Optimal Submission Problem in a Limit Order Book with VaR Constraints

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***Abstract*—We consider an optimal selection problem for bid and ask quotes subject to a value-at-Risk (VaR) constraint when arrivals of the buy and sell orders are governed by a Poisson process. The problem is formulated as a constrained utility maximization problem over a finite time horizon. Using a diffusion approximation to Poisson arrivals of market orders, the dynamic programming principle can be applied here. We propose an efficient procedure to solve this constrained utility maximization problem based on a successive approximation algorithm. Numerical examples with and without the VaR constraint are used to illustrate the effect of the risk constraint on the dealer's choices. We also conduct numerical experiments to analyze the impacts of the risk constraint on dealer's terminal profit.**

***Keywords*-High-frequency trading; Limit Order Book; Diffusion Approximation; HJB equations; VaR;**

I. Introduction

Over the past years, high-frequency trading has been progressively gained a foothold in financial markets. In high-frequency trading, programs analyze market data to capture trading opportunities that may open up for only a fraction of a second to several hours. One set of high-frequency trading strategies is that involve placing a limit order to sell (or ask) or a buy limit order (or bid) in order to earn the bid-ask spread. Due to the most effective developments in information technology (IT), it is possible for the dealers to post limit orders at the price they choose and ensure the availability of high frequency data on the limit order book. To maximize the terminal profit, the dealer faces an inventory risk arising from uncertainty in the stock's price and a transactions risk due to Poisson arrival of market buy and sell orders. To consider these two risk sources, Ho and Stoll [1] developed a model to analyze the optimal prices for a monopolistic dealer in a single stock. Their results show that the optimal bid and ask quotes are around the "true" price of the stock. In [2], Ho and Stoll also pointed out that the bid and ask quotes are related to the reservation prices of the dealers when dealers are under competition. Based on these two papers, Avellaneda and Stoikov [3] studied the optimal submission strategies by assuming the "true" price of the stock is modeled as a Brownian motion.

Our work is diferent from those in the exising literature in two major aspects. Firstly, we consider the presence of a risk constraint to an optimal selection problem of bid and ask quotes. From the lessions of a number of financial turmoils in recent years such as the Asian financial crisis in 1997, the recent global financial crisis and the debt crisis in Europe, we learn that maximizing profits is not the only objective that needs to be taken into account for the market participations. The consideration for risk control is of primal importance. Indeed, the importance of risk measurement and management has captured much attention among academic researchers and market practitioners. Various methods and techniques for measuring, managing and controlling risk have been proposed in the literature. One of the important and widely used tools for risk measurement is Value-at-Risk (VaR). VaR is the maximum loss we might expect with a given probability level over a given holding or horizon period. For an excellent introduction of VaR and its practical implementation, interested readers may refer to Jorion [7], Duffie and Pan [8],[9], Best [10] and J.P. Morgan's Risk Metrics - Technical Document. Basak and Shapiro [14] considered the optimal portfolio allocation problem by maximizing the utility function of an economic agent with the VaR constraint. Yiu et al.[15] considered the optimal portfolio selection problem subject to a maximum value-at-Risk (MVaR) constraint when the market parameters are allowed to switch over time according to a continuous-time, finite-time, observable Markov chain, whose states are interpreted as the states of an economy. In our paper, we extend the model in [3] by considering the risk constraint. The risk is measured by the VaR of a portfolio in a short time duration. We then formulate the optimal submission problem of bid and ask quotes as a stochastic optimal control problem with VaR constraints.

Secondly, we use a diffusion approximation so that the stock inventory level and the wealth dynamics are approximated by the Wiener process. In this case, the dynamic

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programming principle is applicable. The normal distribution is in the core of the space of all observable processes. This distribution often provides a reasonable approximation to a variety of data. From [4], when the intensity is large enough, the Poisson distribution $Poi(\lambda)$ can be well approximated by the normal distribution $N(\lambda, \lambda)$. Hence, we can apply a diffusion approximation to approximate Poisson arrivals of the market buy and sell orders. There are many applications of a diffusion approximation. For example: In Kobayashi's [5] paper, queuing processes of various service stations which interact with each other are approximated by a vector-valued Wiener process; In [6], Nagaev et al. assumed that the stock price evolution is described by a Markov chain. By applying a diffusion approximation to the Markov chain, they obtained a simple but powerful approximate formula for the studied characteristic. In our paper, a diffusion approximation is employed so that the dynamic programming principle is applicable to deduce a set of HJB equations. Then the solution of the optimal submission problem of bid and ask quotes can be obtained by solving the (Hamilton-Jacobi-Bellman) HJB equation. We employ the successive approximation algorithm introduced by Chang and Krishna [16] to solve the HJB equation which is a second-order partial differential equation (PDE) in coupled with an optimization. The successive approximation algorithm separates the optimization problem from the boundary value PDE problem and thus making the problem solvable by some standard numerical techniques.

