



<b>Title</b>	<b>Author's reply to "comments on 'timing estimation and resynchronization for amplify-and-forward communication systems'"</b>
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Considering that [1] seeks to address timing synchronization in AF relaying cooperative networks, it can be concluded that the signal model in [1, eq. (2)] is oversimplified, since in practical cooperative communications systems the timing offsets,  $\epsilon_k^{[sr]}$ , for  $k = 1, 2, \dots, R$ , cannot be perfectly estimated and compensated.

The authors of [1] further assume that at the  $k$ th relay, a second training sequence,  $\mathbf{p}_k$ , can be perfectly superimposed on the received signal (see [1, eq. (3)]). However, this assumption is an oversimplification, since in practical communications systems the source and relays are equipped with different oscillators. Therefore,  $\mathbf{p}_k$  and  $\mathbf{s}$  are affected by different timing offsets and, subsequently, [1, eq. (6)] must be rewritten as

$$r_k(t) = \sum_{k=1}^K h_k f_k \bar{\mathbf{A}}_{\bar{\epsilon}_k} \mathbf{W}_k \Omega_k \mathbf{s} + \sum_{k=1}^K h_k \mathbf{A}_{\epsilon_k} \mathbf{W}_k \mathbf{p}_k + \sum_{k=1}^K h_k \mathbf{A}_{\epsilon_k} \mathbf{W}_k \mathbf{e}_k + \mathbf{v} \quad (8)$$

where  $\bar{\mathbf{A}}_{\bar{\epsilon}_k} \triangleq [\bar{\mathbf{a}}_{-L_g}(\bar{\epsilon}_k), \dots, \bar{\mathbf{a}}_0(\bar{\epsilon}_k), \dots, \bar{\mathbf{a}}_{L_o+L_g-1}(\bar{\epsilon}_k)]$  with  $\bar{\mathbf{a}}_i \triangleq [g(-iT - \bar{\epsilon}_k T), g(-iT + T_s - \bar{\epsilon}_k T), \dots, g(-iT + (L_o Q - 1)T_s - \bar{\epsilon}_k T)]$ ,  $\bar{\epsilon}_k$ , for  $k = 1, 2, \dots, R$ , accounts for the timing offset estimation error at the  $k$ th relay plus the timing offset from the  $k$ th relay to the destination,  $\epsilon_k$ , and the remaining terms in (8) are defined in [1, eq. (6)]. Based on the training design proposed in [1], the received signal at the destination is affected by two sets of timing offset values  $\bar{\epsilon}_k$  and  $\epsilon_k$ , for  $k = 1, 2, \dots, R$ , instead of only the  $\epsilon_k$  as claimed in [1].

Finally, unlike the results in [1], which assume that the signal at the relays is perfectly matched-filtered, AF relaying cooperative communications systems only require the relays to amplify and forward the received signal as shown in prior work in this field [3]–[6]. This is one of the main advantages of AF relaying, which ensures that the relays have a simple structure that can be more easily deployed in practical applications.

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## Author's Reply to "Comments on 'Timing Estimation and Resynchronization for Amplify-and-Forward Communication Systems'"

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In this reply, technical issues in [1] regarding the Cramér–Rao bound (CRB) and the assumption on relay processing are further investigated and justified. The CRB proposed in [1] is an approximate bound by assuming independence between parameters. On the other hand, in this reply, no such assumption is made, and the true CRB is derived. It is shown that the CRB in [1] approximates the true CRB with high accuracy even in low signal-to-noise ratio (SNR). Besides, it is assumed that the signals received at relays are perfectly re-synchronized in time for tractable treatment in [1], and it is admitted that this task can be onerous in practice.

*Cramér–Rao Bound:* In [1], it is assumed that the channels  $h_k$  and  $\xi_k$ ,  $k = 1, \dots, K$  are independent while they are not, since  $\xi_k = f_k h_k$ . Thus, the CRB in the original paper is not the "true" CRB. However, as shown in Section IV in [1] on the resynchronization algorithm, it is the composite channel  $\boldsymbol{\xi}$  and its estimation uncertainty that enter the algorithm. Therefore, it is assumed that  $h_k$  and  $\xi_k$ ,  $k = 1, \dots, K$  are independent unknown vectors for the purpose of estimation and uncertainty analysis. This manipulation is usually employed in amplify-and-forward (AF) systems (e.g., [2]) without jeopardizing the performance of the design.

