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Author(s)	Su, H; Chen, MZ; Wang, XF; Valeyev, NV
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Adaptive Group Consensus of Coupled Harmonic Oscillators with Multiple Leaders

Housheng Su[†]

Michael Z. Q. Chen*

Xiaofan Wang[§]

Najl V. Valeyev[‡]

Abstract— In this paper, we investigate the group consensus of coupled harmonic oscillators with multiple leaders in an undirected fixed network. Unlike many existing algorithms for group consensus of multi-agent systems or cluster synchronization of complex dynamical networks, which require global information of the underlying network such as the eigenvalues of the coupling matrix or centralized control protocols, we propose a novel decentralized adaptive group consensus algorithm for coupled harmonic oscillators. By using the decentralized adaptive group consensus algorithm and without using any global information of the underlying network, all agents in the same group asymptotically synchronize with the corresponding leader even when only one agent in each group has access to the information of the corresponding leader. Numerical simulation results are presented to illustrate the theoretical results.

Keywords: Distributed control, multi-agent systems, group consensus, adaptive control, leader.

I. INTRODUCTION

Recently, there has been increasing interest in the study of multi-agent distributed coordination from diverse fields such as biology, physics and engineering [1], [2], [3], [4], [5], [6], [7], [8], [9], [9], [11], [12]. This is partly due to the great potential applications of multi-agent distributed coordination, such as cooperative control of unmanned aircrafts and mobile robots [13], [14], [15]. Inspired by the pioneering models of Reynolds [3] and Vicsek [4], many versions of distributed coordinated control algorithms have been proposed, including consensus [16], [17], [18], [19], [20], swarming [21], [22], [23], [24], and flocking [12], [25], [26], [27], [28], [29], [30]. Consensus can be classified into complete consensus [16], [17], [18], [19], [20] and group consensus [31], [32], [33].

In particular, group consensus algorithms [31], [32], [33] for multi-agent systems or cluster synchronization algorithms [34], [35] of complex dynamical networks aim to guide all agents or nodes in the same cluster to reach the same state

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[†]Department of Control Science and Engineering, Image Processing and Intelligent Control Key Laboratory of Education Ministry of China, Huazhong University of Science and Technology, Luoyu Road 1037, Wuhan 430074, China.

*Department of Mechanical Engineering, The University of Hong Kong, Hong Kong. Email: mzqchen@hku.hk.

[§]Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, China.

[‡]College of Engineering, Mathematics and Physical Sciences, University of Exeter, Exeter, U.K.

value. Group consensus of multi-agent systems was investigated in undirected networks [31] and directed networks [32], respectively. Furthermore, group consensus was studied with communication delays on switching networks [33]. Cluster synchronization of complex networks was investigated via centralized adaptive pinning control [34] and in networks of coupled nonidentical dynamical systems [35].

Most existing algorithms [31], [32], [33], [34], [35] for group consensus of multi-agent systems or cluster synchronization of complex dynamical networks are only concerned with first-order dynamics. In reality, however, a broad class of agents cannot be described by first-order dynamic models, such as torque motors and gas jets, which are adjusted for their desired motion directly by their acceleration rather than by their speeds. Therefore, second-order type consensus algorithms have been broadly investigated. Flocking algorithms for multi-agent systems were proposed by a combination of velocity consensus term with a local artificial potential term [12], [25], [26], [27], [28], [29], [30]. In order to track a virtual leader or multiple virtual leaders, navigational feedback term was added to the basic flocking algorithm [12], [28], [29], [30]. Second-order consensus of multi-agent systems was presented to achieve both velocity and position consensus [36], [37], [38]. Synchronization problems of coupled harmonic oscillators in both fixed and switching networks were investigated in [39]. Without any explicit assumption on the network connectivity, synchronization of coupled harmonic oscillators in a dynamic proximity network was investigated in [40].

