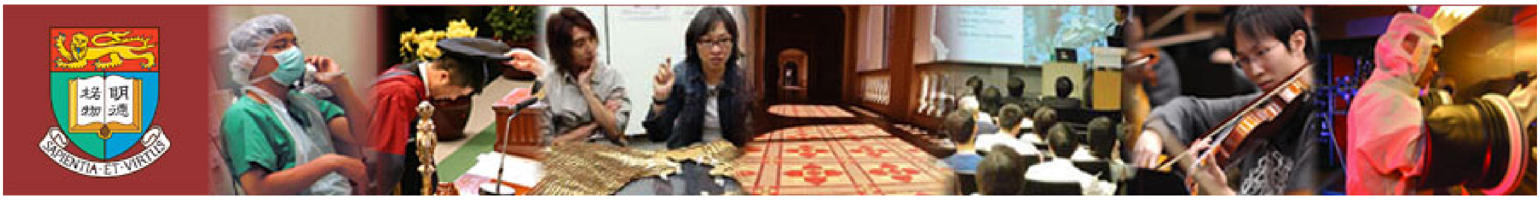

A MODEL-BASED METHOD WITH JOINT SPARSITY CONSTRAINT FOR DIRECT DIFFUSION TENSOR ESTIMATION

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ABSTRACT

Diffusion tensor imaging (DTI) has been widely used for nondestructive characterization of microstructures of myocardium or brain connectivity. It requires repeated acquisition with different diffusion gradients. The long acquisition time greatly limits the clinical application of DTI. In this paper, a novel method, named model-based method with joint sparsity constraint (MB-JSC), effectively incorporates the prior information on the joint sparsity of different diffusion-weighted images in direct estimation of the diffusion tensor from highly undersampled k-space data. Experimental results demonstrate that the proposed method is able to estimate the diffusion tensors more accurately than the existing method when a high net reduction factor is used.

Index Terms—Diffusion tensor imaging (DTI), model-based (MB) method, joint sparsity constraint, distributed compressed sensing

1. INTRODUCTION

Magnetic resonance diffusion tensor imaging (DTI)[1] provides a non-invasive method for in vivo evaluation of tissue water mobility. It has been widely used for nondestructively characterizing microstructures of myocardium or brain connectivity. Typically, a minimum of seven scans of the same images (one reference image plus six diffusion-weighted images) were acquired and reconstructed, from which DTI indices, such as fractional anisotropy (FA) and apparent diffusion coefficient (ADC) representing the extent of diffusion anisotropy and the average diffusion rate, were obtained.

However, long acquisition time greatly limits the practical application of DTI. To accelerate the imaging speed, there are mainly two strategies to obtain diffusion tensor **D** from undersampled k-space data. The first one is to reconstruct all diffusion weighted images (DWI) first and then estimate **D** by conventional least squares fitting [2, 3]. These methods often need additional reference data. Compressed sensing (CS) has also been applied to reconstruct all DW images under the total variation constraint [4]. However, the number of unknowns is usually very large, which may lead to reconstruction errors and thus fitting errors in diffusion tensors. The second strategy is the model-based (MB) method which fits diffusion tensors directly and nonlinearly to the acquired data based on the data consistency in the DTI model without image reconstruction [5, 6]. This strategy is sensitive to the initial diffusion tensor because the measured data correspond to the continuous Fourier transform whereas the estimation is discrete [7]. Therefore, it is necessary to introduce a regularization term. Additionally, most existing methods can only achieve accelerations up to a factor of two with Cartesian undersampling. In this work, we propose a novel model-based method using a joint sparsity constraint [8] (MB-JSC). In addition to the benefit of fewer unknowns and no error propagation in the MB method, the proposed method also greatly reduces the number of measurements and improves the robustness to initial diffusion tensor.

2. PROPOSED METHOD

In DTI, the *j*-th diffusion-weighted image $\mathbf{f}_j$ can be represented as

$$\mathbf{f}_j = \mathbf{I}_0 e^{-b g_j^T \mathbf{D} g_j} e^{i\varphi_j} \qquad (1)$$

where $\mathbf{I}_0$ is the reference image, which was reconstructed separately and then used to estimate the diffusion tensor **D** in this work, *b* is the diffusion weighting factor, g_j is the *j*-th diffusion encoding vector and φ_j is the corrected image phase.

