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Analysis of Chaos in Josephson Junctions With External Magnetic Field for High-Precision Voltage Measurement in Electric Vehicles

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Abstract—This paper deals with analysis and control of chaos for high-precision voltage measurement in electric vehicles (EVs) based on Josephson voltage standards. The key is to take into account the effect of external magnetic field in EV environment on Josephson junctions. Based on a modified equivalent circuit model to characterize the Josephson junction, it can be identified that chaos is elicited by the external magnetic field. Consequently, a backstepping controller is designed to suppress the chaos, and hence improve the voltage measurement. Both mathematical derivation and numerical simulation are provided to validate the occurrence of chaos and the feasibility of the control scheme.

Index Terms—Backstepping control, chaos, electric vehicles, Josephson junction.

I. INTRODUCTION

W ITH ever increasing concern on clean and energy-efficient transportation, the development of electric vehicles (EVs), including land vehicles and marine vehicles, has attracted wide attention [1], such as the energy management of EVs [2] and the high temperature superconductor (HTS) application to EVs [3]. However, the energy management of EVs has a fundamental problem, namely the voltage measurement device for batteries is not accurate enough. For example, the nickel-metal hydride cell varies from 1.25 V to 1.2 V over the 80% change in state-of-charge (SOC), indicating that a resolution of 0.1% SOC needs an accurate measurement of 62.5 μ V at 1.2 V. Thus, a high-precision voltage measurement device is highly desirable for EVs.

Recently, the Josephson voltage standards have been developed for high-precision voltage measurement [4], [5]. This voltage measurement technique is developed based on a series array of thousands of superconductor-insulator-normal- conductor-insulator-superconductor (SINIS) Josephson junctions. However, it exhibits complex nonlinear behavior [6] such as chaos may occur at certain external currents. So, various control schemes are proposed to prevent the Josephson junctions from chaos [7]. Nevertheless, most studies on the Josephson

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Fig. 1. Potential application in EVs.

junctions only deal with the influence of external current, while ignoring the effect of external magnetic field which is inevitable in EV environment.

The purpose of this paper is to analyze the nonlinear behavior of Josephson junctions in the presence of varying external magnetic field, and to propose a control scheme to stabilize chaos for high-precision voltage measurement. Firstly, a modified Josephson junction model will be proposed and analyzed in Section II. Secondly, a backstepping control scheme will be proposed and analyzed in Section III. Finally, conclusion and further discussion will be given in Section IV.

II. MODIFIED JOSEPHSON JUNCTION MODEL

Fig. 1 shows the potential application of Josephson voltage standards for the energy management in EVs in which there are various electric devices emitting electromagnetic radiation. In order to assess the effect of magnetic field on the dynamic performance of Josephson junctions, a modified resistive-capacitive-inductive shunted junction (RCLSJ) model is proposed as shown in Fig. 2. Compared with the traditional one, the proposed model takes into account the external magnetic field in the surroundings.

A. Mathematical Model

As depicted in Fig. 2, the Josephson junction is shunted to a capacitor C and a resistor R_v as well as an inductor L and a resistor R_s that are connected in series, where θ denotes the



Fig. 2. Modified Josephson junction model.

phase difference of the superconducting order parameter across the junction. Usually, R_v represents the junction resistance, and C is the junction capacitance.

From the modified model, a set of circuit equations can be derived as given by [8], [9]:

$$C\frac{dV}{dt} + \frac{V}{R_v} + I_c \sin\left(\theta - \rho f(t)\right) + I_s = I_{ext}$$
(1)

$$I_{ext} = I_{dc} + I_{ac} \sin(\omega t) \quad (2)$$
$$\frac{h}{2} \frac{d\theta}{dt} = V \quad (3)$$

$$L\frac{dI_s}{dt} + I_s R_s = V \tag{4}$$

where $\rho f(t)$ represents the effect of the external magnetic field, V is the voltage across the Josephson junction, h is the Planck's constant, e is the electron charge, I_c is the critical current, I_s is the current in the $L - R_s$ branch, and I_{ext} is the external current which is composed of the DC component I_{dc} and the AC component $I_{ac} \sin(\omega t)$ [9], [10]. The nonlinear R_v is represented by a piecewise linear resistance as defined by:

$$R(V) = \begin{cases} R_n, & if \ |V| > V_g \\ R_{sg}, & if \ |V| \le V_g \end{cases}$$
(5)

where V_g denotes the gap junction voltage, R_n is the normal state resistance when $I_{ext} > I_c$, and R_{sg} is the subgap resistance when $I_{ext} \leq I_c$.

