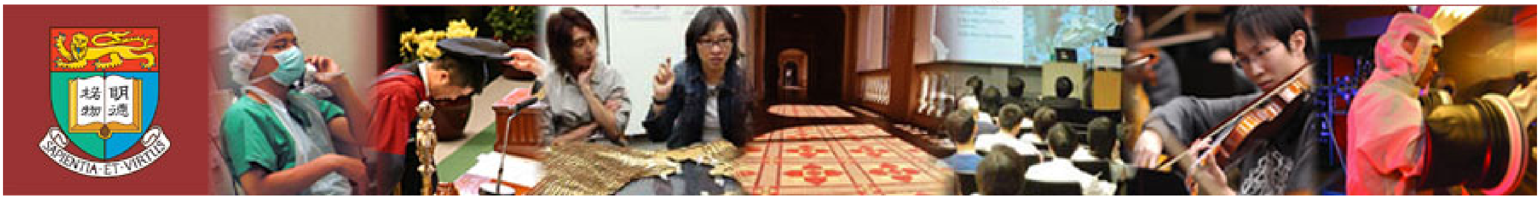

Title	Adaptive flocking of multi-agent systems with locally Lipschitz nonlinearity
Author(s)	Zhang, N; Su, H; Chen, MZ; Wang, XF
Citation	The 31st Chinese Control Conference (CCC 2012), Hefei, China, 25-27 July 2012. In Chinese Control Conference, 2012, p. 6351-6356
Issued Date	2012
URL	http://hdl.handle.net/10722/160293
Rights	Chinese Control Conference. Copyright © Institute of Electrical and Electronics Engineers.

Proceedings of the 31st Chinese Control Conference
July 25-27, 2012, Hefei, China

Adaptive Flocking of Multi-agent Systems with Locally Lipschitz Nonlinearity

Ningzi Zhang[1], Housheng Su[1], Michael Z. Q. Chen[2], Xiaofan Wang[3]

1. Department of Control Science and Engineering, Image Processing and Intelligent Control Key Laboratory of Education Ministry of China, Huazhong University of Science and Technology, Wuhan 430074, P. R. China
E-mail: zhangningzi@smail.hust.edu.cn, houshengsu@gmail.com

2. Department of Mechanical Engineering, The University of Hong Kong, Hong Kong, P. R. China
E-mail: mzqchen@hku.hk

3. Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, P. R. China
E-mail: xfwang@sjtu.edu.cn

Abstract: This paper investigates adaptive flocking of multi-agent systems (MASs) with a virtual leader. All agents and the virtual leader share the same intrinsic nonlinear dynamics, which satisfies a locally Lipschitz condition and depends on both position and velocity information of the agent itself. Under the assumption that the initial network is connected, an approach to preserving the connectivity of the network is proposed. Based on the Lyapunov stability theory, an adaptive flocking control law is derived to make the MASs track the virtual leader without collision. Finally, a numerical example is presented to illustrate the effectiveness of the theoretical results.

Key Words: Adaptive flocking, Multi-agent systems, Locally Lipschitz condition, Nonlinearity, Virtual leader

1 Introduction

Recently, a greater emphasis has been placed on the study of multi-agent systems in a range of fields, such as physics, biology, computer science and control engineering [1, 2]. Flocking phenomenon is that a group of agents move to a coordinated state with only limited information of environment, and in nature, it exists in many forms like flocking of birds, swarming of insects, and schooling of fish [3, 4]. The research on flocking problems of multi-agent systems has essential realistic significance because there are many such systems in nature and the characteristics of these systems can be utilized in cooperative control of cyber-physical systems, such as unmanned air vehicles (UAVs), multi-robots and satellites [5–14].

There are a large number of previous works concentrating on the flocking or consensus problem of systems consisting of agents governed by double-integrator dynamics. However, more and more works turn to study the flocking of multiple agents governed by nonlinear dynamics, because this situation is more realistic. In [15], a flocking algorithm for multi-agent systems with fixed topologies and nonlinear dynamics based on pseudo-leader mechanism is proposed. In a recent paper [16], a flocking problem of multiple agents governed by nonlinear dynamics with a virtual leader is investigated, and an adaptive flocking algorithm is proposed. Under the assumption that the initial network is connected, all the agents in the multi-agent system can asymptotically synchronize with the virtual leader. In some existing works, such as [15, 16], the second-order systems with nonlinear intrinsic dynamics only depending on velocity information of the agent are investigated. In [15–17], a globally Lipschitz-like condition is used to explore the flocking problem, while in the second-order consensus problem [18, 19], such condition is also used.

However, the nonlinear intrinsic dynamics of a system depending on both position and velocity information is more rational in reality. For the generality of systems, in this paper, a system consisting of agents whose dynamics depends on both position and velocity information is taken into consideration. Moreover, many nonlinear systems cannot satisfy the globally Lipschitz-like condition, and they are only locally Lipschitz, such as the Lorenz system and Chen system. In our work, a locally Lipschitz condition is presented for multi-agent systems with locally Lipschitz nonlinearity. And then, an adaptive flocking algorithm is proposed to achieve the control objective without any information of the agents dynamics.

The organization of the remainder of this paper is as follows. Section 2 describes some preliminaries and a second-order flocking problem with locally Lipschitz condition to be solved in this paper. Section 3 states the main results on the problem. Section 4 presents a numerical example on the performance of the proposed flocking algorithms. Section 5 draws conclusions to the paper.

