_k \mathbf{P}_k^H \mathbf{H}_{MR,k}^H}_{\triangleq \mathbf{Q}_k} \}. \quad (22)$$

Putting (22) into (21), the optimization problem becomes

$$\begin{aligned} \min_{\mathbf{Q}_k} \quad & \text{Tr}(\Pi(\mathbf{R}_n^{-1/2} \sum_{k=1}^K \{ \mathbf{H}_{MR,k} \mathbf{Q}_k \mathbf{H}_{MR,k}^H \} \mathbf{R}_n^{-1/2} + \mathbf{I}_{N_R})^{-1}) \\ \text{s.t.} \quad & \text{Tr}(\mathbf{Q}_k) \leq P_{s,k}, \quad k = 1, \dots, K, \quad \mathbf{Q}_k = \mathbf{P}_k \mathbf{P}_k^H \succeq \mathbf{0}. \end{aligned} \quad (23)$$

Using the Schur-complement lemma [13], the optimization problem (23) can be further formulated as a standard semi-definite programming (SDP) optimization problem [15]

$$\begin{aligned} \min_{\mathbf{X}, \mathbf{Q}_k} \quad & \text{Tr}(\mathbf{X}) \\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{X} & \Pi^{1/2} \\ \Pi^{1/2} & \mathbf{R}_n^{-1/2} \sum_k \{ \mathbf{H}_{MR,k} \mathbf{Q}_k \mathbf{H}_{MR,k}^H \} \mathbf{R}_n^{-1/2} + \mathbf{I}_{N_R} \end{bmatrix} \succeq \mathbf{0} \\ & \text{Tr}(\mathbf{Q}_k) \leq P_{s,k}, \quad k = 1, \dots, K, \quad \mathbf{Q}_k \succeq \mathbf{0}. \end{aligned} \quad (24)$$

The SDP problems can be efficiently solved using interior-point polynomial algorithms [14].

In summary, the uplink beamforming design alternates between the design of $\tilde{\mathbf{F}}$ in (18) and \mathbf{Q}_k in (24). The algorithm stops when $\|\text{MSE}_U^I - \text{MSE}_U^{I+1}\| \leq \mathcal{T}_U$, where MSE_U^I is the total MSE in the I^{th} iteration and \mathcal{T}_U is a threshold value. After convergence, $\mathbf{P}_k = \mathbf{Q}_k^{1/2}$, $\mathbf{F} = \tilde{\mathbf{F}}\mathbf{\Xi}^{-1/2}\mathbf{R}_n^{-1/2}$ and $\mathbf{B} = (\mathbf{H}_{RB}\mathbf{F}\mathbf{H}_{MR}\mathbf{P})^H(\mathbf{H}_{RB}\mathbf{F}\mathbf{R}_x\mathbf{F}^H\mathbf{H}_{RB}^H + \mathbf{R}_\xi)^{-1}$.

The computation complexity of the proposed algorithm is much lower compared to that in [11]. Firstly, there is no need to compute equalizer at each iteration. Secondly, unlike the algorithm given by [11], for the forwarding matrix design there exists a closed-form solution and it does not need to numerically search a scalar variable as Lagrange multiplier. Finally, for the source precoder design, there is no need to transform the matrix variables into long vectors. It means that the dimensionality of the SDP problem to be solved (thus the complexity) in our proposed algorithm is much smaller than that of the algorithm in [11].

Remark 1: In case $N_{M,k} > L_k$, there is an additional constraint $\text{Rank}\{\mathbf{Q}_k\} \leq L_k$ in (23). In this case, as rank constraints are nonconvex, transition from (23) to (24) involves a relaxation on the rank constraint. Then the objective function of (24) is a lower bound of that of (23). However, this relaxation has been widely adopted in the design of MU MIMO uplink beamforming [12]. Notice that when $N_{M,k} \leq L_k$ (known as fully-loaded or overloaded MIMO systems [16]), there is no relaxation involved.

Remark 2: In order to guarantee the objective function monotonically decreases at each iteration, the proposed algorithm alternates between \mathbf{F} and \mathbf{Q}_k 's. It means that in the computation process we do not recover \mathbf{P}_k 's from the corresponding \mathbf{Q}_k 's since only \mathbf{Q}_k 's appear in the optimization problem. How to recover \mathbf{P}_k 's from \mathbf{Q}_k 's when the iterative algorithm stops has been discussed in detail in [13].