By substituting (2) and (3) into (1) and replacing R_v by R(V), it yields a standard 2nd-order differential equation:

$$\frac{d^2\theta}{dt^2} + \beta \frac{d\theta}{dt} + \Omega_0^2 \sin\left(\theta - \rho f(t)\right) + \frac{2\pi e}{hC} I_s = A_0 + A_1 \sin(\omega t)$$
(6)

where $\beta = 1/(R(V)C)$ is the so-called damping factor of the junction, $\Omega_0 = (2\pi e I_c/hC)^{1/2}$ is the so-called natural frequency of the junction, $A_0 = (2\pi e I_{dc}/hC)$ and $A_1 = (2\pi e I_{ac}/hC)$.

Selecting the state variables as $x_1 = \theta$, $x_2 = V$ and $x_3 = I_s$, and then taking the derivatives with respect to τ , the system equation can be transformed as:

$$\dot{x}_1 = x_2 \tag{7}$$

$$\dot{x}_2 = \frac{1}{1} \left[L_x + L_x \sin(\omega \tau / \omega_z) - G(x_2) x_2 - x_2 \right]$$

$$x_{2} = \frac{\beta_{C}}{\beta_{C}} \frac{[I_{dc} + I_{ac} \sin(\omega t / \omega_{o}) - G(x_{2})x_{2} - x_{3}]}{-\sin(x_{1} - \rho f(t))]$$
(8)

$$\dot{x}_3 = \frac{x_2 - R_s x_3}{\beta_L} \tag{9}$$

where $\beta_C = (hC/2\pi eI_c)$ and $\beta_L = (2\pi eI_c L/hR_s)$ are the capacitive and inductance constants respectively,



Fig. 3. Phase portrait of x_2 versus x_3 with $\rho = 0.1$.

G(V) = 1/R(V) is the junction conductance, $\omega_0 = 2\pi e/h$ and $\tau = \omega_0 t$.

When $\rho f(t)$ is very small, namely $|\rho f(t)| \ll 1$, it yields:

$$\sin\left(\theta - \rho f(t)\right) \approx \sin(\theta) - \rho f(t)\cos(\theta) \tag{10}$$

By using (10), (8) can be further simplified as given by:

$$\dot{x}_{2} = \frac{1}{\beta_{c}} \left[I_{dc} + I_{ac} \sin(\omega \tau / \omega_{o}) - G(x_{2})x_{2} - x_{3} - \sin(x_{1}) + \rho f(t) \cos(x_{1}) \right]$$
(11)

B. Nonlinear Analysis

When applying the Josephson voltage standards to EVs, the Josephson junctions take the definite advantage of high-precision voltage measurement but their stability is inevitably affected by external magnetic fields emitted by electric motor drives and vehicular wireless communication. In order to assess the nonlinear dynamics of the modified RCLSJ model under varying external magnetic field, the magnetic field magnitude ρ is naturally chosen as the bifurcation parameter.

Due to the aforementioned nonlinear behavior, the system exhibits complex oscillations including periodic, multi-periodic and chaotic oscillations along with the increasing external magnetic field. The Runge-Kutta method is used to numerically simulate the modified model. Based on the available data of Josephson junctions [11], it can deduce $\beta = 0.707$, $\beta_C = 0.707$, $\beta_L = 2.6$, $G(x_2) = 0.061$ when $|x_2| \le 2.9$ and $G(x_2) = 0.366$ when $|x_2| > 2.9$.

