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STUDENT'S CLASS WORKS: AN EXAMPLE IN HONG KONG 8TH GRADE MATHEMATICS CLASSROOM

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The student's class work of an 8th grade mathematics classroom in Hong Kong has been studied. The data were obtained from the Learning Perspective Study (LPS). Five consecutive lessons for the topic of "solving simultaneous equation by the graphical method and the method of substitution" were analyzed. The tasks in the lessons were differentiated as the teacher's examples and the student's class work. The cognitive domains of the student's class work were further classified. All the tasks were classified as either knowing or applying, and no reasoning domain was identified. Results showed that most of students imitated the teacher's examples completely or partly. Only two of the tasks showed modification of teacher's method and one task solved by student's own method. Finally, we argue that the strong direct role of teacher might help the students master their mathematical content in a relatively short time but may have the danger of limiting the students' opportunity for independent thinking.

INTRODUCTION

Learning takes place in a mathematics classroom by different kinds of activities. Besides teacher's instruction, opportunities for the students working either in their seats individually or in a small group are important. Students' work in the lessons often is recorded in their notebooks. For example, Fried and Amit (2003) investigated the students' notebooks in an Israel Grade 8 mathematics class and the work in the notebook was in fact "the rehearsals for public display" (p.97). Therefore, in the study by Fried and Amit, the students' notebooks were public as they were open for inspection. On the other hand, in the case of Hong Kong mathematics classrooms, student's class work is often individual and private. Sometimes in Hong Kong lessons the student's work is selected to show on the board. Therefore, the student's class work can be categorized as public or private depending how they are used in the lessons or the culture established in the class.

The examination of the student's class work can help in understanding the students' mathematics learning outcome. The purpose of this study focused on two questions:

- What are the features of the student's class work?
- What are the similarities or differences between teacher's examples and students' class work?

A FRAMEWORK OF THE STUDY

Agency of knowledge generation

Knowledge can be generated by the interactions of the teacher and the students during the engagement of the tasks. Clarke and Xu (2008) have been studied the responsibility for knowledge generation in Australian, China, Japan, Korea and the USA mathematics classroom. The voices of mathematical ideas in the teacher-student interactions in classroom activity have been analysed. The relative contributions of teacher and students to the mathematical terms could be provided as an indication of the responsibility for knowledge generation. In this study, both the teacher and the students were responsible for the knowledge generation.

Cognitive domain of tasks

Doyle (1988, 1986) studied the work in the mathematics classrooms and found that familiar work system usually lead to high levels of student engagement and production and a well-managed classroom environment. In contrast, novel works would lead to high risk of failure. Carpenter and Lehrer (1999) described how task aids in learning. Tasks can range from simple drill-and-practice exercises to complex problem-solving tasks set in rich contexts. However, it is not the content of the tasks alone which determine the opportunity for learning. A challenging problem can be taught in such way that students simply followed some routine procedures, whereas a simple task for some fundamental basic skills can be taught in a culture fostering mathematical understanding. Therefore, the nature of the tasks used in the lessons is important factor for the students' learning. The framework developed in TIMSS study is applied in this study to classify the cognitive domains of the student's class work.

> The first domain, knowing, covers the facts, concepts, and procedures students need to know, while the second, applying, focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The third domain, reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems. (Mullis, et al., 2009, p.40)

DESIGN OF THE LEARNER'S PERSPECTIVE STUDY (LPS)

Ten consecutive mathematics lessons of a Hong Kong 8th grade class (HK3) were obtained from the Learner's Perspective Study (LPS). An integrated system of three cameras were used to collect data in which one is for the whole class, one is for the teacher; one is for a group of two focus students and one on-site mixing of teacher's and focus students' cameras. Each group of students was videotaped once and post-lesson video-stimulated interviews of focus students conducted. Focus students' notebooks and worksheets including those notes on that particular lesson were photocopied and collected. (Clarke et al., 2006)

Method of analysis

Five consecutive lessons (HK3_L05 to HK3_L09) were selected and analysed in this study. The five lessons covered the topic of "solving a system of simultaneous equations by the graphical method (HK3_L05 to HK3_L07) and by the method of substitution (HK3_L08 to HK3_L09)". The two sub-topics were chosen as the methods are contrastive.