2 Preliminaries and problem statement

Consider a multi-agent system with N agents, labeled as $1, \cdots, N$. The dynamics of each agent is characterized by

$$\begin{aligned} \dot{q}_i &= p_i \\ \dot{p}_i &= f(p_i, q_i) + u_i, \quad i = 1, \cdots, N \end{aligned} \tag{1}$$

where $q_i \in \Re^n$, $p_i \in \Re^n$ are the position vector and the velocity vector of the ith agent, respectively, $f(q_i, p_i) = (f_1(q_i, p_i), f_2(q_i, p_i), ..., f_n(q_i, p_i))^T \in \Re^n$ is the ith agents intrinsic nonlinear dynamics, and $u_i \in \Re^n$ is the control input.

A virtual leader has been taken into consideration in our

This work was supported in part by the National Natural Science Foundation of China under Grant No. 61104140, the Fundamental Research Funds for the Central Universities (HUST: Grant No. 2011JC055), the Research Fund for the Doctoral Program of Higher Education (RFDP) under Grant No. 20100142120023, the Natural Science Foundation of Hubei Province of China under Grant No. 2011CDB042.

approach, and its dynamics is characterized by

$$\begin{aligned}\dot{q}_r &= p_r,\\ \dot{p}_r &= f(q_r, p_r),\end{aligned} \tag{2}$$

where $q_r, p_r \in \Re^n$ are respectively the leader's position and velocity vector. An assumption has been made in this paper that the virtual leader and the agents have the same intrinsic dynamics.

Remark 1 It should be noted that we choose the dynamics in the form of $f(q_i, p_i)$ instead of $f(p_i)$ selected in some existing papers, and obviously, it is more general and rational.

In this flocking problem of multi-agent systems, the aims of designing the control law are stated as follows:

i) $\lim_{t\to\infty} \|p_i(t) - p_r(t)\| = 0$ for any $i = 1, 2, ..., N$,
ii) The distance between any two agents is stabilized asymptotically,
iii) There is no collision among the systems.

Assumption 1 The nonlinear function f in system (1) is locally Lipschitz, that is, for any compact set $S \in \Re^{Nn}$, there exist a positive constant matrix $B(S, N(0))$ whose elements depend on the space and the initial network $N(0)$, and a constant $\delta(S) > 0$ whose value depend on the space S, such that

$$\tilde{y}^T \tilde{f}(x, y) \le \tilde{y}^T (B(S, N(0)) \otimes I_n)\tilde{x} + \delta(S)\|\tilde{y}\|^2,$$
$$\forall \tilde{x} = x - x_r, \tilde{y} = y - y_r \in S, \tag{3}$$

where $\tilde{f}(x, y) = f(x, y) - f(x_r, y_r)$ and $B(S, N(0))$ is a $N \times N$ matrix whose element b_{ij} satisfies that when $i \neq j$, $b_{ij} < 0$ if there is an edge between agent i and agent j in the initial network, while $b_{ij} = 0$ otherwise. $b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}$.

Remark 2 The local Lipschitz condition is weaker than that global Lipschitz condition. However, for some specific systems, the global Lipschitz condition may not hold. It is clear that if the Jacobian matrix of $f(x, y)$ is continuous for any $x \in \Re^{Nn}$, $y \in \Re^{Nn}$, $f(x, y)$ is at least locally Lipschitz. Accordingly, the local Lipschitz condition is more reasonable.

Assumption 2 The virtual leader's signal (q_r, p_r) is bounded, that is, there exists a compact set $S = S(q_r(0), p_r(0)) \subset \Re^n \times \Re^n$, such that the trajectory of system (2) starting from $(q_r(0), p_r(0))$ is always in the set S.

In this paper, it is supposed that only a small fraction of the agents can receive information from the virtual leader, owing to which, certain network connectivity-preserving rules are required to guarantee the convergence of the coordinated motion. Assume that the connectivity of the network can be always maintained and all agents have the same sensing radius $R > 0$. Let $\varepsilon \in (0, R]$ be a given constant. Let $G(t) = (V, E(t))$ denote an undirected dynamic graph consisting of a set of vertices $V = \{1, 2, ..., N\}$ whose elements represent agents in the group, and a time-varying set of edges $E(t) = \{(i, j) : i, j \in V\}$, containing unordered pairs of vertices that represent neighboring relations among agents. The connectivity-preserving rules of the network is described as follows [20]:

i) Initial edges are generated by

$$E(0) = \{(i, j) : \|q_i(0) - q_j(0)\| < r, \quad i, j \in V\};$$

ii) $\varphi(i, j)(t)$ represents whether there is an edge between agent i and j, and
$\varphi(i, j)(t) =$

$$\begin{cases} 0, \begin{cases} if((\varphi(i,j)(t^-) = 0 \cap \|q_i(t) - q_j(t)\|) \ge R - \varepsilon) \\ \cup \ ((\varphi(i,j)(t^-) = 1 \cap \|q_i(t) - q_j(t)\|) \ge R), \end{cases} \\ 1, \begin{cases} if((\varphi(i,j)(t^-) = 1 \cap \|q_i(t) - q_j(t)\|) < R) \\ \cup \ ((\varphi(i,j)(t^-) = 0 \cap \|q_i(t) - q_j(t)\|) < R - \varepsilon); \end{cases} \end{cases}$$

iii) $r < R$.