The rest of the paper is organized as follows. In Section II, we formulate the constrained optimal submission problem in a limit order book. By applying the diffusion approximation to the Poisson arrival of market orders, we deduced the HJB equation according to the dynamic programming principle. In Section III, the successive approximation algorithm is introduced to solve the HJB equation with VaR constraint. The results of the numerical experiments are presented in Section IV. We then summarize the main results in the final section.

II. The Model

A. Problem Formulation

We consider an optimal bid and ask quotes selection problem by extending the model in [3]. In the securities market, the dealers provide liquidity on the exchange by submitting the limit order. A limit order is an order to buy a security at no more than a specific price p_b, or to sell a security at no less than a specific price p_a. A buy limit order can only be executed at the bid price p_b or lower, and a sell limit order can only be executed at the ask price p_a or higher. The limit order can only be filled if the stock market price reaches the limit price. We define the distances

$$\delta_b = s - p_b \quad \text{and} \quad \delta_a = s - p_a.$$

In the security market, execution of limit orders is determined by the dealer's submission of the limit orders and arrival of market orders.We can assume the dealer's buy limit order will be executed at Poisson rate $\lambda_b(\delta_b)$ since a buy limit order can only be executed at the bid price p_b or lower. The Poisson rate $\lambda_b(\delta_b)$ should be a decreasing function of δ_b. Similarly, the Poisson rate $\lambda_a(\delta_a)$ for the executed sell limit order is also a decreasing function of δ_a. According to [3] and the results in the econophysics literature, for example, [11], [12] and [13], the Poisson intensity can be derived as

$$\lambda_a(\delta_a) = Ae^{-k\delta_a} \quad \text{and} \quad \lambda_b(\delta_b) = Ae^{-k\delta_b}.$$

Then the inventory level of the stock at time t should be:

$$q_t = N_t^b - N_t^a.$$

where N_t^b and N_t^a are Poisson processes with intensities λ_b and λ_a. And N_t^b is the amount of stocks bought by the dealer and N_t^a is the amount of stocks sold. Then the wealth is also a stochastic process and determined by the executed limit orders:

$$dX_t = rX_t + p_a dN_t^a - p_b dN_t^b$$

where r is the risk-free interest rate. The stock price in the market is modeled as a Brownian motion which is same as the model in [3]: $S_t = s + \sigma W_t$. The dealer wants to maximize his terminal utility of the wealth. Then this optimal submission problem in a limit oder book can be formulated as:

$$\max_{\delta_a, \delta_b} E_t[-e^{-\gamma(X_T + q_T S_T)}] \tag{1}$$

subject to:

$$\begin{cases} dS_t &= \sigma dW_t, \quad S_0 = s\ , \\ dX_t &= rX_t + p_a dN_t^a - p_b dN_t^b\ , \\ q_t &= N_t^b - N_t^a. \end{cases}$$

where γ is the coefficient for exponential utility which represents the degree of risk aversion.

B. The diffusion approximation

From [4] we know that if $X \sim Poisson(\lambda)$, then $X \approx N(\mu = \lambda, \sigma = \sqrt{\lambda})$ for $\lambda > 20$, and approximation improves as λ increases. Hence, we apply the diffusion approximation to the uncertainty sources with Poisson nature. Then the constraints for the optimal submission in a limit oder book can be rewritten as:

$$\begin{cases} dS_t &= \sigma dW_{t_1}, \quad S_0 = s\ , \\ dX_t &= (rX_t + p_a\lambda_a - p_b\lambda_b)dt \\ & \quad +p_a\sqrt{\lambda_a}dW_{t_2} - p_b\sqrt{\lambda_b}dW_{t_3}\ , \\ dq_t &= (\lambda_b - \lambda_a)dt + \sqrt{\lambda_b}dW_{t_3} - \sqrt{\lambda_a}dW_{t_2}. \end{cases} \tag{2}$$