The teaching materials in the lessons were the two worksheets and the examples not in the worksheets. Each worksheet was the major teaching material of each sub-topic, worksheet 1 was for solving a system of simultaneous equation by graphical method and worksheet 2 was for the method of substitution. Additional examples were demonstrated by the teacher and these examples were excluded from the worksheet. The tasks in the lessons were classified as either the teacher’s examples or the student’s work. The teacher’s examples were demonstrated in teacher’s exposition. The student’s work was sub-categorized into student’s class work and student’s unfinished class work. Student’s class work consisted of those questions solved by the students in the class. Student’s unfinished class work was that incomplete class work and the teacher assigned to complete them at home due to insufficient time of the lesson (table 1). Each task was then coded with a cognitive domain with sub-category. Three cognitive domains: knowing, applying and reasoning were applied to the tasks. Since students copied the teacher’s examples directly and did not require any cognitive skills, those teacher’s examples were coded as copying (table 2).

The similarities and differences between the student’s class work and the corresponding teacher’s examples were compared. The class work of two focus students in each lesson were collected and categorized. The degrees of imitation of teacher’s examples on student’s class works were categorized as imitation, partial imitation, modified method, students’ own method and others (table 3).

Table 1: Classification of teacher’s and student’s work

Teacher/student work	Description
Teacher’s example	The teacher demonstrated the examples in exposition
Student’s work	
Student’s class work	Students worked in the class
Student’s unfinished class work	Students did not complete in the class and continue to work at home

Example of knowing – compute

5. Solve the following simultaneous equations

(a) 5 – x- 2y = 7x + 5y + 3 = -2

(b) 2x – y + 4 = 25 – 3x + 2y = 12

(c) 6x + 5y – 4 = 3y – 8x -14 = 0 (Question 5 in worksheet 2)

It required students to carry out routine algebraic procedures and thus was categorized as knowing – compute.

Example of knowing – retrieve

2(b) Use the graph drawn in (a) to answer the following questions:

(i) Does the point A (2, 2) lie on the line?

(ii) If B (b, 0) is a point on the line, what is the value of b?

(iii) If C (0, c) is a point on the line, what is the value of c?

(iv) If D (-1, d) is a point on the line, what is the value of d? (Question 2b in worksheet 1)

The questions required students to retrieve information from the graph drawn in 2(a), it was thus classified as knowing – retrieve.

Example of applying – implement

> 7. Solve the following pair of simultaneous linear equations graphically:
>
> $$\begin{cases} x + 2y - 4 = 0 \\ 2x + 4y = -8 \end{cases}$$
>
> (Question 7 in worksheet 1)

The question asked students to implement a set of mathematical instruction to drawing the graphs, so it belongs to applying – implement.

Example of Copying

> Draw the graph of the linear equation in two unknowns
>
> y = 2x + 3 where x takes values from -2 to 2. (Question 1 in worksheet1)

Students copied the notes from the teacher's examples, so it was classified as copying.

Table 2: Codes for cognitive domains (modified from TIMSS 2011 framework, p. 40-46)

Code	Description
	Knowing
Recall	Recall definitions; terminology; number properties; geometric properties; and notation
Recognize	Recognize mathematical objects, e.g. shapes, numbers, expressions, and quantities. Recognize mathematical entities that are mathematically equivalent
Compute	Carry out algorithmic procedures for+, − ,× , ÷, or a combination of these with whole numbers, fractions, decimals and integer. Approximate numbers to estimate computations. Carry out routine algebraic procedures.
Retrieve	Retrieve information from graphs, tables, or other sources; read simple scales
Measure	Use measuring instruments; choose appropriate units of measurement
Classify/Order	Classify/group objects, shapes, numbers, and expressions according to common properties; make correct decisions about class membership; and order numbers and objects by attributes.
	Applying
Select	Select an efficient/appropriate operation, method, or strategy for solving problems where there is a knowing procedure, algorithm or method of solution
Represent	Display mathematical information and data in diagrams, tables, charts, or graphs, and generate equivalent representation for a given mathematical entity or relationship
Model	Generate an appropriate model, such as an equation, geometric figure, or diagram for solving a routine problem
Implement	Implement a set of mathematical instructions
Solving routine problems	Solve standard problems similar to those encountered in class. The problems can be in familiar contexts or purely mathematical
	Reasoning
Analyse	Determine, describe, or use relationships between variables or objects in mathematics situations, and make valid inferences from given information.
Generalize/Specialize	Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms
Integrate/Synthesize	Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas. Combine mathematical facts, concepts, and procedures to establish results, and combine results to procedure a further result.
Justify	Provide a justification by reference to known mathematical results or properties
Solving non-routine problems	Solve problem set in mathematical or real life contexts where students are unlikely to have encountered closely items, and apply mathematical facts, concepts, and procedures in unfamiliar or complex contexts.
	Others (new code)
Copying	Copy the teacher's examples from teacher's demonstration