In this section, the "true" CRB corresponding to the new set of parameters  $\boldsymbol{\beta} \triangleq [\boldsymbol{\epsilon}, \Re\{\mathbf{f}\}^T, \Im\{\mathbf{f}\}^T, \Re\{\mathbf{h}\}^T, \Im\{\mathbf{h}\}^T]^T$  is computed as a comparison to the approximate CRB in [1]. Denoting  $\boldsymbol{\mu} \triangleq \mathbf{M}(\boldsymbol{\epsilon})\mathbf{F}\mathbf{h} + \mathbf{N}(\boldsymbol{\epsilon})\mathbf{h} = \mathbf{M}(\boldsymbol{\epsilon})\mathbf{H}\mathbf{f} + \mathbf{N}(\boldsymbol{\epsilon})\mathbf{h}$  with  $\mathbf{F} \triangleq \text{diag}[f_1, \dots, f_K]$ ,  $\mathbf{H} \triangleq \text{diag}[h_1, \dots, h_K]$ ,  $\mathbf{M}(\boldsymbol{\epsilon}) \triangleq [\mathbf{A}_{\epsilon_1} \Omega_1 \mathbf{s}_t, \dots, \mathbf{A}_{\epsilon_K} \Omega_K \mathbf{s}_t]$ , and  $\mathbf{N}(\boldsymbol{\epsilon}) \triangleq [\mathbf{A}_{\epsilon_1} \mathbf{p}_1, \dots, \mathbf{A}_{\epsilon_K} \mathbf{p}_K]$ , the  $(i, j)$ th entry of the "true" Fisher information matrix (FIM)  $\mathbf{I}$  is calculated as [3]

$$\mathbf{I}_{i,j} = 2\Re\left\{\frac{\partial \boldsymbol{\mu}^H}{\partial \beta_i} \mathbf{R}_z^{-1} \frac{\partial \boldsymbol{\mu}}{\partial \beta_j}\right\} + \text{tr}\left(\mathbf{R}_z^{-1} \frac{\partial \mathbf{R}_z}{\partial \beta_i} \mathbf{R}_z^{-1} \frac{\partial \mathbf{R}_z}{\partial \beta_j}\right)$$

where  $\beta_i$  is the  $i$ th element in vector  $\boldsymbol{\beta}$  and the corresponding components are computed as in the paper, except for the terms  $\partial \boldsymbol{\mu}^H / \partial \Re\{f_i\} = \mathbf{j} \cdot \partial \boldsymbol{\mu}^H / \partial \Im\{f_i\} = \left(h_i \cdot \mathbf{A}_{\epsilon_i} \Omega_i \mathbf{s}_t\right)^H$  and  $\partial \boldsymbol{\mu}^H / \partial \Re\{h_i\} = \mathbf{j} \cdot \partial \boldsymbol{\mu}^H / \partial \Im\{h_i\} = \left(f_i \cdot \mathbf{A}_{\epsilon_i} \Omega_i \mathbf{s}_t + \mathbf{A}_{\epsilon_i} \mathbf{p}_i\right)^H$ .

With  $\boldsymbol{\Pi}$  defined in the original paper [1], the CRB matrix of the original set of complex-valued parameters  $(\boldsymbol{\epsilon}, \mathbf{f}, \mathbf{h})$  can be evaluated as  $\text{CRB}(\boldsymbol{\epsilon}, \mathbf{f}, \mathbf{h}) = \boldsymbol{\Pi} \cdot \mathbf{I}^{-1} \cdot \boldsymbol{\Pi}^H$

$$\text{CRB}(\boldsymbol{\epsilon}, \mathbf{f}, \mathbf{h}) = \begin{bmatrix} \mathbf{Q}_{\epsilon,\epsilon} & \mathbf{Q}_{\epsilon,\mathbf{f}} & \mathbf{Q}_{\epsilon,\mathbf{h}} \\ \mathbf{Q}_{\mathbf{f},\epsilon} & \mathbf{Q}_{\mathbf{f},\mathbf{f}} & \mathbf{Q}_{\mathbf{f},\mathbf{h}} \\ \mathbf{Q}_{\mathbf{h},\epsilon} & \mathbf{Q}_{\mathbf{h},\mathbf{f}} & \mathbf{Q}_{\mathbf{h},\mathbf{h}} \end{bmatrix} \quad (1)$$