In this paper, we choose $\varepsilon = R$. The definition of adjacency matrix of system (1) on graph $G(t)$ is

$$A(t) = (a_{ij}(t)),$$

where $a_{ij}(t) = \begin{cases} 1, & if \quad (i, j) \in E(t), \\ 0, & \text{otherwise}. \end{cases}$

Then the corresponding Laplacian matrix is $L(t) = D(A(t)) - A(t)$, where $D(A(t)) = \text{diag}(d_1(t), d_2(t), ..., d_N(t))$ is the degree matrix with $d_i(t) = \sum_{j=1, j\neq i}^{N} a_{ij}(t)$. The eigenvalues of $L(t)$ are depicted by $\lambda_1(L(t)), \lambda_2(L(t)), \cdots,$ $\lambda_N(L(t))$, satisfying $\lambda_1(L(t)) \le \lambda_2(L(t)) \le \cdots \le \lambda_N(L(t))$. $H = \text{diag}(h_1, h_2, ..., h_N)$ is the matrix which describes the connectivity of every agent and the virtual leader, where $h_i > 0$ if the ith agent can get the information of virtual leader and $h_i = 0$ otherwise.

Lemma 1[21] If L is the symmetric Laplacian matrix of a connected undirected graph G, and the matrix $E = diag(e_1, e_2, ..., e_n)$ with $e_i \ge 0$ for $i = 1, 2, ..., n$, and at least one element in E is positive, then $L + E > 0$.

Lemma 2[16] If G_1 is a connected undirected graph and G_2 is a graph generated by adding some edge(s) into the graph G_1, then $\lambda_1(L_2 + E) \ge \lambda_1(L_1 + E) > 0$, where $E = diag(e, 0, \cdots, 0)$ with $e > 0$, and L_1 and L_2 are the symmetric Laplacians of graphs G_1 and G_2, respectively.

3 Main results

In this paper, the potential function $\Psi(\|q_i - q_j\|)$ is proposed as a nonnegative function of the distance between agent i and agent j represented by $\|q_i - q_j\|$, which is differentiable with respect to $\|q_i - q_j\| \in (0, R)$, such that

i) $\Psi(\|q_i - q_j\|) \to +\infty$ as $\|q_i - q_j\| \to 0$,
ii) $\Psi(\|q_i - q_j\|)$ achieves its unique minimum when $\|q_i - q_j\|$ comes to a desired distance,
iii) $\Psi(\|q_i - q_j\|) \to +\infty$ as $\|q_i - q_j\| \to R$.

One example of such a potential function is the following

$$\Psi(\|q_i - q_j\|) = \begin{cases} +\infty, & \|q_i - q_j\| = 0, \\ \frac{R}{\|q_i - q_j\|(R - \|q_i - q_j\|)}, & \|q_i - q_j\| \in (0, R) \\ +\infty, & \|q_i - q_j\| = R. \end{cases} ,$$

Let

$$\begin{aligned} d_0 &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(0)}^{N} \Psi(\|q_i(0) - q_j(0)\|), \\ \tilde{d}_0 &= \frac{1}{2}\sum_{i=1}^{N} \|p_r(0) - p_i(0)\|^2, \end{aligned}$$

and a space is formed:

$$S(d_0, \tilde{d}_0, q_r, p_r)$$

$$= \left\{ q,p \in \Re^{nN} | \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(t)} \Psi\left(\|q_i - q_j\|\right) + \frac{1}{2}\sum_{i=1}^{N}\|q_r - q_i\|^2 \le \sigma_1 d_0 + \sigma_2 \tilde{d}_0 \right\},$$

where $q = (q_1^T, q_2^T, ..., q_N^T)^T$, $p = (p_1^T, p_2^T, ..., p_N^T)^T$, and σ_1, σ_2 are scalars to be chosen later. Then, the local Lipschitz condition is that there exists constant matrix $B(S, G(0))$ whose elements depend on both the space S and the initial network $G(0)$ and constant $\delta(S) > 0$ depending on the space S, such that, for any $\tilde{q} = q_i - q_r \in S(d_0, \tilde{d}_0, q_r, p_r)$, and $\tilde{p} = p_i - p_r \in S(d_0, \tilde{d}_0, q_r, p_r)$, $\tilde{p}^T(\tilde{f}(q,p)) \le \tilde{p}^T(B(S, G(0)) \otimes I_n)\tilde{q} + \delta(S)\|\tilde{p}\|^2$, where $\tilde{f}(q,p) = f(q,p) - f(q_r, p_r)$, $B(S, G(0))$ is a $N \times N$ matrix whose element b_{ij} satisfies that when $i \neq j$, $b_{ij}\begin{cases} < 0, & if \quad (i,j) \in E(0), \\ = 0, & otherwise, \end{cases}$ while $b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}$.

The control law is specified as:

$$\begin{aligned} u_i &= -\sum_{j\in N_i(t)} \nabla_{q_i}\Psi(\|q_i - q_j\|) - \sum_{j\in N_i(t)} m_{ij}(q_i - q_j) \\ &\quad - \sum_{j\in N_i(t)} \tilde{m}_{ij}(p_i - p_j) - h_i c_i (q_i - q_r) \\ &\quad - h_i \tilde{c}_i (p_i - p_r), \\ \dot{m}_{ij} &= k_{ij}(p_i - p_j)^T (q_i - q_j), \\ \dot{\tilde{m}}_{ij} &= \tilde{k}_{ij}(p_i - p_j)^T (p_i - p_j), \\ \dot{c}_i &= w_i (p_i - p_r)^T (q_i - q_r), \\ \dot{\tilde{c}}_i &= \tilde{w}_i (p_i - p_r)^T (p_i - p_r), \end{aligned} \tag{4}$$

where $N_i(t)$ is the set of agents that can connect with agent i at time t, $m_{ij} > 0$, $\tilde{m}_{ij} > 0$ respectively represent the position and velocity coupling strengths satisfying $m_{ij} = m_{ji}$, $\tilde{m}_{ij} = \tilde{m}_{ji}$, and c_i, $\tilde{c}_i$ are the position and velocity navigation feedback weights respectively. Suppose that there is only a small fraction of agents can get the information of the virtual leader, and $h_i = 1$ if the ith agent is informed, while $h_i = 0$ otherwise.