According to Eq. (1), we can see that the diffusion weightings only modulate the intensity of each diffusion-weighted image. It means if a pixel in the reference image $\mathbf{I}_0$ is nonzero, the corresponding pixels in DWIs should also be nonzero. This joint sparsity property along the diffusion directions of $\mathbf{f}_j$'s motivates the application of distributed compressed sensing (DCS) [5] to DTI. Specifically, the transform sparsness in the spatial domain is enforced using the L_1 norm minimization and the information of common non-zero locations along the diffusion directions (i.e., non-sparseness) is exploited using the L_2 norm minimization. With this prior, the cost function is

$$\boldsymbol{\Phi}(\mathbf{D}) = \left\{ \sum_j \left\| \mathbf{d}_j - \mathbf{P}_j \mathbf{F} \mathbf{f}_j \right\|_2^2 + \lambda \left\| \mathbf{C} \right\|_{1,2} \right\}, \mathbf{C} = \boldsymbol{\Psi}[\mathbf{f}_1, \mathbf{f}_2, \cdots, \mathbf{f}_J] \quad (2)$$

where $\mathbf{d}_j$ is the measured k-space of $\mathbf{f}_j$, $\mathbf{P}_j$ is the undersampling matrix for the j-th diffusion-weighted k-space and is different for each diffusion direction. This scheme causes artifacts to be largely incoherent across diffusion tensor directions. **F** is Fourier transform. **C** is the sparse coefficients matrix with size N (#image pixel) × J (#diffusion direction), $\| \|_{1,2}$ is the mixed L_1-L_2 norm of matrix, which applies the L_2 norm to rows of C first (to promote nonsparsity) and then applying the L_1 norm to the resulting vector (to promote sparsity). $\boldsymbol{\Psi}$ is the sparsifying transform. λ is the regularization parameter whose value is determined empirically. The direct fitting of diffusion tensor to the undersampled k-space data using the joint sparsity constraint can be formulated as $\arg\min_{\mathbf{D}} \boldsymbol{\Phi}(\mathbf{D})$.

The diffusion tensor is represented as

$$\mathbf{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{bmatrix} \quad (3)$$

Similar to Ref.[6], we expand the first exponential term in Eq.(1) as

$$\begin{aligned} bg_n^T D g_n &= b_{xxn} D_{xx} + b_{yyn} D_{yy} + b_{zzn} D_{zz} \\ &\quad + 2b_{xyn} D_{xy} + 2b_{yzn} D_{xy} + 2b_{xzn} D_{xz} \end{aligned} \quad (4)$$

where $b_{xxn} = bg_{xn}^2$, $b_{yyn} = bg_{yn}^2$, $b_{xyn} = bg_{xn} g_{yn}$, $b_{zzn} = bg_{zn}^2$, $b_{yzn} = bg_{yn} g_{zn}$, $b_{xzn} = bg_{xn} g_{zn}$, which are scalar values.

A nonlinear conjugate gradient descent algorithm is used to solve the minimization of Eq. (2). The derivative of Eq.(2) is deduced to estimate the diffusion tensors. As $\| \|_1$ is the sum of absolute values, which is not a smooth function and the derivative could not be deduced, we exploit the approximation $|x| = \sqrt{x^* x + \mu}$ used in Ref. [9], where μ is a positive smoothing parameter, * represents the complex conjugate. The derivative with respect to each of the diffusion coefficients can be found to be

$$\boldsymbol{\Phi}'(\mathbf{f}_j) = 2(\mathbf{F}^{-1}(\mathbf{d}_j) - \mathbf{F}^{-1}(\mathbf{P}\mathbf{F}\mathbf{f}_j)) + \lambda \boldsymbol{\Psi}^* \mathbf{W}^{-1} \boldsymbol{\Psi} \mathbf{f}_j, \quad (5)$$

where **W** is a diagonal matrix with the v-th diagonal elements $\mathbf{W}_v = \sqrt{\left(\|\mathbf{C}\|_2\right)_v^* \left(\|\mathbf{C}\|_2\right)_v + \mu}$

$$\boldsymbol{\Phi}'(\tilde{D}_{xx}) = -\sum_j (b_{xxn} \mathbf{f}_j)^* \boldsymbol{\Phi}'(\mathbf{f}_j)$$

$$\boldsymbol{\Phi}'(\tilde{D}_{yy}) = -\sum_j (b_{yyn} \mathbf{f}_j)^* \boldsymbol{\Phi}'(\mathbf{f}_j)$$

$$\boldsymbol{\Phi}'(\tilde{D}_{zz}) = -\sum_j (b_{zzn} \mathbf{f}_j)^* \boldsymbol{\Phi}'(\mathbf{f}_j) \quad (6)$$

$$\boldsymbol{\Phi}'(\tilde{D}_{xy}) = -2\sum_j (b_{xyn} \mathbf{f}_j)^* \boldsymbol{\Phi}'(\mathbf{f}_j)$$

$$\boldsymbol{\Phi}'(\tilde{D}_{yz}) = -2\sum_j (b_{yzn} \mathbf{f}_j)^* \boldsymbol{\Phi}'(\mathbf{f}_j)$$

$$\boldsymbol{\Phi}'(\tilde{D}_{xz}) = -2\sum_j (b_{xzn} \mathbf{f}_j)^* \boldsymbol{\Phi}'(\mathbf{f}_j)$$