Special cases

Notice that (18) has a more general form than the water-filling solution in traditional point-to-point MIMO systems. On the other hand, (24) is a SDP problem frequently encountered in multiuser MIMO systems. In particular, they include the following existing algorithms as special cases.

- If $\mathbf{H}_{RB} = \mathbf{I}_L$ and $\mathbf{R}_\xi = \mathbf{0}_{L,L}$, we have $\mathbf{\Pi} = \mathbf{I}_L$ in (23), and the SDP optimization problem (24) reduces to that of the uplink multiuser MIMO systems [12].
- Substituting $K = 1$ and $\mathbf{P} = \mathbf{I}_{L_1}$ into (18), it reduces to the solution proposed for LMMSE joint design of relay forwarding matrix and destination equalizer in AF MIMO relay systems without source precoder [3].
- Notice that when there is only one mobile terminal ($K = 1$), the optimization problem (21) is in the same form as (14). Defining $\mathbf{H}_{MR}^H\mathbf{R}_n^{-1}\mathbf{H}_{MR} = \mathbf{U}_{MR}\mathbf{\Lambda}_{MR}\mathbf{U}_{MR}^H$, and $\mathbf{\Pi} = \mathbf{U}_{\Pi}\mathbf{\Lambda}_{\Pi}\mathbf{U}_{\Pi}^H$, a closed-form solution can be derived using the

same procedure as for $\tilde{\mathbf{F}}$, and we have

$$\mathbf{P} = \mathbf{U}_{MR,L} \left[\left(\frac{1}{\sqrt{\mu_p}} \tilde{\mathbf{\Lambda}}_{MR}^{-1/2} \tilde{\mathbf{\Lambda}}_{\Pi}^{1/2} - \tilde{\mathbf{\Lambda}}_{MR}^{-1} \right)^+ \right]^{1/2} \quad (25)$$

where the $\tilde{\mathbf{\Lambda}}_{MR}$ and $\tilde{\mathbf{\Lambda}}_{\Pi}$ are the $L \times L$ principal submatrices of $\mathbf{\Lambda}_{MR}$ and $\mathbf{\Lambda}_{\Pi}$, respectively, and the matrix $\mathbf{U}_{MR,L}$ is the first L columns of \mathbf{U}_{MR} . The scalar μ_p is the Lagrange multiplier which makes $\text{Tr}(\mathbf{P}\mathbf{P}^H) = P_{s,1}$ hold. In this case, the solution given by (25) corresponds to the source precoder design for AF MIMO relaying systems with single user [9].

- Furthermore, substituting $\mathbf{H}_{RB} = \mathbf{I}_L$ and $\mathbf{R}_\xi = \mathbf{0}_{L,L}$ into (25), it becomes the closed-form solution for LMMSE transceiver design in point-to-point MIMO systems [17].

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we investigate the performance of the proposed uplink beamforming algorithm. In the simulations, there is one BS, one relay station and two mobile terminals. For each mobile terminal, two independent data streams will be transmitted in the uplink simultaneously. For each data stream, 10000 independent QPSK symbols are transmitted. The elements of MIMO channels between BS and relay station and between relay station and mobile terminals are generated as independent complex Gaussian random variables with zero mean and unit variance. Each point in the following figures is an average of 500 independent channel realizations. In order to solve SDP problems, the widely used optimization Matlab toolbox CVX is adopted [18]. The threshold for terminating the iterative algorithm is set at $\mathcal{T}_U = 0.0001$.

In the considered uplink case, the noise covariance matrices at relay station and BS are $\mathbf{R}_n = \sigma_n^2 \mathbf{I}_{N_R}$ and $\mathbf{R}_\xi = \sigma_\xi^2 \mathbf{I}_{N_B}$, respectively. We define the first hop SNR at the relay station as P_s/σ_n^2 , where $P_s = \sum_{k=1}^K P_{s,k}$. The second hop SNR at the BS is defined as P_r/σ_ξ^2 .

Fig. 2 shows the convergence behavior of the proposed algorithm when $N_B = 4$, $N_R = 4$ and $N_{M,k} = 2$. A simple variation of the brute force iterative algorithm in [11] is shown as benchmark algorithm. Notice that in this case, at each mobile terminal, the number of antennas equals to that of the data streams, and the proposed algorithm involves no relaxation. Initial precoder and forwarding matrices are set as identity matrices. From the figure, it can be seen that the proposed algorithm converges much faster, indicating its superior performance than the brute force iterative algorithm.