To the best of our knowledge, less attention has been paid to the group consensus of multi-agents described by second-order dynamics. In this paper, we will investigate the group consensus problems for second-order linear harmonic oscillator models in undirected fixed networks. Different from many existing investigations on coupled harmonic oscillators and group consensus, there are multiple leaders in this paper. Unlike many existing algorithms for group consensus of multi-agent systems [31], [32], [33] or cluster synchronization of complex dynamical networks [34], [35], which require global information of the underlying network such as eigenvalues of the coupling matrix or centralized control protocols, we propose a novel decentralized adaptive group consensus algorithm for coupled harmonic oscillators. By using the decentralized algorithm and without using any global information of the underlying network, all agents in the same group asymptotically synchronize with the corresponding leader even when only one agent in each group has access to the information of the corresponding leader.

The rest of the paper is organized as follows. Section II states the problems to be solved in this paper. Section III establishes the new decentralized adaptive group consensus algorithm. Section IV presents the results of simulation studies. Finally, Section V draws the conclusions.

II. PROBLEM STATEMENT

We consider N agents moving in an n-dimensional Euclidean space. The behavior of each agent is described by a harmonic oscillator in the following form

$$\dot{q}_i = p_i,$$

 $\dot{p}_i = -\omega^2 q_i + u_i, \quad i = 1, 2, \dots, N,$
(1)

where $q_i \in \mathbf{R}^n$ is the position of agent $i, p_i \in \mathbf{R}^n$ is its velocity vector, $u_i \in \mathbf{R}^n$ is its control input and ω is the frequency of the oscillator. For notational convenience, we also define

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix}, \quad p = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix}.$$

We define the neighboring graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ to be an undirected graph consisting of a set of vertices $\mathcal{V} = \{1, 2, \ldots, N\}$, whose elements represent agents in the group, and a set of edges $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} : i \sim j\}$, containing unordered pairs of vertices that represent neighboring relations. Vertices *i* and *j* are said to be adjacent if $(i, j) \in$ \mathcal{E} . Suppose there are *d* leaders and *d* nonempty subsets $\{G_1, \ldots, G_d\}$ is a partition of the index set $\{1, 2, \ldots, N\}$, where $\cup_{l=1}^d G_l = \{1, 2, \ldots, N\}$ and $G_l \neq \emptyset$. The problem of group consensus with multiple leaders is to design a control input u_i such that

$$\lim_{t \to \infty} \sum_{l=1}^{d} \sum_{i \in G_l} \|q_i(t) - q_{\gamma_l}(t)\| = 0$$

and

$$\lim_{t \to \infty} \sum_{l=1}^d \sum_{i \in G_l} \|p_i(t) - p_{\gamma_l}(t)\| = 0$$

for all *i*, where $q_{\gamma l}$ and $p_{\gamma l}$ are the position and velocity of the *l*th leader, respectively. The dynamics of the leaders satisfy

$$\dot{q}_{\gamma_l} = p_{\gamma_l}, \dot{p}_{\gamma_l} = -\omega^2 q_{\gamma_l}, \quad l = 1, 2, \dots, d.$$

$$(2)$$

III. ADAPTIVE GROUP CONSENSUS OF COUPLED HARMONIC OSCILLATORS WITH MULTIPLE LEADERS

A. Algorithm Description

The proposed adaptive strategy for agent i is designed as

$$u_{i} = -\sum_{j=1, j \neq i}^{N} c_{ij} a_{ij} (p_{i} - p_{j}) - h_{i} c_{i} (p_{i} - p_{\gamma_{\hat{i}}}),$$

$$\dot{c}_{ij} = h_{ij} a_{ij} k_{ij} (p_{i} - p_{j})^{T} (p_{i} - p_{j}),$$

$$\dot{c}_{i} = h_{i} k_{i} (p_{i} - p_{\gamma_{\hat{i}}})^{T} (p_{i} - p_{\gamma_{\hat{i}}}),$$
(3)