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where  $\mathbf{Q}_{\epsilon, \epsilon}$ ,  $\mathbf{Q}_{\mathbf{f}, \mathbf{f}}$ , and  $\mathbf{Q}_{\mathbf{h}, \mathbf{h}}$  are the  $K \times K$  CRB matrices for  $\epsilon$ ,  $\mathbf{f}$  and  $\mathbf{h}$  respectively. Now that the CRB for  $\mathbf{h}$  and  $\mathbf{f}$  are derived, the CRB of the composite channel  $\xi$  are still yet to be quantitatively analyzed. According to [3], the CRB of a transformed vector  $\xi$  can be obtained as follows:

$$\mathbf{Q}_{\xi, \xi} = \left( \frac{\partial \xi}{\partial \beta} \right) \cdot \text{CRB}(\epsilon, \mathbf{f}, \mathbf{h}) \cdot \left( \frac{\partial \xi}{\partial \beta} \right)^H. \quad (2)$$

It can be readily computed that  $\partial \xi / \partial \beta = [\mathbf{0}_{K \times K} \ \mathbf{H} \ \mathbf{F}]$ , and therefore using the block structure of  $\text{CRB}(\epsilon, \mathbf{f}, \mathbf{h})$ , we have

$$\mathbf{Q}_{\xi, \xi} = \mathbf{F} \mathbf{Q}_{\mathbf{h}, \mathbf{h}} \mathbf{F}^H + \mathbf{H} \mathbf{Q}_{\mathbf{f}, \mathbf{f}} \mathbf{H}^H + \mathbf{F} \mathbf{Q}_{\mathbf{h}, \mathbf{f}} \mathbf{H}^H + \mathbf{H} \mathbf{Q}_{\mathbf{f}, \mathbf{h}} \mathbf{F}^H.$$

Then, the asymptotic distribution of  $\xi$  can be obtained accordingly as  $\xi \sim \mathcal{CN}(\hat{\xi}, \mathbf{Q}_{\xi, \xi})$ , where  $\hat{\xi} = \hat{\mathbf{f}} \odot \hat{\mathbf{h}}$ .

Hereby the  $\text{CRB}(\epsilon, \xi, \mathbf{h})$  in the original paper [1] for  $\theta \triangleq [\epsilon, \Re\{\xi\}^T, \Im\{\xi\}^T, \Re\{\mathbf{h}\}^T, \Im\{\mathbf{h}\}^T]^T$  is numerically compared against the “true” CRB above. In Fig. 1 above, we plot the MSE of  $\zeta_{\text{true}}(\xi) \triangleq \text{tr}(\mathbf{Q}_{\xi, \xi})$  against the ones obtained from  $\text{CRB}(\epsilon, \xi, \mathbf{h})$  in [1]. It can be seen that the difference between the MSE derived from the “true” CRB and that predicted by the “approximate” CRB in [1] is negligibly small, especially in the high signal-to-noise ratio (SNR) region. Similar observations are found in the MSE of the second hop channel  $\mathbf{h}$ , and the figure is omitted here due to space limitation. Since the CRB is mainly derived to replace the asymptotic uncertainty of the estimation algorithms, both the algorithms and the CRB analysis are consistent in the assumption. This also explains why the simulation results for the proposed method reach the CRB in Figs. 2 and 3 in the original paper [1].

*Relay Synchronization:* It was assumed in [1] that the first hop timing offsets can be perfectly estimated and compensated to simplify the problem so that tractable preliminary solutions can be obtained. In practice, this relay synchronization can indeed be onerous. The design

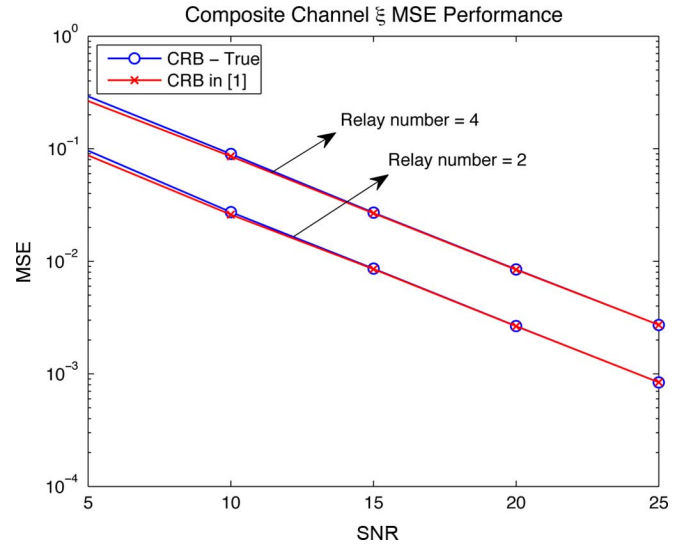


Fig. 1. MSE performance of the composite channel estimate  $\hat{\xi}$  predicted by the “true” CRB and the CRB in [1].

of a comprehensive algorithm, which takes into account the imperfect synchronization at relays, is left for future works.

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