It should be noted that the damping factor of the adopted Josephson functions is $\beta = 0.707$ [11]. When $\beta < 1$, the junctions are over-damped; otherwise, they are under-damped ones. So, the type of Josephson system for this EV application is based on the programmable Josephson voltage standards.

Since this paper aims to investigate the effect of external magnetic field, the effect of I_{ext} is ignored so that $I_{dc} = 0$ and $I_{ac} = 0$. Also, the time-varying pattern of magnetic field variation is taken as $f(t) = \cos(t)$. When the magnetic field magnitude is selected as $\rho = 0.1$, the phase portrait of x_2 versus x_3 is simulated as depicted in Fig. 3. It can be seen that the system exhibits a periodic solution, namely the state variables oscillate with a constant period. By increasing the value of ρ , multi-periodic oscillations can be observed as shown in Figs. 4 and 5. When ρ is further increased to $\rho = 5.6$, the system exhibits



Fig. 4. Phase portrait of x_2 versus x_3 with $\rho = 2.5$.



Fig. 5. Phase portrait of x_2 versus x_3 with $\rho = 4.8$.

chaos as characterized by the highly fluctuating time waveform in Fig. 6 and very messy phase portrait in Fig. 7.

In order to prove the existence of chaos in the modified RCLSJ model mathematically, the maximum Lyapunov exponent λ_{max} is computed as given by:

$$\lambda_i(i=1 \sim d) = \lim_{h \to \infty} \frac{1}{h\Delta t} \sum_{j=0}^{h-1} \log \frac{\left\| T^{\Delta t} \left(X_j + e_i^j \right) - T^{\Delta t}(X_j) \right\|}{\left\| e_i^j \right\|}$$
(12)

where Δt is the evolution time, $T^{\Delta t}$ is the map describing the time evolution of X, e_i^j is the *i*-th base vector of the *d*-dimension state space at the *j*-th step, and λ_i is the Lyapunov exponent which indicates the separating rate of close trajectories of a dynamical system. Consequently, the maximum value of λ_i can be calculated, namely $\lambda_{max} = 1.137$ when $\rho = 5.6$. The positive value of λ_{max} mathematically confirms that chaos occurs in the RCLSJ model when $\rho = 5.6$. Similarly, chaos that occurs at other values of ρ can readily be proved.

III. CHAOS CONTROL

Differing from those backstepping control design for linear systems [12], the backstepping control scheme is modified and extended to nonlinear systems, with emphasis on suppressing chaotic oscillations elicited by external magnetic field in Josephson junctions.

The design procedure for the proposed backstepping controller is described by the following 4 steps:



Fig. 6. Time waveform of x_2 with $\rho = 5.6$.



Fig. 7. Phase portrait of x_2 versus x_3 with $\rho = 5.6$.

• *Step* I: By applying the control signal *u* to the state equation involving the bifurcation parameter which governs the occurrence of chaotic oscillations, (11) can be rewritten as:

$$\dot{x}_{2} = \frac{1}{\beta_{c}} \left[I_{dc} + I_{ac} \sin(\omega \tau / \omega_{o}) - G(x_{2})x_{2} - x_{3} - \sin(x_{1}) + \rho f(t) \cos(x_{1}) \right] + u \quad (13)$$

Step II: By taking x_d = 0 as the desired value as well as defining e₁ = x₁-x_d, e₂ = x₂-c₁e₁ and e₃ = x₃-c₂e₁, a new virtual model is obtained as given by:

$$\dot{e}_{1} = e_{2} + c_{1}e_{1} - x_{d}$$

$$\dot{e}_{2} = \dot{x}_{2} - c_{1}\dot{e}_{1}$$

$$= \frac{1}{\beta_{C}} \left[-G(x_{2})(e_{2} + c_{1}e_{1}) - e_{3} - c_{2}e_{1} - \sin(e_{1} + x_{d}) + \rho\cos(t)\cos(e_{1} + x_{d}) \right]$$

$$- c_{1}e_{2} - c_{1}^{2}e_{1} + c_{1}x_{d} + u$$
(15)

$$\dot{e}_3 = \dot{x}_3 - c_2 \dot{e}_1 = \frac{1}{\beta_L} (e_2 + c_1 e_1 - c_2 e_1) - c_2 e_2 - c_1 c_2 e_1 + c_2 x_d$$
(16)

where c_1 and c_2 are constant coefficients.