The diffusion coefficients are estimated through iteratively updating each parameter, D_{xx}, D_{yy} etc. until the diffusion parameters converge. The pseudo-code of the proposed method is shown in Algorithm 1.

Algorithm 1. Pseudocode for MB-JSC

Input: **d** – k-space measurements
    P – the undersampling matrix
Optional parameters:
TolGrad – stopping criteria by gradient magnitude
MaxIter – strpping criteria by number of iterations
α, β – line search parameters
Output:
D – the numerical approximation of diffusion tensor to Eq.(2)
%initialization
$\mathbf{f}_u = \mathbf{F}^{-1}(\mathbf{Pd})$
D = tensor fitted $\mathbf{f}_u$ using least square
$k = 0; \mathbf{q}_0 = \nabla\boldsymbol{\Phi}(\mathbf{D}); \Delta\mathbf{D} = -\mathbf{q}_0$
%Iterations
$while(\|\mathbf{q}_k\|_2 < \text{TolGrad and } k > \text{maxIter})\{$
  $t = 0.1;$
  $while(\boldsymbol{\Phi}(\mathbf{D}_k + t\Delta\mathbf{D}_k) > \boldsymbol{\Phi}(\mathbf{D}_k) + \alpha t \bullet \text{Real}(\mathbf{q}_k^* \Delta\mathbf{D}_k))$
    $\{t = \beta t\}$
  $\mathbf{D}_{k+1} = \mathbf{D}_k + t\Delta\mathbf{D}_k$
  $\mathbf{q}_{k+1} = \nabla\boldsymbol{\Phi}(\mathbf{D}_{k+1})$
  $\gamma = \frac{\|\mathbf{q}_{k+1}\|_2^2}{\|\mathbf{q}_k\|_2^2}$
  $\Delta\mathbf{D}_{k+1} = -\mathbf{q}_{k+1} + \gamma\Delta\mathbf{D}_k$
  $k = k + 1\}$

To demonstrate the performance of the proposed method, a set of fully-sampled Cartesian k-space DTI data acquired on a 7T Bruker Scanner (Bruker BioSpin) was used to simulate the undersampling k-space data. The spin-echo diffusion tensor imaging (SE-DTI) was performed on an adult SD rat to acquire one reference image and six diffusion-weighted images, with b value = 1000 s/mm^2, TR/TE = 1500/29ms, NEX=10, matrix size = 256×256.

The variable density (MBVD) sampling scheme, usually used in compressed sensing, was applied to simulate net reduction factors of R = 2, 3 and 4 on the phase encoding direction. $\boldsymbol{\Psi}$ represents finite-difference here. The Fractional anisotropy (FA) map and mean diffusivity (MD) map calculated from the reconstructions of full data (see Fig.1) were used as the gold standard. The performance of the proposed method, compared with the model-based method, was quantitatively assessed by calculating the root mean squared error (RMSE) of FA and MD. All methods were implemented in MATLAB. The reconstruction iteration converges within 5 outer loops and 20 inner iterations.

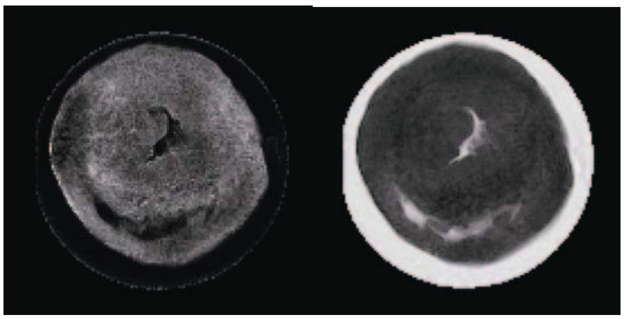


Fig.1 The Fractional anisotropy (FA) map and mean diffusivity (MD) map calculated from the reconstructions of full data.