An energy function $V(t)$ is proposed to demonstrate the flocking results:

$$\begin{aligned} &V(q,p,q_r,p_r,m,\tilde{m},c,\tilde{c}) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(t)} \Psi\left(\|q_i - q_j\|\right) + \frac{1}{2}\sum_{i=1}^{N}(p_i - p_r)^T(p_i - p_r) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(t)} \frac{(m_{ij} - \theta_{ij})^2}{2k_{ij}} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(t)} \frac{(\tilde{m}_{ij} - \tilde{\theta})^2}{2\tilde{k}_{ij}} \\ &+ \frac{1}{2}\sum_{i=1}^{N} h_i \frac{\left(c_i - \sum_{j\in N_i(t)}\theta_{ij}\right)^2}{k_i} + \frac{1}{2}\sum_{i=1}^{N} h_i \frac{(\tilde{c}_i - \tilde{\theta})^2}{\tilde{k}_i}, \end{aligned} \tag{5}$$

where, θ_{ij} is the element of $N \times N$ matrix Θ,which satisfies that when $i \neq j$, $\theta_{ij} = \theta_{ji}\begin{cases} > 0, & if \quad (i,j) \in E(0), \\ = 0, & otherwise, \end{cases}$ and $\theta_{ii} = 0$, $L(\Theta) = D(\Theta) - \Theta$, where $D(\Theta) = diag(D_1, D_2,$ $..., D_N)$ with $D_i = \sum_{j=1, j\neq i}^{N} \theta_{ij}$, and $L(\Theta) + HD(\Theta) = B(S, G(0))$.

Theorem 1

Consider a system of N agents with dynamics (1) and a virtual leader with dynamics (2), and the controller (4) is applied. Suppose that Assumption 1 and 2 hold, and the initial network $G(0)$ is connected.

i) $G(t)$ will not change for all $t \ge 0$;
ii) For all $t \in [0, +\infty)$, the combined trajectories of the agents and the control parameters $(q, p, m, \tilde{m}, c, \tilde{c})$ in (1), (2) and (4) belong to a compact hyper-ellipsoid

$$\begin{aligned} &\Omega(\sigma_1 d_0, \sigma_2 \tilde{d}_0, \theta, \tilde{\theta}, p_r, q_r) = \\ &\{(q,p,m,\tilde{m},c,\tilde{c}) \in \Re^{Nn} \times \Re^{Nn} \times \Re^{NN} \times \Re^{NN} \\ &\times \Re^N \times \Re^N | V < \sigma_1 d_0 + \sigma_2 \tilde{d}_0\}, \end{aligned} \tag{6}$$

where σ_1 and σ_2 are scalars satisfying $\sigma_1 > 1, \sigma_2 > 1$, if the initial value is selected from

$$\begin{aligned} &\Omega_0(\sigma_1 d_0, \sigma_2 \tilde{d}_0, \theta, \tilde{\theta}, p_r, q_r) = \\ &\{(q,p,m,\tilde{m},c,\tilde{c}) \in \Re^{Nn} \times \Re^{Nn} \times \Re^{NN} \times \Re^{NN} \\ &\times \Re^N \times \Re^N | V(0) < \sigma_1 d_0 + \sigma_2 \tilde{d}_0\}. \end{aligned} \tag{7}$$

Proof

Let $q_{ir} = q_i - q_r$, $p_{ir} = p_i - p_r$, $q_{ij} = q_i - q_j = q_{ir} - q_{jr}$, $p_{ij} = p_i - p_j = p_{ir} - p_{jr}$. Then (4) can be rewritten as

$$\begin{aligned} u_i &= -\sum_{j\in N_i(t)} \nabla_{q_{ir}}\Psi(\|q_{ij}\|) - \sum_{j\in N_i(t)} m_{ij}(q_{ir} - q_{jr}) \\ &\quad - \sum_{j\in N_i(t)} \tilde{m}_{ij}(p_{ir} - p_{jr}) - h_i c_i q_{ir} - h_i \tilde{c}_i p_{ir}, \\ \dot{m}_{ij} &= k_{ij}(p_{ir} - p_{jr})^T (q_{ir} - q_{jr}), \\ \dot{\tilde{m}}_{ij} &= \tilde{k}_{ij}(p_{ir} - p_{jr})^T (p_{ir} - p_{jr}), \\ \dot{c}_i &= w_i {p_{ir}}^T q_{ir}, \\ \dot{\tilde{c}}_i &= \tilde{w}_i {p_{ir}}^T p_{ir}. \end{aligned} \tag{8}$$

Then, the energy function (5) can be rewritten as

$$\begin{aligned} &V(q,p,q_r,p_r,m,\tilde{m},c,\tilde{c}) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(t)} \Psi\left(\|q_{ij}\|\right) + \frac{1}{2}\sum_{i=1}^{N} p_{ir}^T p_{ir} \\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(t)} \frac{(m_{ij} - \theta_{ij})^2}{2k_{ij}} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i(t)} \frac{(\tilde{m}_{ij} - \tilde{\theta})^2}{2\tilde{k}_{ij}} \\ &+ \frac{1}{2}\sum_{i=1}^{N} h_i \frac{\left(c_i - \sum_{j\in N_i(t)}\theta_{ij}\right)^2}{w_i} + \frac{1}{2}\sum_{i=1}^{N} h_i \frac{(\tilde{c}_i - \tilde{\theta})^2}{\tilde{w}_i}. \end{aligned} \tag{9}$$