where $A = (a_{ij})_{N \times N}$ is the adjacent matrix, $c_{ij}(0) = c \ge 0$, $c_i(0) \ge 0$ and \hat{i} is the subscript of the subset for which $i \in G_{\hat{i}}$. The positive constants $k_{ij} = k_{ji}$ and k_i are the weights of the adaptive laws for parameters $c_{ij}(t)$ and $c_i(t)$, respectively. If agent *i* is the neighbor of the *l*th leader, then, $h_i = 1$; otherwise, $h_i = 0$. If agents *i* and *j* are in the same cluster, then $h_{ij} = 1$; otherwise, $h_{ij} = 0$. Clearly, the adaptive parameters $c_{ij}(t)$ for agent *i* only contain the state information of its neighbors.

It is assumed in [31], [32], [33], [34] that for an $N \times N$ symmetric matrix

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1d} \\ A_{21} & A_{22} & \cdots & A_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ A_{d1} & A_{d2} & \cdots & A_{dd} \end{bmatrix},$$
(4)

where each block $A_{uv}(u, v = 1, ..., d)$ is a zero-row-sum matrix, and each block A_{uu} is irreducible which satisfies $a_{ii} = -\sum_{j=1, j \neq i}^{k_u} a_{ij}$ where $a_{ij} = a_{ji} \ge 0 (i \neq j)$ and k_u is the number of nodes in the subset u.

Remark 1: From Equation (4), the undirected graph is not assumed to be connected, but each undirected subgraph should be connected. Note also that, unlike the traditional definition of coupling matrix of the network, the element a_{ij} with $i \in G_u$ and $j \in G_v$ in (4) may be negative here, which is called an inhibitory coupling [31], [32], [33], [34]. This provides a mechanism to desynchronize two nodes belonging to two different clusters.

B. Main Results and Theoretical Analysis

The following assumption and lemmas are needed for our main result.

Lemma 1: [41] If $L = (l_{ij}) \in \mathbf{R}^{N \times N}$ is a symmetric irreducible matrix with $l_{ii} = -\sum_{j=1, j \neq i}^{N} l_{ij}$, $l_{ij} = l_{ji} \ge 0 (i \neq j)$, then for any matrix $E = \text{diag}(e, 0, \dots, 0)$ with e > 0, all eigenvalues of the matrix L - E are negative.

Lemma 2: [34] For any $x \in \mathbf{R}^p$, $y \in \mathbf{R}^q$, and matrix $M = (m_{ij}) \in \mathbf{R}^{p \times q}$,

$$x^{\mathsf{T}}My \leq (1/2) \max[p,q] \cdot \max_{i \neq j} |m_{ij}| (x^{\mathsf{T}}x + y^{\mathsf{T}}y).$$

Lemma 3: Let matrix A be given as in Equation (4). Then, under the adaptive strategy (3), one has

$$B = (a_{ij}c_{ij}(t))$$

=
$$\begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1d} \\ B_{21} & B_{22} & \cdots & B_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ B_{d1} & B_{d2} & \cdots & B_{dd} \end{bmatrix} \in \mathbf{R}^{N \times N}$$

is a symmetric matrix, in which each block $B_{uv}(u, v = 1, ..., d)$ is a zero-row-sum matrix and each block B_{uu} is with $a_{ii}c_{ii}(t) = -\sum_{j=1, j\neq i}^{k_u} a_{ij}c_{ij}(t)$, $a_{ij}c_{ij}(t) = a_{ji}c_{ji}(t) \ge 0 (i \neq j)$ where k_u is the number of nodes in the subset u.