We assume that the three Brownian motions W_{t_1}, W_{t_2} and W_{t_3} are independent. Recall that the dealer's objective is given by the value function:

$$v(S, X, q, t) = \max_{\delta_a, \delta_b} E_t[-e^{-\gamma(X_T + q_T S_T)}].$$

According to the dynamic programming principle we can deduce the following Hamilton-Jacobi-Bellman equation to select the optimal bid and ask prices in a limit order book.

$$
\begin{aligned}
v_t + \max_{\delta_a,\delta_b} \quad & \Bigg\{\frac{1}{2}\Big((p_a^2\lambda_a + p_b^2\lambda_b)\frac{\partial^2 v}{\partial x^2} - 2(p_a\lambda_a + p_b\lambda_b)\frac{\partial^2 v}{\partial x\partial q} \\
& +(\lambda_a+\lambda_b)\frac{\partial^2 v}{\partial q^2} + \sigma^2\frac{\partial^2 v}{\partial S^2}\Big) \\
& +(rX_t + p_a\lambda_a - p_b\lambda_b)\frac{\partial v}{\partial x} + (\lambda_b - \lambda_a)\frac{\partial v}{\partial q}\Bigg\} = 0\ ,
\end{aligned}
$$

with terminal condition $v(T,\cdot) = -e^{-\gamma(X_T+q_T S_T)}$. For this type of HJB equation, we make the assumption that $v = -e^{-\gamma X}e^{-\gamma u(S,q,t)}$ to simplify the problem. Then we obtain

$$
\begin{aligned}
u_t + \max_{\delta_a,\delta_b} \quad & \Bigg\{\frac{1}{2}\Big((p_a^2\lambda_a + p_b^2\lambda_b)(-\gamma) + 2\gamma(p_a\lambda_a + p_b\lambda_b)\frac{\partial u}{\partial q} \\
& +(\lambda_a+\lambda_b)(\frac{\partial^2 u}{\partial q^2} - \gamma\frac{\partial u}{\partial q}) + \sigma^2(\frac{\partial^2 u}{\partial S^2} - \gamma\frac{\partial u}{\partial S})\Big) \\
& +(rX_t + p_a\lambda_a - p_b\lambda_b) + (\lambda_b - \lambda_a)\frac{\partial u}{\partial q}\Bigg\} = 0, \quad (3)
\end{aligned}
$$

with the terminal condition $u(T,\cdot) = q_T S_T$.

C. The VaR constraint

In this subsection, we present the VaR constraint of the optimal selection problem in a limit order book. It is reasonable to assume that the dealer submit the limit orders at the beginning of the small time intervals discretely. The stock price and the arrival of the market orders are approximately constants in the small time interval $[t, t+h]$. Firstly we define $V(t) = X_t + q_t S_t$. According to (2),

$$
\begin{aligned}
dq_t S_t = \quad & q_t\sigma dW_{t_1} + S_t(\lambda_b - \lambda_a)dt \\
& + S_t\sqrt{\lambda_b}dW_{t_3} - S_t\sqrt{\lambda_a}dW_{t_2} + d[S_t, q_t].
\end{aligned}
$$

We have assumed that W_{t_1}, W_{t_2} and W_{t_3} are independent, hence $d[S_t, q_t] = 0$.
Then, in the small time interval $[t, t+h]$:

$$
\begin{aligned}
& \Delta V(t,h) \\
= \ & (X_t + q_t S_t) - e^{-rh}(X_{t+h} + q_{t+h}S_{t+h}) \\
= \ & e^{rt}\Big(\int_t^{t+h} re^{-r\tau}q_t S_t d\tau - \int_t^{t+h} e^{-r\tau}q_t\sigma dW_{\tau_1} \\
& - \int_t^{t+h} e^{-r\tau}(p_a\lambda_a - p_b\lambda_b + S_t(\lambda_b - \lambda_a))d\tau \\
& + \int_t^{t+h} e^{-r\tau}(S_t\sqrt{\lambda_a} - p_a\sqrt{\lambda_a})dW_{\tau_2} \\
& + \int_t^{t+h} e^{-r\tau}(p_b\sqrt{\lambda_b} - S_t\sqrt{\lambda_b})dW_{\tau_3}\Big)
\end{aligned}
$$