• *Step* III: By defining the Lyapunov function as:

$$Y = \frac{1}{2} \sum_{i=1}^{3} k_i e_i^2 \tag{17}$$



Fig. 8. Time response of x_2 with $\rho = 5.6$ under chaos control.



Fig. 9. Phase portrait of x_2 versus x_3 with $\rho = 5.6$ under chaos control.

where $k_i \ge 0$, it yields:

$$Y = k_1 e_1 \dot{e}_1 + k_2 e_2 \dot{e}_2 + k_3 e_3 \dot{e}_3$$

= $k_1 e_1 (e_2 + c_1 e_1 - x_d)$
+ $k_2 e_2 \left\{ \frac{1}{\beta_C} \left[-G(x_2)(e_2 + c_1 e_1) - e_3 - c_2 e_1 - \sin(e_1 + x_d) + \rho \cos(t) \cos(e_1 + x_d) \right] - c_1 e_2 - c_1^2 e_1 + c_1 x_d + u \right\}$
+ $k_3 e_3 \left\{ \frac{1}{\beta_L} (e_2 + c_1 e_1 - c_2 e_1) - c_2 e_2 - c_1 c_2 e_1 + c_2 x_d \right\}$ (18)

Step IV: In order to stabilize the system, (18) should be a negative-definite function, namely Y < 0. Thus, the corresponding parameters are chosen as k₁ = 0, k₂ = 1, k₃ = 0 and c₁ = 0, and the control signal is designed as:

$$u = x_2 \left[-\frac{1}{\beta_C G(x_2)} + \frac{G(x_2)}{\beta_C} \right] + \frac{(x_3 - c_2 x_1)}{\beta_C} + \frac{c_2 x_1}{\beta_C} + \frac{\sin(x_1)}{\beta_C}$$
(19)

To assess the feasibility of the proposed backstepping controller, the dynamic performance of the system is simulated. Namely, when $\rho = 5.6$, the controller is initially inactive and then activated after time t = 40 s. Fig. 8, shows the time response of x_2 which verifies that the controller can successfully stabilize the chaotic oscillation to the stable periodic oscillation. Additionally, the amplitude of this periodic oscillation is greatly suppressed. The phase portrait of x_2 versus x_3 is plotted in Fig. 9 which also indicates that the system is free from the chaos.

Therefore, by using this backstepping controller, the Josephson junctions can avoid the occurrence of chaos even under a large magnitude of external magnetic field. It indicates that the Josephson voltage standards can be employed for high-precision voltage measurement in EVs. Consequently, by providing high-precision voltage feedback of EV batteries, the performance of the EV energy management system can be significantly improved.

Finally, it should be noted that there are other techniques such as a careful shielding to avoid chaos in the Josephson junctions elicited by an external magnetic field.

IV. CONCLUSION

In this paper, analysis and control of chaos for adopting Josephson voltage standards in EVs have been presented. The key is to propose the modified RCLSJ model to characterize the Josephson junction in the presence of external magnetic field. Both mathematical derivation and numerical simulation confirms that complex behaviors including chaotic oscillations can be elicited along with the increase of external magnetic field. Consequently, a backstepping controller has been proposed and designed to suppress the chaos occurred at Josephson junctions, and hence improve the accuracy of voltage measurement. Dynamic performance of the system has also been simulated to validate the feasibility of the proposed control scheme.

Because of the costly Josephson voltage standards and the associated energy consumption for cryogenic cooling, a precise voltage measurement based on Zener diodes may be a more economical way to provide the desired measurement. It is anticipated that the application of Josephson voltage standards will be justifiable only for those military or large EVs such as electric tanks or electric vessels.

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