Table 3: The methods employed in the student's work

Degree of imitation	Description
Imitation	Students reproduced the teacher's examples exactly in solving the problem.
Partial imitation	Students reproduced the teacher's examples in solving the problem, but some of steps stated in the teacher's examples were missed or some of the steps were not the same.
Modified method	Students reproduced the teacher's examples with modification
Student's own method	Students used a method different from the teacher's examples
Others	The work was completed partly or no work was done

RESULTS

The mathematical tasks observed in the lessons were categorized as the teacher's examples and the student's work. 17 mathematical tasks were identified and some of the tasks were further divided as sub-tasks resulting in a total of 23 tasks. For instance, Question 4 in worksheet 2 consisted of three sub-questions in which Question 4a was the teacher's example while Question 4b and Question 4c were the student's class work. In the ten teacher's examples, seven of them were from the worksheets while the remaining three questions were excluded from the worksheets; these questions were aimed to review some terminologies. For instance, the teacher used an equation of $y = 2x - 1$ as an example of "an equation of two unknowns". The other 13 tasks were the student's work in which 12 of them were student's class works and 1 was student's unfinished class work.

The lessons often arranged as 1 or 2 teacher's examples followed by 1 or 2 student's class works (expect HK3_L07, no teacher's example was found in this lesson). The tasks of the teacher's examples and the subsequent student's class work were often in the same phenotype. Students copied the teacher's examples and/or listened to the teacher's discourse during the teacher's expositions. The copies of teacher's examples were found in eight out of ten students' worksheets or notebooks. Since copying the teacher's examples did not require cognitive skills, they were not further analysed but acted as a measure of the degree of the imitation. The student's work was categorized with cognitive domains and the degree of the imitation of student's work was investigated.

Feature of student's class work

The numbers and categories of the tasks were summarized in table 4. Two types of cognitive main domains, knowing and applying were found. Most of tasks were categorized knowing – compute (53.8%, 7 out of 13) or applying – implement (38.5%, 5 out of 13; 4 of them were student's class work and the other one was the student's unfinished class work). The remaining task was categorized as knowing – retrieve (7.7%, 1 out of 13). No reasoning domain was found.

The cognitive domain of the tasks may due to the nature of the sub-topic and the design of the tasks. The tasks in the worksheet 1 (the graphical method) were categorized into three sub-domains. The tasks required students to carry out algorithmic procedures to prove the points on a line without plotting a graph (knowing – compute), to read the information from a graph (knowing – retrieve) or to apply a set of procedures to plot a graph (applying – implement). On the other hand, all the tasks in worksheet 2 (the method of substitution) were identified as knowing – compute. The tasks asked students to perform algorithmic procedures for operation in solving a system of simultaneous equations. The tasks of the worksheets were seemed to design for student practice.

According to the TIMSS 2007 report, both 4th and 8th grades of Hong Kong students got the highest average scales scores for Mathematics in knowing among the three cognitive domains. The second was applying and the third was reasoning. Students may familiarize to work on the questions of knowing domain and thus resulted in a better performance. The majority of tasks in our study were in knowing and applying domains. Thus students had more practices in solving these two domains of questions and familiarized to work on them.