3. RESULTS AND DISCUSSION

Figures 2 and 3 show the FA and MD maps estimated using MB and MB-JSC methods with R=2, 3 and 4, respectively. We can see that the maps estimated using two methods at R=2 are in good agreement with the gold standard. The FA and MD maps from the proposed MB-JSC method exhibits less noise than those from the full data and MB method due to the additional constraint. When R= 3, the artifacts appear in the maps from MB, but are negligible in those from MB-JSC. When the k-space was heavily undersampled with R=4, the maps estimated using the MB-JSC method still only show negligible artifacts while the MB method presents severe artifacts (indicated by red boxes). The improvement of MB-JSC over MB is also demonstrated in the RMSE of FA and MD listed in Table 1. RMSE of MD is reported in $10^{-3} cm^2/s$. These values are consistent with the above observations.

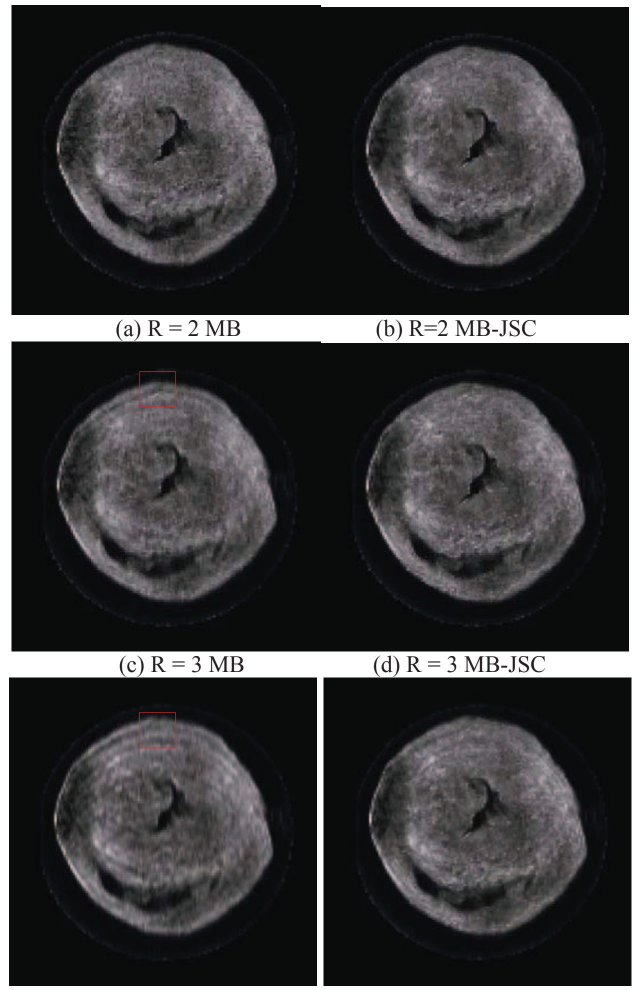
The estimated maps using a random initial D with R=2 are shown in Fig. 4. The MB method presents obvious artifacts. It agrees with the observations in [6] that the MB method is sensitive to the initial D. In contrast, MB-JSC doesn't exhibit large variations with different initial D. It suggests that the introduction of the joint sparsity constraint can improve the robustness of model-based methods.

The main advantage of the proposed method is that the joint sparse constraint allows the intra-signal correlations to be exploited in all diffusion-weighted images. The scan time could thereby be reduced with the improved model-based reconstruction method by acquiring less. Even when the reduction factor R reaches 4, the directly reconstructed parameters are still acceptable. The benefit will be more appealing in 3D cases.

For multichannel coil data, complex coil sensitivity maps $\mathbf{S}_l$ can be incorporated into the method:

$$\boldsymbol{\Phi}(\mathbf{D}) = \left\{ \sum_l \sum_j \left\| \mathbf{d}_{l,j} \text{-} \mathbf{PF}\left(\mathbf{S}_l \mathbf{f}_j\right) \right\|_2^2 + \lambda \left\| \mathbf{C} \right\|_{1,2} \right\} \tag{7}$$

where $\mathbf{d}_{l,j}$ is the measured k-space data from coil l at the j-th diffusion-weighted image .