Suppose that the graph $G(t)$ switches at time $t_k (k = 1, 2, \ldots)$ which means that $G(t)$ is a fixed graph in each time-interval $[t_{k-1}, t_k)$. Note that $V(0)$ is finite and time derivative of $V(t)$ in $[t_0, t_1)$ is shown as follows

$$
\begin{aligned}
\dot{V} = & \sum_{i=1}^{N} p_{ir}^T \sum_{j \in N(t)} \nabla_{q_{ir}} \Psi(\|q_{ij}\|) \\
& + \sum_{i=1}^{N} p_{ir}^T (f(q_i, p_j) - f(q_r, p_r)) \\
& - \sum_{i=1}^{N} p_{ir}^T \sum_{j \in N_i(t)} \nabla_{q_{ir}} \Psi(\|q_{ij}\|) \\
& - \sum_{i=1}^{N} p_{ir}^T \sum_{j \in N_i(t)} m_{ij}(q_{ir} - q_{jr}) \\
& - \sum_{i=1}^{N} p_{ir}^T \sum_{j \in N_i(t)} \tilde{m}_{ij}(p_{ir} - p_{jr}) \\
& - \sum_{i=1}^{N} p_{ir}^T h_i c_i q_{ir} - \sum_{i=1}^{N} p_{ir}^T h_i \tilde{c}_i p_{ir} \\
& + \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i(t)} (m_{ij} - \theta_{ij})(p_{ir} - p_{jr})^T (q_{ir} - q_{jr}) \\
& + \frac{1}{2} \sum_{i=1}^{N} \sum_{j \in N_i(t)} (\tilde{m}_{ij} - \tilde{\theta})(p_{ir} - p_{jr})^T (p_{ir} - p_{jr}) \\
& + \sum_{i=1}^{N} h_i (c_i - \sum_{j \in N_i(t)} \theta_{ij}) p_{ir}^T q_{ir} + \sum_{i=1}^{N} h_i (\tilde{c}_i - \tilde{\theta}) p_{ir}^T p_{ir} \\
= & \sum_{i=1}^{N} p_{ir}^T (f(q_i, p_j) - f(q_r, p_r)) \\
& - \sum_{i=1}^{N} p_{ir}^T \sum_{j \in N_i(t)} \theta_{ij}(q_{ir} - q_{jr}) \\
& - \sum_{i=1}^{N} p_{ir}^T \sum_{j \in N_i(t)} \tilde{\theta}(p_{ir} - p_{jr}) \\
& - \sum_{i=1}^{N} h_i \sum_{j \in N_i(t)} \theta_{ij} p_{ir}^T q_{ir} - \sum_{i=1}^{N} h_i \tilde{\theta} p_{ir}^T p_{ir} \\
\leq & -\tilde{p}^T ((L(\Theta) + HD(\Theta) - B(S, G(0))) \otimes I_n) \tilde{q} \\
& -\tilde{p}^T ((\tilde{\theta}(L(t) + H) - \delta(S) I_N) \otimes I_n) \tilde{p} \\
= & -\tilde{p}^T ((\tilde{\theta}(L(t) + H) - \delta(S) I_N) \otimes I_n) \tilde{p}.
\end{aligned}
$$

By Lemma 1, we can see that $L(0) + H > 0$, and with the condition $\tilde{\theta} \geq \frac{\delta(S)}{\lambda_{\min}(L(0)+H)}$, one has $\dot{V}(t) \leq 0$, therefore, the energy function $V(t)$ is non-increasing on $[t_0, t_1)$, which implies that $V(t) \leq V(0) < \infty$ for $t \in [t_0, t_1)$.

From the definition of the potential function, we have $\lim_{\|q_i - q_j\| \to R} \Psi(\|q_i - q_j\|) = +\infty$. Therefore, no distance of existing edges will tend to R for $t \in [t_0, t_1)$, which also implies that no existing edges will be lost before time t_1. According to the network connectivity-preserving rules and the definition of potential function, since we choose $\varepsilon = R$, if $\varphi(i,j)(t^-) = 0$, new edge is impossible to add between agent i and j, so no new edge will add to the network, and then the associated potentials remain finite. Thus, $V(t_1)$ is finite.

Similar to the above analysis, the time derivative of $V(t)$ in every $[t_{k-1}, t_k)$ is

$$V(t) \leq -\tilde{p}^T ((\tilde{\theta}(L(t) + H) - \delta(S) I_N) \otimes I_n) \tilde{p}.$$

By Lemma 2, it follows from $\tilde{\theta} \geq \frac{\delta(S)}{\lambda_{\min}(L(0)+H)}$ that

$$\tilde{\theta} \geq \frac{\delta(S)}{\lambda_{\min}(L(0) + H)} \geq \frac{\delta(S)}{\lambda_{\min}(L(t_{k-1}) + H)}.$$

Then, it can be obtained that

$$V(t) \leq V_{k-1} < \infty \; for \; t \in [t_{k-1}, t_k),$$
$$k = 1, 2, \ldots$$

Thus, no distance of existing edges will tend to R for $t \in [t_{k-1}, t_k)$ and no edges will be lost before t_k and $V(t_k)$ is finite.