Proof: If nodes i and j are not in the same cluster, then by the adaptive strategy (3), one has

$$\dot{c}_{ij}(t) = 0.$$

If nodes i and j are in the same cluster, then by the adaptive strategy (3), one has

$$\dot{c}_{ij}(t) = \dot{c}_{ji}(t).$$

Therefore, *B* is a symmetric irreducible matrix. Similar to (4), each block $B_{uv}(u, v = 1, ..., d)$ is a zero-row-sum matrix and each block B_{uu} is with $a_{ii}c_{ii}(t) = -\sum_{j=1, j \neq i}^{k_u} a_{ij}c_{ij}(t), a_{ij}c_{ij}(t) = a_{ji}c_{ji}(t) \ge 0 (i \neq j)$, where k_u is the number of nodes in the subset u.

Lemma 4: Let matrix A be given as in Equation (4). Then

$$\sum_{j=1, j \neq i}^{N} a_{ij} c_{ij} (p_i - p_j) = \sum_{j=1, j \neq i}^{N} a_{ij} c_{ij} (\tilde{p}_i - \tilde{p}_j),$$

where $\tilde{p}_i = p_i - p_{\gamma_i}$.

Proof: From Lemma 3, each block $B_{uv}(u, v = 1, ..., d)$ is a zero-row-sum matrix.

$$\sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ij}c_{ij}(p_i - p_j) = \sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ij}c_{ij}(\tilde{p}_i - \tilde{p}_j),$$

and

$$\sum_{j=1,\hat{i}\neq\hat{j}}^{N} a_{ij}c_{ij}(\tilde{p}_{i}-\tilde{p}_{j}) = \sum_{j=1,\hat{i}\neq\hat{j}}^{N} a_{ij}c_{ij}(p_{i}-p_{j})$$
$$-\sum_{j=1,\hat{i}\neq\hat{j}}^{N} a_{ij}c_{ij}p_{\gamma\hat{i}} + \sum_{j=1,\hat{i}\neq\hat{j}}^{N} a_{ij}c_{ij}p_{\gamma\hat{j}}$$
$$= \sum_{j=1,\hat{i}\neq\hat{j}}^{N} a_{ij}c_{ij}(p_{i}-p_{j}).$$

Thus, this completes the proof of Lemma 4.

Lemma 5: Let matrix A be given as in Equation (4). Then

$$\sum_{i=1}^{N} (p_i - p_{\gamma_i})^{\mathsf{T}} \sum_{j=1, i=j, j \neq i}^{N} a_{ij} c_{ij} (p_i - p_j) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} h_{ij} a_{ij} c_{ij} (p_i - p_j)^{\mathsf{T}} (p_i - p_j).$$

Proof: Note that, if $\hat{i} = \hat{j}$, then nodes i and j are in the same cluster, therefore

$$\sum_{i=1}^{N} (p_i - p_{\gamma_{\hat{i}}})^{\mathsf{T}} \sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ij}c_{ij}(p_i - p_j) = \sum_{i=1}^{N} \tilde{p}_i^{\mathsf{T}} \sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ij}c_{ij}(\tilde{p}_i - \tilde{p}_j),$$

and

$$\frac{1}{2} \sum_{i=1}^{N} \sum_{\substack{j=1, j\neq i \\ j=1, j\neq i}}^{N} h_{ij} a_{ij} c_{ij} (p_i - p_j)^{\mathrm{T}} (p_i - p_j) = \frac{1}{2} \sum_{i=1}^{N} \sum_{\substack{j=1, \hat{i}=\hat{j}, j\neq i}}^{N} a_{ij} c_{ij} (\tilde{p}_i - \tilde{p}_j)^{\mathrm{T}} (\tilde{p}_i - \tilde{p}_j).$$

For the symmetry of the matrix A, $c_{ij}(0) = c_{ji}(0)$ and $k_{ij} = k_{ji}$,

$$a_{ii}c_{ii} = -\sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ij}c_{ij} = -\sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ji}c_{ji}.$$