Under the measure $\mathcal{P}$, the conditional probability distribution of $\Delta V(t,h)$ given $\mathcal{F}_t$ is a normal distribution with the conditional mean:

$$
\begin{aligned}
E[\Delta V(t,h) \mid \mathcal{F}_t] \ = \ & q_t S_t(e^{-rh} - 1) \\
& + \Big(p_a\lambda_a - p_b\lambda_b + S_t(\lambda_b - \lambda_a)\Big)\frac{1 - e^{-rh}}{r}.
\end{aligned}
$$

and the conditional variance of $\Delta V(t,h)$

$$
Var[\Delta V(t,h) \mid \mathcal{F}_t] = \frac{1 - e^{-2rh}}{2r}\Big(\lambda_a(p_a - S_t)^2 + q_t^2\sigma^2 + \lambda_b(S_t - p_b)^2\Big).
$$

The VaR of the wealth with confidence level α is given by

$$
\begin{aligned}
& VaR_\alpha(\Delta V(t,h) \mid \mathcal{F}_t) \\
= \ & \inf\{x \in R \mid P(\Delta V(t,h) > x \mid \mathcal{F}_t) \leq 1 - \alpha\} \\
= \ & E[\Delta V(t,h) \mid \mathcal{F}_t] + \phi^{-1}(\alpha)\sqrt{Var[\Delta V(t,h) \mid \mathcal{F}_t]}
\end{aligned}
$$

which depends on the bid and ask prices p_b, p_a we submit at time t.
We define the risk constraint at the level G for this optimal submission problem in a limit order book as

$$
VaR_\alpha(\Delta V(t,h) \mid \mathcal{F}_t) \leq G
$$

Then the optimal submission problem in a limit order book with the VaR constraint is summarized as

$$
\max_{\delta_a,\delta_b} E_t[-e^{-\gamma(X_T+q_T S_T)}]
$$

subject to:

$$
\left\{
\begin{aligned}
dS_t \ &= \ \sigma dW_{t_1}, \quad S_0 = s\ , \\
dX_t \ &= \ (rX_t + p_a\lambda_a - p_b\lambda_b)dt \\
& \quad + p_a\sqrt{\lambda_a}dW_{t_2} - p_b\sqrt{\lambda_b}dW_{t_3}\ , \\
dq_t \ &= \ (\lambda_b - \lambda_a)dt + \sqrt{\lambda_b}dW_{t_3} - \sqrt{\lambda_a}dW_{t_2}.
\end{aligned}
\right. \quad (4)
$$

and

$$
E[\Delta V(t,h) \mid \mathcal{F}_t] + \phi^{-1}(\alpha)\sqrt{Var[\Delta V(t,h) \mid \mathcal{F}_t]} \leq G. \quad (5)
$$

III. Numerical Experiments and Discussions

In this section, we first present the iterative algorithm to solve the HJB equation with VaR constraint. Then by figuring out the optimal selection of the bid and ask prices with risk constraint we can find how the risk constraint affects dealer's choices. We also conduct numerical experiments to analyze the influence of risk constraint to dealer's terminal profit.

A. The Iterative Algorithm

As a necessary condition for an optimal solution of a stochastic control problem, the HJB equation is a second-order nonlinear partial differential equation. Analytical solutions can be obtained only for some special cases with simple state equations. In this sub section we shall apply the successive approximation algorithm to solve the HJB equation numerically which was introduced in [16]. According to the successive approximation algorithm, the problem of solving the HJB equation numerically has been separated into two sub-problems:
(1) Solving the PDE numerically, and
(2) Optimization of the nonlinear function over δ_a and δ_b.
To solve the PDE, we employ a finite difference scheme introduced in [18]. According to the finite difference scheme, we divide the domain of the computation into a grid of $N_t \times N_q \times N_X$ mesh points, where N_t, N_q and N_X represent the number of mesh points in the time and the space domains. For a function of u defined on the grid we write $u_{l,m,n}$ for the value of u at the grid point (t_l, q_m, X_n). Then the steps in the iterative algorithm are presented as follows:

Step I: For each $l = N_t - 1, \ldots, 0, m = 1, \ldots, N_q, n = 1, \ldots, N_X$, the initial value $\delta_a = 0.5, \delta_b = 0.5$. Then $u^0_{l,m,n}$ are computed from the following equation:

$$\begin{aligned} u^0_{l,m,n} &= u^0_{l+1,m,n} + \Delta t \Big\{ \frac{1}{2} \Big((p_a^2 \lambda_a + p_b^2 \lambda_b)(-\gamma) + r X_t \\ &\quad + p_a \lambda_a - p_b \lambda_b \Big) + \Big((p_a \lambda_a + p_b \lambda_b - \frac{\lambda_a + \lambda_b}{2}) \gamma \\ &\quad + \lambda_b - \lambda_a \Big) \frac{u^0_{l+1,m+1,n} - u^0_{l+1,m,n}}{\Delta q} \\ &\quad + \frac{\lambda_a + \lambda_b}{2} \frac{u^0_{l+1,m+1,n} - 2u^0_{l+1,m,n} + u^0_{l+1,m-1,n}}{\Delta q^2} \\ &\quad + \frac{\sigma^2}{2} \Big(\frac{u^0_{l+1,m,n+1} - 2u^0_{l+1,m,n} + u^0_{l+1,m,n-1}}{\Delta S^2} \\ &\quad - \gamma \frac{u^0_{l+1,m,n+1} - u^0_{l+1,m,n}}{\Delta S} \Big) \Big\}, \end{aligned}$$

Step II: With the constraint, according to (3) and (5), our optimization problem is given by

$$\begin{aligned} \max_{\delta_a, \delta_b} \quad & \Big\{ \frac{1}{2} \Big((p_a^2 \lambda_a + p_b^2 \lambda_b)(-\gamma) + 2\gamma (p_a \lambda_a + p_b \lambda_b) \frac{\partial u}{\partial q} \\ & + (\lambda_a + \lambda_b)(\frac{\partial^2 u}{\partial q^2} - \gamma \frac{\partial u}{\partial q}) + \sigma^2 (\frac{\partial^2 u}{\partial S^2} - \gamma \frac{\partial u}{\partial S}) \Big) \\ & + (r X_t + p_a \lambda_a - p_b \lambda_b) + (\lambda_b - \lambda_a) \frac{\partial u}{\partial q} \Big\}, \end{aligned}$$

subject to

$$E[\Delta V(t,h) \mid \mathcal{F}_t] + \phi^{-1}(\alpha) \sqrt{Var[\Delta V(t,h) \mid \mathcal{F}_t]} \le G$$

Furthermore we compute $u^k_{l,m,n}$

$$\begin{aligned} u^k_{l,m,n} &= u^k_{l+1,m,n} + \Delta t \Big\{ \frac{1}{2} \Big((p_a^2 \lambda_a + p_b^2 \lambda_b)(-\gamma) + r X_t \\ &\quad + p_a \lambda_a - p_b \lambda_b \Big) + \Big((p_a \lambda_a + p_b \lambda_b - \frac{\lambda_a + \lambda_b}{2}) \gamma \\ &\quad + \lambda_b - \lambda_a \Big) \frac{u^k_{l+1,m+1,n} - u^k_{l+1,m,n}}{\Delta q} \\ &\quad + \frac{\lambda_a + \lambda_b}{2} \frac{u^k_{l+1,m+1,n} - 2u^k_{l+1,m,n} + u^k_{l+1,m-1,n}}{\Delta q^2} \\ &\quad + \frac{\sigma^2}{2} \Big(\frac{u^k_{l+1,m,n+1} - 2u^k_{l+1,m,n} + u^k_{l+1,m,n-1}}{\Delta S^2} \\ &\quad - \gamma \frac{u^k_{l+1,m,n+1} - u^k_{l+1,m,n}}{\Delta S} \Big) \Big\}, \end{aligned}$$

Step III: Return to Step II with $k = k + 1$ until

$$\| u^{k-1} - u^k \| < \epsilon$$

where ϵ is a small positive number. The proof of the convergence of this Successive Approximation Algorithm can be found in [17].