Since $G(0)$ is connected, while no edge in $E(0)$ will lost and no new edge can add to $E(0)$, $G(t)$ will not change for all $t \geq 0$, then $G(t) = G(0)$, $L(t) = L(0)$.

Since $V(t) \leq V(0) < \infty$, we can draw the conclusion that if the initial value is selected form $\Omega_0(\sigma_1 d_0, \sigma_2 \tilde{d}_0, \theta, \tilde{\theta}, p_r, q_r)$, the combined trajectories of the agents and the control parameters $(q, p, m, \tilde{m}, c, \tilde{c})$ in (1), (2) and (4) belong to a compact hyper-ellipsoid $\Omega(\sigma_1 d_0, \sigma_2 \tilde{d}_0, \theta, \tilde{\theta}, p_r, q_r)$.

To ensure the set $\Omega_0(\sigma_1 d_0, \sigma_2 \tilde{d}_0, \theta, \tilde{\theta}, p_r, q_r)$ is well defined, we choose $\sigma_1 > 1$ and $\sigma_2 > 1$.

Theorem 2 Consider a system of N agents with dynamics (1) and a virtual leader with dynamics (2). Suppose that the initial network $G(0)$ is connected, Assumption 1 and 2 hold, and the initial value defined in (7). Then, when the controller (4) is applied, the following statements hold:

i) All agents asymptotically move with the same velocity;
ii) Almost every final configuration locally minimizes each agent's global potential;
iii) Collisions among agents are avoided.

Proof

All the lengths of edges are not longer than $\Psi^{-1}(V(0))$. Therefore, the set

$$\Omega' = \{(q, p, m, \tilde{m}, c, \tilde{c}) \in \Omega(\sigma_1 d_0, \sigma_2 \tilde{d}_0, \theta, \tilde{\theta}, p_r, q_r) | V \leq V(0)\}$$

is positively invariant.

Since $G(t)$ is connected and unchanged for all $t \geq 0$, it is clear that $\|q_{ir} - q_{jr}\| < (N-1)R$ for all i and j. Because $V(t) \leq V(0)$, one has $p_{ir}^T p_{ir} \leq 2V(0)$, $\|p_{ir}\| \leq \sqrt{2V(0)}$. Therefore, the set Ω is closed and bounded, hence compact. According to the fact that system (1) with control input (4) is an autonomous system for all $t \geq 0$, the LaSalle Invariance Principle [22] can be applied, so the corresponding trajectories will converge to the largest invariant set inside the region:

$$\Omega_f = \{(q, p, m, \tilde{m}, c, \tilde{c}) \in \Omega(\sigma_1 d_0, \sigma_2 \tilde{d}_0, \theta, \tilde{\theta}, p_r, q_r) | \dot{V} = 0\}$$

From Lemma 1 and the fact that $\dot{V}(t) \leq 0$, $\dot{V} = 0$ if and only if $\tilde{p} = 0_{nN}$, that is, $p_1 = p_2 = \cdots = p_N = p_r$, so $\lim_{t \to \infty} \|p_i(t) - p_r(t)\| = 0$, all the agents asymptotically move with the same velocity.

Hence, one has

$$\begin{aligned}
\dot{p}_{ir} &= -\sum_{j\in N_i(t)} \nabla_{q_{ir}}\Psi(\|q_{ij}\|) - \sum_{j\in N_i(t)} m_{ij}(q_{ir}-q_{jr}) \\
&\quad -h_i c_i q_{ir} \\
&= -\sum_{j\in N_i(t)} \frac{\partial\Psi(\|q_{ij}\|)}{\partial\|q_{ij}\|}\frac{1}{\|q_{ij}\|}(q_i-q_j) \\
&\quad -\sum_{j\in N_i(t)} m_{ij}(q_{ir}-q_{jr}) - h_i c_i q_{ir} \\
&= 0
\end{aligned}$$

Generally, unless the initial configuration of the agents is close enough to the global minimum, almost every final configuration locally minimizes each agent's global potential. From the definition of potential function, it is obvious that $\lim_{\|q_{ij}(t)\|\to 0} \Psi(\|q_{ij}\|) = +\infty$. Although $V(t) \le V(0)$ for all $t \ge 0$, collisions among agents are avoided.

4 Numerical example

In this section, a numerical example is given to illustrate the effectiveness of our theoretical results. We take a group of 10 agents which move in a 3-dimensional space in this example. Consider the intrinsic nonlinear dynamic term $f(q,p)$ given as

$$f(q,p) = \begin{pmatrix} \xi_1(p_y - p_x) - q_x \\ \xi_2 p_x - p_x p_z - p_y - q_y \\ p_x p_y - \xi_3 p_z - q_z \end{pmatrix},$$

where the parameter q, p are respectively defined as $q = [q_x, q_y, q_z]^T$, $p = [p_x, p_y, p_z]^T$. In particular, the parameters are chosen as $\xi_1 = 10$, $\xi_2 = 28$ and $\xi_3 = \frac{8}{3}$.