(a) R = 2 MB (b) R=2 MB-JSC

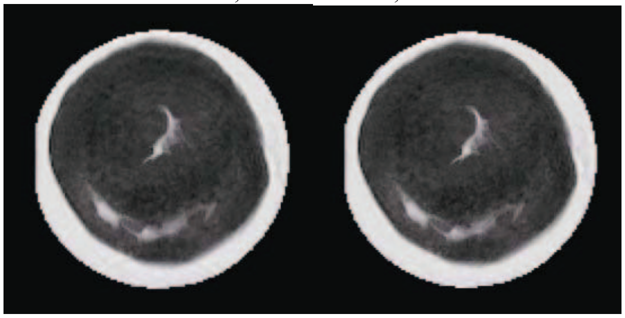
(c) R = 3 MB (d) R = 3 MB-JSC

(e) R = 4 MB (f) R = 4 MB-JSC

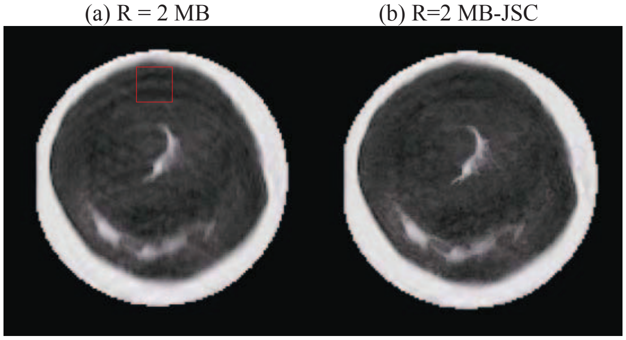
Fig.2 FA maps reconstructed using MB (left column) and MB-JSC (right column) methods.

4. CONCLUSION

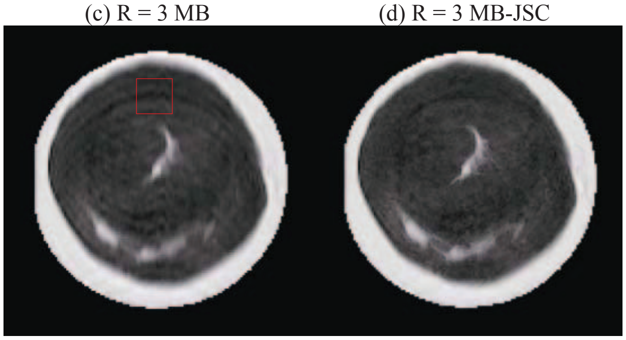
A novel model-based method with joint sparse constraint is proposed to directly estimate the diffusion tensor from undersampled k-space data. Compared with the traditional model-based method, the proposed method can significantly reduce the artifacts due to undersampling, especially when the acceleration factor becomes high. The quantified performance metrics demonstrate that the proposed method can improve the estimation accuracy of the model-based method. The method has potential to be applied in biological tissue characterization, such as neural, muscle and heart.



(a) R = 2 MB (b) R=2 MB-JSC



(c) R = 3 MB (d) R = 3 MB-JSC



(e) R = 4 MB (f) R = 4 MB-JSC

Fig 3. Mean diffusivity maps reconstructed using different methods. The left column is for model-based method and the right column is for MB-JSC.

Table 1. The quantified performance metrics in the root-mean-squared errors (RMSE)

	MB		MB-JSC	
RMSE	FA	MD	FA	MD
R=2	0.0245	5.0941e-5	0.0320	5.2712e-5
R=3	0.0447	8.1461e-5	0.0422	7.0220e-5
R=4	0.0546	9.2823e-5	0.0489	8.0143e-5

5. ACKNOWLEDGMENT

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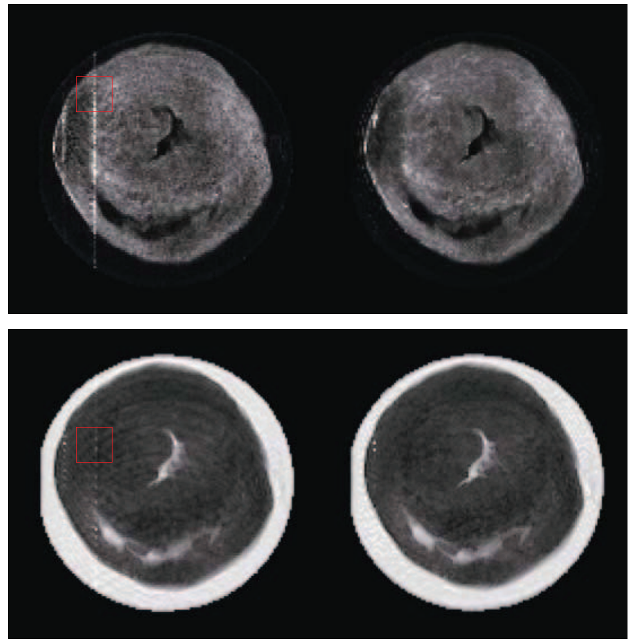

MB MB-JSC

Fig 4. FA and MD maps using random initial D with R = 2.

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