B. Optimal Bid and Ask Prices with VaR Constraint

In this subsection, we conduct the numerical experiments to compare the optimal submission in a limit order book arising from the model with VaR constraint with that obtained from the model without VaR constraint. We implement the above iterative algorithm by MATLAB. Figure 1 shows the simulated path with the parameters $T = 1, \gamma = 0.1, r = 0.02, s = 100, \sigma = 2, dt = 0.01, k = 1.5, A = 140$ for the model with VaR constraint. From (1) we know that the dealer's profit consists of two parts: the terminal value of the inventory stock and the terminal wealth obtained from the transaction. From Figure 1 we can observe that the dynamic of stock price has a significant impact on the dealer's choice. For example, in the time interval $(0.30, 0.33)$, the stock price is increasing, the dealer should hold more stocks to increase his inventory value by submitting a higher bid price. He can also sell the stock he holds at a higher price by submitting a higher ask price. And as the stock price increases, the increase rate of the ask price is greater than that of bid price. This makes intuitive sense as the dynamic of stock price affect the dealers submission from two perspectives. On one hand, to minimize the inventory risk, it is wise for the dealer to submit a higher ask price if his inventory level is positive, meanwhile, he should lower the ask price if he is a short seller. On the other hand, to consider the transaction risk and maximize his terminal wealth, he should sell the stock at a higher price and buy the stock in a lower price to earn more bid-ask spread. The dealer's incentive to hold more stock results in a higher intensity of market buy orders, until $t = 0.33$, where the stock price stars to decrease. We consider the inventory risk throughout the submission process, hence, when the terminal time is approaching, the stock price is easier to predict. Then the optimal bid and ask prices seem to be symmetric with respect to the stock price since the inventory risk is small.

Figure 2 shows the simulated path with the same parameters for the model without VaR constraint. By comparing Figure 1 with Figure 2, we can find that the dealer behaves more conservative when we consider the risk constraint. For example, in the time interval $(0.40, 0.43)$, the optimal ask price at $t = 0.41$ in Figure 1 is 103.8 while $p_a = 104$ in Figure 2 at the same time. It seems that when the stock price moves drastically, the dealer's strategy is more modest by considering the risk constraint which is reasonable since the dealer should not be that anxious or excited to make a profit when there is an opportunity to invest in the market.

After 200 times simulations we obtained the average and the standard deviations of the profit from the two models. The strategy with VaR constraint has a lower profit (of \$63.32 versus \$75.46) and lower standard deviation (\$4.32 versus \$7.14) which is illustrated in the histogram in Figure 3.

IV. Conclusion

We consider the optimal bid and ask quotes selection problem subject to a value-at-Risk (VaR) constraint when the arrival of the buy and sell orders are governing by Poisson processes. By considering diffusion approximation,

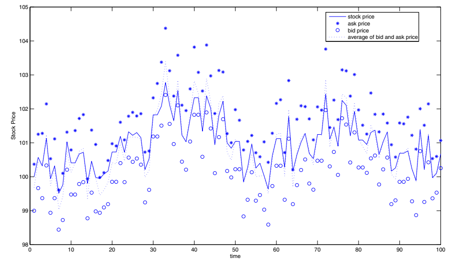

Figure 1. Simulation Results with risk constraint

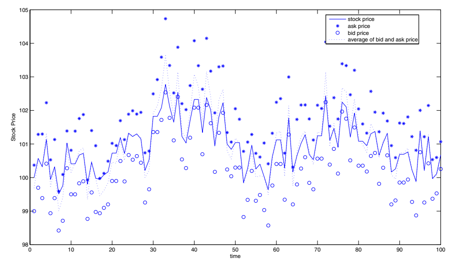

Figure 2. Simulation Results without risk constraint

the dynamic programming principle is applicable. We obtain the numerical solutions by solving the HJB equation. We can find that the dealer behaves more conservative when we consider the risk constraint and the profit from our model has a lower expectation and lower standard deviation than that from the model without the VaR constraint.

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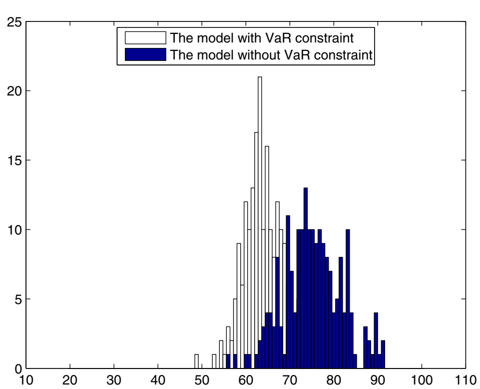

Figure 3. 200 simulations with $\gamma = 0.1$

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