The initial positions and initial velocities of the 10 agents are generated from cubes $[0,5]\times[0,5]\times[0,5]$ and $[0,3]\times[0,3]\times[0,3]$, respectively. The initial position of the virtual leader is chosen as $q_r(0) = [6,7,8]^T$, while its initial velocity is $p_r(0) = [1,2,3]^T$. The initial edges are generated by $E(0) = \{(i,j) : \|q_i(0) - q_j(0)\| < r, \quad i,j \in V\}$, where r is chosen as $r = 5$. Let $\varepsilon = R = 10$. The potential function is chosen as

$$\Psi(\|q_{ij}\|) = \begin{cases} +\infty, & \|q_{ij}\| = 0, \\ \frac{R}{\|q_{ij}\|(R-\|q_{ij}\|)}, & \|q_{ij}\| \in (0,R) \\ +\infty, & \|q_{ij}\| = R. \end{cases},$$

The initial value of position coupling strengths is chosen as $m_{ij}(0) = 0$, with $k_{ij} = 0.1$ for all i and j, while the initial value of velocity coupling strengths is selected as $\tilde{m}_{ij}(0) = 0$, with $\tilde{k}_{ij} = 0.1$ for all i and j. The initial weights on the position and velocity navigational feedbacks are chosen as $c_i(0) = 0$, $\tilde{c}_i(0) = 0$, respectively, with $w_i = 0.1$, $\tilde{w}_i = 0.1$ for all i. Suppose that there is only one agent can get the information of the virtual leader, and without loss of generality, we assume that the first agent is informed, that is, $h_1 = 1$, and $h_i = 0$ for $i = 2, 3, ..., 10$. The initial network is connected. Fig. 1 shows the initial states of the 10 agents and the virtual leader. Fig. 2 depicts the final state of the network after 50 seconds with the control law (4). Fig. 3 describes the velocity differences between every agent and the virtual leader on the x-axis, y-axis, z-axis. From Fig. 3, it is illustrated that each agent finally moves with the same velocity as the virtual leader. Fig. 4 and Fig. 5 show the weights of navigational feedbacks and coupling strengths of position and velocity, respectively, and all converge to constants.

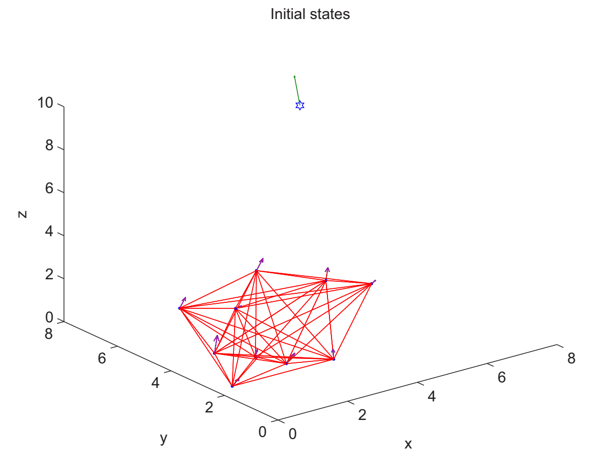

Fig. 1: Initial states.

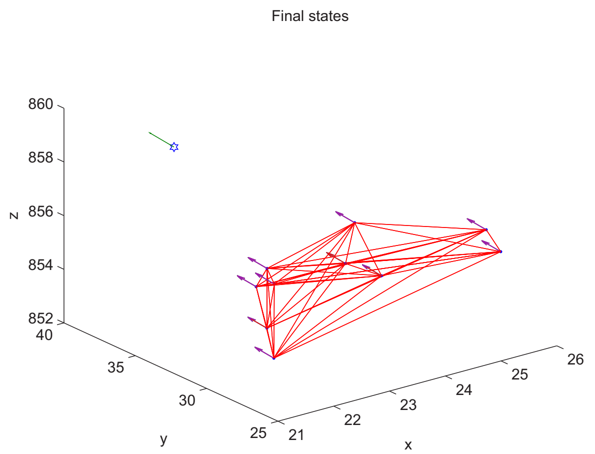

Fig. 2: Final states.

5 Conclusions

In this paper, the adaptive flocking problem of multi-agent systems with local Lipschitz nonlinearity has been investigated. Different from most existing works, we consider the situation where all the agents and the virtual leader share the same intrinsic nonlinear dynamics, which depends on both position and velocity information and is assumed to be only locally Lipschitz. A connectivity-preserving adaptive flocking control law has been proposed to synchronize each agent's velocity with the virtual leader's velocity, without requirement on any information of the agents dynamics.

References

[1] J. Ferber, *Multi-agent Systems: An Introduction to Distributed Artificial Intelligence*. New York: Addison-Wesley, 1999.

[2] R. Olfati-Saber, J. A. Fax, and R. M. Murray, Consensus and cooperation in networked multi-agent systems, *Proceedings of the IEEE*, 95(1): 215–233, 2007.

[3] C. W. Reynolds, Flocks, Herds, and Schools, A Distributed

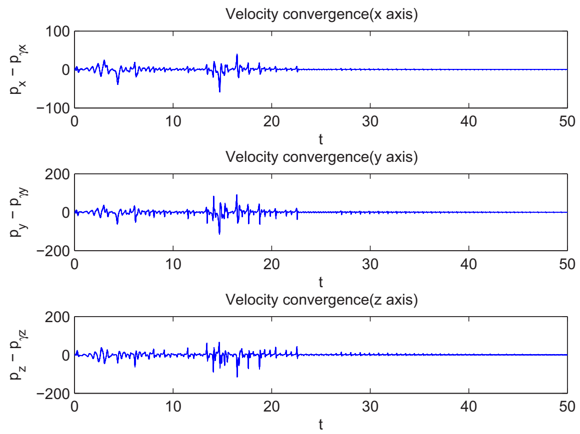

Fig. 3: Velocity convergence.