Thus,

$$\sum_{\substack{j=1,\hat{i}=\hat{j}, j\neq i \\ \frac{1}{2} \left\{ \sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ji}c_{ji} + \sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ij}c_{ij} \right\} \tilde{p}_{i}^{\mathrm{T}}\tilde{p}_{i}}$$

and

$$a_{ii}c_{ii} = -\sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ij}c_{ij} = -\sum_{j=1,\hat{i}=\hat{j}, j\neq i}^{N} a_{ji}c_{ji}.$$

Thus,

$$\begin{aligned} a_{ij}c_{ij}\tilde{p}_i^{\mathsf{T}}\tilde{p}_j + a_{ji}c_{ji}\tilde{p}_j^{\mathsf{T}}\tilde{p}_i &= \\ \frac{1}{2}(a_{ij}c_{ij}\tilde{p}_i^{\mathsf{T}}\tilde{p}_j + a_{ij}c_{ij}\tilde{p}_j^{\mathsf{T}}\tilde{p}_i + a_{ji}c_{ji}\tilde{p}_i^{\mathsf{T}}\tilde{p}_j + a_{ji}c_{ji}\tilde{p}_j^{\mathsf{T}}\tilde{p}_i). \end{aligned}$$

From the analysis above, one has

$$\sum_{i=1}^{N} (p_i - p_{\gamma_i})^{\mathsf{T}} \sum_{\substack{j=1, i=j, j\neq i}}^{N} a_{ij} c_{ij} (p_i - p_j) = \frac{1}{2} \sum_{i=1}^{N} \sum_{\substack{j=1, j\neq i}}^{N} h_{ij} a_{ij} c_{ij} (p_i - p_j)^{\mathsf{T}} (p_i - p_j).$$

Thus, this completes the proof of Lemma 5.

Our main results can be summarized in the following theorem.

Theorem 1: Consider a system of N agents with dynamics (1) and d leaders with dynamics (2), where each follower is steered by the adaptive strategy (3). Suppose that at least one agent in each cluster is selected to be informed by the corresponding leader. Let matrix A be given as in Equation (4). Then, group consensus can be achieved, namely,

$$\lim_{t \to \infty} \sum_{l=1}^d \sum_{i \in G_l} \|q_i(t) - q_{\gamma_l}(t)\| = 0$$

and

$$\lim_{t \to \infty} \sum_{l=1}^d \sum_{i \in G_l} \|p_i(t) - p_{\gamma_l}(t)\| = 0$$

for all i.

Proof: From Equations (1), (2) and (3), the error dynamics is given by

$$\begin{aligned} &\dot{\tilde{q}}_{i} = \tilde{p}_{i}, \\ &\dot{\tilde{p}}_{i} = -\omega^{2} \tilde{q}_{i} - \sum_{j=1, j \neq i}^{N} c_{ij} a_{ij} (\tilde{p}_{i} - \tilde{p}_{j}) - h_{i} c_{i} \tilde{p}_{i}, \\ &\dot{c}_{ij} = h_{ij} a_{ij} k_{ij} (\tilde{p}_{i} - \tilde{p}_{j})^{T} (\tilde{p}_{i} - \tilde{p}_{j}), \\ &\dot{c}_{i} = h_{i} k_{i} \tilde{p}_{i}^{T} \tilde{p}_{i}, \end{aligned}$$
(5)

For notational convenience, we also define

$$\tilde{q} = \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \vdots \\ \tilde{q}_N \end{bmatrix}, \quad \tilde{p} = \begin{bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \\ \vdots \\ \tilde{p}_N \end{bmatrix}.$$

Consider a candidate for the Lyapunov function

$$\begin{aligned} Q(\tilde{q}, \tilde{p}, c_{ij}, c_i) &= \frac{1}{2} \omega^2 \tilde{q}^{\mathsf{T}} \tilde{q} + \frac{1}{2} \tilde{p}^{\mathsf{T}} \tilde{p} \\ + \frac{1}{2} \sum_{i=1}^{N} \left[\sum_{j=1, j \neq i}^{N} \frac{(c_{ij} - m)^2}{2k_{ij}} + \frac{(c_i - m)^2}{k_i} \right], \end{aligned}$$

which is a positive definite function and radially unbounded with respect to \tilde{p} and \tilde{q} . The positive constant

$$m > \frac{(d-1)\max_{u,v}[k_u, k_v] \cdot \max_{i,j} |a_{ij}c_{ij}|}{\min_u \lambda(H_{uu} - B_{uu})}$$