Table 4: Numbers and categories of the tasks

Cognitive domain	Numbers	Percentage (%)
Knowing – Compute (student's class work)	7	53.8%
Knowing – Retrieve (student's class work)	1	7.7%
Applying – Implement (student's class work)	4	30.8%
Applying – Implement (student's unfinished class work)	1	7.7%
Total	13	100%

Comparison on work of teacher's examples and student's class work

The class works of two focus students were collected in each lesson. The work of each task was compared with the corresponding teacher's example. A total of 30 tasks were classified as some of the sub-questions were further divided because of their different degree of imitation. Table 5 showed the degree of imitation of student's work. Although the teacher demonstrated the procedures explicitly, about 16.7% (5 out 30) of tasks imitated the teacher's examples completely and 43.3% (12 out of 30) imitated the teacher's examples with missing parts. Most of the missing procedures in the student's work were labelling equations and solution statements. The same student repeated the missing procedures in solving the same kind of tasks. For instance, Jennifer who was the focus student in HK3_L08 missed the labelling on the equations in all tasks. 6.7% (2 out of 30) of student's work were classified as modified method and one work (3.3%) was solved by student's method. Unattempted and incomplete work contributed 30.0% (9 out of 30) of the total tasks. When the unfinished work was excluded, 18 out of 21 tasks (85.7%) were imitated the teacher's examples completely or partly and 3 tasks (14.3%) were either modified method or student's method. This indicated that most of the student's work was solved by imitation from the teacher's examples.

Table 5: The degree of imitation of student's work

Code	Number	Percentage
Imitation	5	16.7%
Partial imitation	13	43.3%
Modified method	2	6.7%
Student's own method	1	3.3%
Incomplete/Unattempted work	9	30%
Total	30	100%

Case 1: Imitation

Figure 1 showed Janet's class work where part (a) was the teacher's example and part (b) was her class work. By comparing with her copy of the teacher's example, she reproduced the teacher's procedures exactly in solving the class work. Since the teacher's example and the class work belong to the same question, the student imitated the teacher's example as expected.

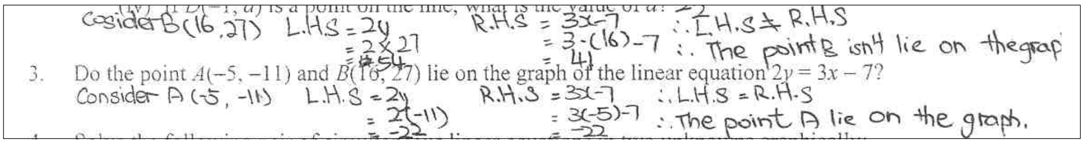

Figure 1: Example of imitation: the class work of Janet (HK3_L05). The top (point B) was the class work and the bottom (point A) was the copy of teacher's example.

Case 2: Degree of imitation

The teacher used three points to plot a line by using the values of 1, 3 and 5 as x coordinate with the corresponding y values. By comparing the class work of two focus students, Janice and Gary, their works showed different degree of imitation. Gary followed the teacher's examples by using the 1, 3 and 5 as the values of x coordinate in plotting the two lines. He imitated the teacher's example exactly. On the other hand, Janice also used three points in drawing the lines but she used 1, 2 and 3 as the values of x coordinate in plotting one of the line. Although she imitated the teacher's example by using three points, she chose different values of x. In her interview, she explained the choice of different values of x because of her birthday.

Int: Mm … have you think about how to choose?

Janice: No, just any number.

Int: Just pick it randomly.

Janice: Choose one, two, and three.

Int: You choose one, two, three, what is it for? Why have to choose one, two, and three?

Janice: My birthday.

When we look back on the teacher's discourse, the teacher used three points to plot the lines but did not explain the selection of points and the importance of the range. Strictly speaking both Gary and Janice followed the teacher's instruction and imitated the teacher's method. However, Gary's work did not suggest whether he could use other points or not. Janice's work showed an awareness of the feasibility of using points other than those suggested by the teacher. However, an important aspect for plotting the graph in practice was to consider the range while choosing the point. Janice probably had not realize the importance of the range in drawing lines, while she chose the points with her "birthday" preference.

Case 3: Modified method: different number of points in plotting line (HK3_L05)

In the corresponding teacher's example, the teacher demonstrated the standard procedures of solving an equation by the graphical method. The procedures involved using three pairs (-2, -1), (0, 3) and (2, 7) of x and y values to plot the graph. Since the question set the range from -2 to 6, the range was the same in Jane's and Janet's work. In Jane's work, although she reproduced the teacher' example in using the even numbers (-2, 0, 2, 4 and 6) as the values of x coordinate, she used five points to draw the line. By contrast, Janet set nine points to draw the line in which the values of x were -2, -1, 0, 1, 2, 3, 4, 5 and 6. When they discussed their work, they found that 3 points were enough in plotting lines.

Janet: Negative two to six.

Jane: You may choose any three of them.