When $L_k < N_{M,k}$, the proposed algorithm involves a relaxation. Fig. 3 shows the total data MSEs of the benchmark algorithm and the proposed algorithm, when $L_k = 2$ and $N_{M,k} = 4$. The SNR at BS is fixed at $P_r/\sigma_\xi^2 = 20$ dB. The joint relay forwarding matrix and destination equalizer design in [3] is also shown for comparison. It can be viewed as a design without source precoders at mobile terminals. From Fig. 3, it can be seen that the benchmark and proposed algorithms, which involve the joint design of precoder, forwarding matrix and equalizer, perform better than the algorithm in [3]. This indicates the importance of source precoder design in AF

relaying cellular networks. Furthermore, although the proposed algorithm involves a relaxation, its performance is still promising, and is close to that of the benchmark algorithm, but the proposed algorithm has a much lower complexity. Finally, it can also be observed that increasing the number of antennas at relay station greatly improves the performance of uplink beamforming design for all algorithms.

V. CONCLUSIONS

In this paper, LMMSE beamforming design for uplink AF relaying cellular networks has been investigated. The precoder matrices at mobile terminals, forwarding matrix at relay station and equalizer matrix at the base station were jointly designed via an iterative algorithm. The proposed algorithm covers several existing algorithms for MU MIMO and AF MIMO relaying systems as the special cases. Simulation results demonstrated that our proposed algorithm performs well with low complexity, even when a relaxation is adopted to tackle the case where the number of data streams is smaller than that of transmit antennas.

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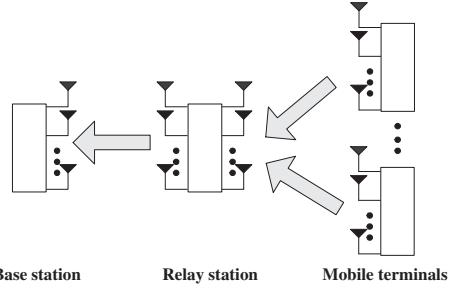


Fig. 1. Uplink amplify-and-forward MIMO relaying cellular networks.

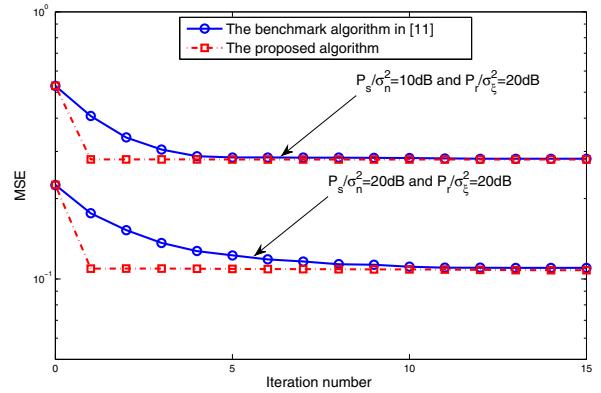


Fig. 2. The convergence behaviors of the benchmark algorithm and the proposed algorithm when $N_B = 4$, $N_R = 4$ and $N_{M,k} = 2$.

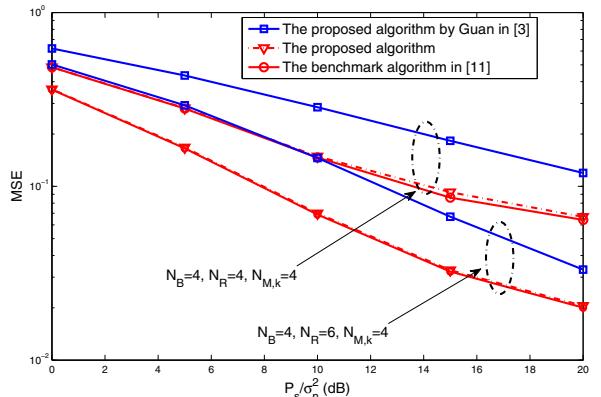


Fig. 3. Total MSEs of the detected data of the benchmark algorithm, the proposed algorithm with relaxation and the algorithm proposed by Guan.