Differentiating Q(t) gives

$$\begin{split} \dot{Q}(t) &= \sum_{i=1}^{N} \tilde{p}_{i}^{\mathsf{T}} \left[-\sum_{j=1, j \neq i}^{N} a_{ij} c_{ij} (\tilde{p}_{i} - \tilde{p}_{j}) - h_{i} c_{i} \tilde{p}_{i} \right] \\ &+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} h_{ij} a_{ij} (c_{ij} - m) (\tilde{p}_{i} - \tilde{p}_{j})^{\mathsf{T}} (\tilde{p}_{i} - \tilde{p}_{j}) \\ &+ \sum_{i=1}^{N} h_{i} (c_{i} - m) \tilde{p}_{i}^{\mathsf{T}} \tilde{p}_{i} \\ &= \sum_{r=1}^{n} \sum_{i=1}^{N} \tilde{p}_{i}^{rT} \left[-\sum_{j=1, j \neq i}^{N} a_{ij} c_{ij} (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r}) - h_{i} c_{i} \tilde{p}_{i}^{r} \right] \\ &+ \frac{1}{2} \sum_{r=1}^{n} \sum_{i=1}^{N} \tilde{p}_{i}^{rT} \left[-\sum_{j=1, j \neq i}^{N} a_{ij} a_{ij} c_{ij} (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r}) - h_{i} c_{i} \tilde{p}_{i}^{r} \right] \\ &+ \frac{1}{2} \sum_{r=1}^{n} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} h_{ij} a_{ij} c_{ij} (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r})^{\mathsf{T}} (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r}) \\ &- \frac{1}{2} \sum_{r=1}^{n} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} h_{ij} a_{ij} m (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r})^{\mathsf{T}} (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r}) \\ &+ \sum_{r=1}^{n} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} h_{ij} a_{ij} m (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r})^{\mathsf{T}} (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r}) \\ &+ \sum_{r=1}^{n} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} h_{ij} a_{ij} m (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r})^{\mathsf{T}} (\tilde{p}_{i}^{r} - \tilde{p}_{j}^{r}) \\ &+ \sum_{r=1}^{n} \sum_{u=1}^{n} \sum_{u=1}^{N} p^{urT} (B_{uu} - H_{uu}) p^{ur} \\ &+ \sum_{r=1}^{n} \sum_{u=1}^{n} \sum_{v \neq u}^{d} p^{urT} (mB_{uu} - mH_{uu}) p^{ur} \\ &+ (1/2) \max_{u,v} [k_{u}, k_{v}] \cdot \max_{i,j} |a_{ij} c_{ij}| | \\ &+ \sum_{r=1}^{n} \left[\sum_{u=1}^{d} p^{urT} (mB_{uu} - mH_{uu}) p^{ur} \right] \\ &+ (d-1) \cdot \max_{u,v} [k_{u}, k_{v}] \cdot \max_{i,j} |a_{ij} c_{ij}| | \\ &+ \sum_{r=1}^{n} \left[\sum_{u=1}^{d} p^{urT} p^{ur} \right], \end{split}$$

where $p^r = [p_1^r, \dots, p_N^r]^T = [p^{1r}, \dots, p^{ur}, \dots, p^{dr}]^T \in \mathbf{R}^N$, $p^{ur} \in \mathbf{R}^{k_u}$ is the vector in the *u*th cluster, and

$$H = \begin{bmatrix} H_{11} & 0 & \cdots & 0 \\ 0 & H_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H_{dd} \end{bmatrix}.$$