Janet: I see. You should have told me earlier.

Similar to Janice, they did not show that they knew the reason of using three points in drawing lines.

Case 4: The students' method

Joanne did her class work without imitation the teacher's instruction. In the lesson, Joanne did Question 6a before the teacher's instruction. She applied the same method as using in the previous questions in the same worksheet and did not enlarge the LCM of the denominators before the transformation of one of the equations (the teacher's instruction).

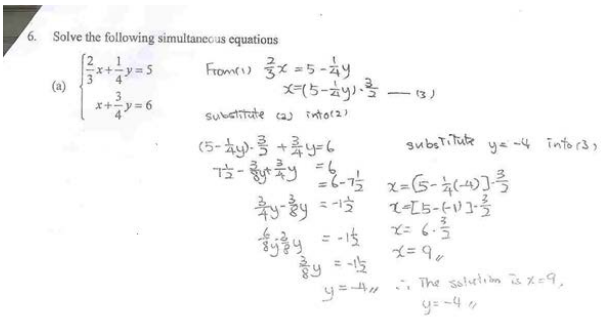

Figure 4: Joanne's class work of Question 6a in worksheet 2.

When we examined her worksheet in details, she did one of sub-question in each question (except question 1). She explained,

> "The same calculation method, but not the same numbers, just for familiarizing, see whether you understand it or not."

Although the same cognitive domains of tasks seemed to limit the variation of the tasks, the levels of difficulty of the tasks were increased. Modification of the method was suggested in the teacher's instruction. Another feature of Joanne's class work was the absence of the teacher's examples. However, she valued the teacher's examples as a model for imitation; "I don't know how to do it without examples", she said.

Students' view on teaching practice

Mok, Kuar, Zhu and Yau (in press) studied a comparison on the student's view on teacher practice in Hong Kong and in Singapore classrooms. The findings of both Hong Kong and Singapore had three main instructional practice, exposition, seatwork and review and feedback. Hong Kong students valued the teacher's explanation and teacher's demonstration, individual work and giving feedback were most important sub-categories in exposition, seatwork and review and feedback respectively.

The same coding scheme was applied to the students' interviews. The most frequent categories were exposition – teacher's demonstration (11 out of 32 segments, 34.3%), exposition – new knowledge (5 out of 32 segments, 15.6%), seatwork – individual class work (6 out of 32 segments, 18.8%) and review and feedback – giving feedback (4 out of 32 segment, 12.5%). The teacher's demonstration and the new knowledge were the most important as they eager to learn the methods and concepts from the teacher's demonstration and pay attention to those concepts that were new to them. Students valued individual seatwork as importance as they believed that they understood the concepts by their own work. They appreciated on checking the answers and the corrective feedback was also an important part of their learning process. The similar results of both Hong Kong schools were expected as the lessons shared similar teaching and learning features.

Conclusion

The teachers play a significant guiding role in the mathematics classroom in the culture of in Asian regions (e.g., Mok and Morris, 2001; Leung and Park, 2002). For example, in an exemplary mathematics lesson studied by Mok (2009), the teaching in the lessons followed a directive approach. The teacher demonstrated the standard procedures in the teacher's examples were the major input component of teaching. The strong guidance was observed in the teacher's exposition and in between-desk-instruction of student's class work and it was welcome by the students.

In our study, the student's class work of an 8^{th} grade mathematics classroom in Hong Kong has been examined. The cognitive domains of the tasks were restricted to the knowing and applying domains and thus may limit the student's opportunities in reasoning development in these two sub-topics. By comparing the student's class work with the corresponding teacher's examples, most of students imitated teacher's examples completely or partly. Students believed that the teacher's examples were the learning model and they could understand the methods and the concepts through the imitation. The lessons were in directive teaching approach with strong teacher's guidance. Students consequently seemed to be able to master the mathematical content efficiently in a relative short time. However, when we looked into the students' class work in their notebooks in detail (e.g., the work of Janet, Gary and Janice), their work showed little evidence of efforts going beyond imitation. In the case of Joanne, even though she chose to use a method other than the teacher's suggestion, there was little sign of exploring the alternative methods at a higher cognitive level. The observation was alarming: the directive teaching approach with strong teacher's guidance may help the students get the answers for a short time, but there may be a high trade off for limiting the opportunities for independent thinking and the exploration.

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