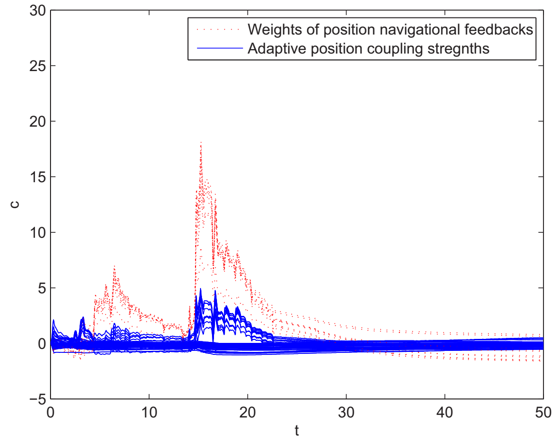

Fig. 4: Weights of position navigational feedbacks and adaptive position coupling strengths.

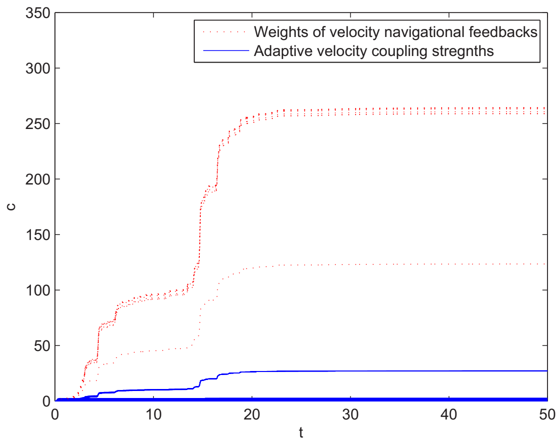

Fig. 5: Weights of velocity navigational feedbacks and adaptive velocity coupling strengths.

Behavioral Model, *Computer Graphics, ACM SIGGRAPH 87 Conference Proceedings*, 21(4): 25–34, 1987.

[4] T. Vicsek, A. Czirook, E. Ben-Jacob, O. Cohen and I. Shochet, Novel type of phase transition in a system of self-deriven particles, *Physical Review Letters*, 75(6): 1226–1229, 1995.

[5] A. Jadbabaie, J. Lin, and A. S. Morse, Coordination of groups of mobile agents using nearest neighbor rules, *IEEE Transactions on Automatic Control*, 48(6):988–1001, 2003.

[6] R. Olfati-Saber, Flocking for multi-agent dynamic systems: algorithms and theory, *IEEE Trans. on Automatic Control*, 51(3): 401–420, 2006.

[7] H. Su, X. Wang, and Z. Lin, Synchronization of coupled harmonic oscillators in a dynamic proximity network, *Automatica*, 45(10): 2286–2291, 2009.

[8] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, Flocking in fixed and switching networks, *IEEE Transactions on Automatic Control*, 52(5): 563–868, 2007.

[9] H. G. Tanner, A. Jadbabaie and G. J. Pappas, Stable flocking of mobile agents, part I: fixed topology, in *Proc. of the 42nd IEEE Conference on Decision and Control*, 2003: 2010–2015.

[10] H. G. Tanner, A. Jadbabaie and G. J. Pappas, Stable flocking of mobile agents, part II: dynamic topology, in *Proc. of the 42nd IEEE Conference on Decision and Control*, 2003: 2016–2021.

[11] H. Su, X. Wang and Z. Lin, Flocking of multi-agents with a virtual leader, *IEEE Trans. on Automatic Control*, 54(2): 293–307, 2009.

[12] H. Su, X. Wang and W. Yang, Flocking in multi-agent systems with multiple virtual leaders, *Asian Journal of Control*, 10(2), 238–245, 2008.

[13] H. Su, X. Wang and G. Chen, A connectivity-preserving flocking algorithm for multi-agent systems based only on position measurements, *International Journal of Control*, 82(7): 1334–1343, 2009.

[14] L. Wang, H. Dai, H. Dong, Y. Cao, and Y. Sun, Adaptive synchronization of weighted complex dynamical networks through pinning, *The European Physical Journal B*, 61(3): 335–342, 2008.

[15] J. Zhou, W. Yu, X. Wu, M. Small and J. Lu, Flocking of multi-agent dynamical systems based on pseudo-leader mechanism, *Systems and Control Letters*, 61(1): 195–202, 2012.

[16] H. Su, G. Chen, X. Wang, and Z. Lin, Adaptive Flocking with a Virtual Leader of Multiple Agents Governed by Nonlinear Dynamics, in *Proceedings of the 29th Chinese Control*, Conference, 2010: 5827–5832.

[17] W. Yu, G. Chen, and M. Cao, Distributed Leader-follower flocking control for multi-agent dynamical systems with time-varying Velocities, *Systems and Control Letters*, 59(9): 543–552, 2010.

[18] W. Yu, G. Chen, M. Cao, and J. Kurths, Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics, *IEEE Transactions on Systems, Man and Cybernetics-Part B*, 40(3): 881–891, 2010.

[19] H. Su, G. Chen, X. Wang, and Z. Lin, Adaptive second-order consensus of networked mobile agents with nonlinear dynamics, *Automatica*, 47(2): 368–375, 2011.

[20] H. Su, X. Wang, and G. Chen, Rendezvous of multiple mobile agents with preserved network connectivity, *Systems and Control Letters*, 59(5): 313–322, 2010.

[21] Y. Hong, J. Hu, and L. Gao, Tracking control for multi-agent consensus with an active leader and variable topology, *Automatica*, 42(7): 1177–1182, 2006.

[22] H. K. Khalil, *Nonlinear Systems, Third Edition*. Upper Saddle, New Jersey: Prentice Hall, 2002.