According to (4), and at least one node in each cluster is selected to be informed by the corresponding leader, from Lemma 1 and Lemma 3, it can be verified that $B_{uu} - H_{uu}$ is negative-definite. Since the positive constant

$$m > \frac{(d-1)\max_{u,v}[k_u, k_v] \cdot \max_{i,j} \mid a_{ij}c_{ij} \mid}{\min_u \lambda(H_{uu} - B_{uu})},$$

 $\dot{Q}(t) < 0.$

one has

Thus,

$$\lim_{t \to \infty} \sum_{l=1}^{d} \sum_{i \in G_l} \|q_i(t) - q_{\gamma_l}(t)\| = 0$$

and

$$\lim_{t \to \infty} \sum_{l=1}^d \sum_{i \in G_l} \|p_i(t) - p_{\gamma_l}(t)\| = 0$$

for all *i*. This completes the proof of Theorem 1.

Remark 2: A common feature of the works presented in [33], [34] is that there are certain convergence conditions that require global information on the underlying network such as the eigenvalues of the coupling matrix of the network or a centralized adaptive strategy. In protocol (3), each agent only has the state information of its neighbors and only selects those few agents that have access to the information of the corresponding leader. Without using any global information of the underlying network and under the control input (3), all agents in the same group asymptotically synchronize with the corresponding leader even when only one agent in each group has access to the information of the corresponding leader.

IV. SIMULATION STUDY

The simulation is performed with 8 agents on a real line whose initial positions and velocities are randomly chosen within [-4, 4] and [-2, 2], respectively. Suppose 2 nonempty subsets $\{G_1, G_2\}$ is a partition of the index set $\{1, 2, \ldots, N\}$, where $G_1 = \{1, 2, 3, 4, 5\}$ and $G_2 =$ $\{6, 7, 8\}$. The network topology of the coupled harmonic oscillators are shown in Fig. 1. The adjacency matrix A = (a_{ij}) of Fig. 1 is

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$



Fig. 1. The coupled harmonic oscillators example with ${\cal N}=8$ and 2 clusters.

There are 2 leaders whose initial positions and velocities are $q_{\gamma_1} = 1$, $q_{\gamma_2} = 2$, $p_{\gamma_1} = 1$ and $p_{\gamma_2} = 2$, respectively. Suppose that agent 1 and agent 6 are selected to be informed by leader 1 and leader 2, respectively.

In the simulation, the parameter $\omega = 3.162$, $c_{ij}(0) = 0$, $c_i(0) = 0$, $k_{ij} = 1$ and $k_i = 1$. Fig. 2 shows the group consensus of coupled harmonic oscillators under the control input (3). Plots 2(a) and 2(b) are respectively the evolutions of the positions and velocities of the eight agents. It is obvious from these plots that the control input (3) is capable of achieving stable group consensus motion. Plot 2(c) depicts the adaptive weights of velocity feedback gains which eventually tend to constant; Plot 2(d) presents the adaptive coupling strengths that eventually tend to constant.

V. CONCLUSION

In this paper, we have investigated group consensus of coupled harmonic oscillators with multiple leaders. By introducing local adaptation strategies for both the weights on the velocity navigational feedback and the velocity coupling strengths, we proposed an adaptive group consensus algorithm and showed that all agents in the same group asymptotically synchronize with the corresponding leader even when only one agent in each group has access to the information of the corresponding leader. Compared with other relevant results in the literature, the results obtained in this paper only require local information of the underlying network and the decentralized adaptive strategy, and are therefore easier to apply. Other topics such as the directed and switching network topology, the effects of time delay and disturbance may warrant further studies.

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Fig. 2. Group consensus of 8 coupled harmonic oscillators and 2 leaders under the control input (3). (a) Positions, (b) velocities, (c) adaptive weights of velocity feedback gains, (d) adaptive coupling strengths.

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