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Author(s)	Chu, Y; Chan, SC; Zhang, Z; Tsui, KM
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A NEW REGULARIZED TVAR-BASED ALGORITHM FOR RECURSIVE DETECTION OF NONSTATIONARITY AND ITS APPLICATION TO SPEECH SIGNALS

Y. J. Chu, S. C. Chan, Z. G. Zhang, and K. M. Tsui

Department of Electrical and Electronic Engineering, The University of Hong Kong E-mail: {yjchu; scchan; zgzhang; kmtsui}@eee.hku.hk

ABSTRACT

This paper develops a new recursive nonstationarity detection method based on time-varying autoregressive (TVAR) modeling. A local likelihood estimation approach is introduced which gives more weights to observations near the current time instant but less to those distance apart. It thus allows the Wald test to be computed based on RLStype algorithms with low computational cost. A reliable and efficient state regularized variable forgetting factor (VFF) QR decomposition (QRD)-based RLS (SR-VFF-QRRLS) algorithm is adopted for estimation for its asymptotically unbiased property and immunity to lacking of excitation. Advantages of the proposed approach over conventional approaches are 1) it provides continuous parameter estimates and the corresponding stationary intervals with low complexity, 2) it mitigates low excitation problems using state regularization, and 3) stationarity at different scales can be detected by appropriately choosing a certain window size. The effectiveness of the proposed algorithm is evaluated by testing vocal tract changes in real speech signals.

Index Terms—Nonstationarity detection, local likelihood, Wald test, TVAR, RLS, state regularization

1. INTRODUCTION

In many digital signal applications, the signals under study are often assumed to be wide sense stationary (WSS) Gaussian random sequences. Nonstationarity, which is frequently encountered, will pose significant difficulties in further analysis, since these analyses usually assume that the data record is stationary. It would therefore be important to determine the duration in which a signal is stationary at a desired scale. Much effort has been spent on nonstationarity detection. The statistics based on Fourier transform [1] may not be viable for the short data record such as speeches. For speech detection, early work usually involves a piece-wise AR model assumption [2], where the AR coefficients are estimated as constants within a short time interval using maximum likelihood estimation (MLE) [3]. This implies that within each interval, the vocal tract is assumed to be stationary while jump happens abruptly at the end of each interval. In reality, however, vocal tracts are continually changing, either slowly or rapidly. To this end, the TVAR model was proposed for a better approximation of vocal tracts [4]-[6]. Recently, based on the TVAR model, a new nonstationary detector using the Rao test was proposed in [7]. It requires the MLE under the null hypothesis, which is usually easier to compute. On the other hand, the most widely used test is the generalized likelihood ratio test (GLRT), which requires MLEs under both hypotheses. It was first developed and applied to speech processing in [8] for detecting between an AR and TVAR models. The arithmetic complexity of GLRT is usually higher and the MLE may lead to ill-posed problems [6] due to insufficient samples. Consequently, it may lead to significantly higher false alarms (FAs) in detection. As an alternative, the Wald test [9], which is asymptotically equivalent to GLRT, can be used when the estimate under the alternative hypothesis is available. Though, it has a lower computational complexity, the use of this test is rather limited due to the difficulties in estimating the TVAR parameters that may again suffer from the insufficient sample problem.

In this paper, a new recursive Wald test is proposed for detecting nonstationarity of signals. It is based on the TVAR model and a local likelihood function (LLF), which can be implemented recursively in a RLS-type estimation algorithm with low computational cost. The LLF gives more weights to observations near the current time instant but less to those far away from it as in conventional RLS algorithms. It thus allows the Wald test to be computed based on RLStype algorithms. Since the recursive estimation of TVAR parameters is a crucial step in using the Wald test, a reliable and efficient state regularized VFF QRD RLS algorithm [10] is employed which possesses the asymptotically unbiased property and is less sensitive to lacking of input excitation. Moreover, it can be implemented by the QRD structure which has low roundoff error and efficient hardware realization. In summary, the advantages of the proposed approach over conventional approaches are 1) it provides a continuous parameter estimates and the corresponding interval detections with order stationary $O(p^2)$ computational complexity (the test is O(p)), 2) it mitigates the low excitation problem using the state regularization, and 3) stationarity at different scales can be detected by choosing appropriately a certain window size. The effectiveness of the proposed algorithm is evaluated by testing vocal tract changes in real speech signals.

2. THE PROPOSED RECURSIVE WALD TEST

In the TVAR model, the nonstationary discrete-time signal $\{x(n)\}$ at time instant *n* is given by

$$x(n) = \sum_{i=1}^{p} a_i(n) x(n-i) + g(n) = \boldsymbol{a}^{T}(n) x(n) + g(n)$$
(1)

where *p* is the order of the model, $a(n)=[a_1(n),...,a_p(n)]^T$ is the model coefficient vector of length *p*, $x(n)=[x(n-1),...,x(n-p)]^T$ is the corresponding signal vector, and $\{g(n)\}$ is the excitation which is assumed to be a zero-mean white Gaussian process with variance σ_n^2 . Normally, the process is assumed stationary and all the data record will be used to estimate the model parameters. However, real-world data may be nonstationary and we need to test whether the data at hand is stationary or not. We can formulate this problem as the following hypothesis test:

$$\mathcal{H}_{0}: \boldsymbol{\theta} = \boldsymbol{\theta}_{0}$$

$$\mathcal{H}_{1}: \boldsymbol{\theta} \neq \boldsymbol{\theta}_{0}$$

$$(2)$$

where θ_0 is the *p* by 1 known vector. Assuming *N*+1 samples of the observations $\{x(n)\}$ whose components are continuous random variables, we let $p(x;\theta)$ be its probability density function (pdf), which is assumed to be continuously differentiable with respect to θ . Since g(n) is white Gaussian, the LF of the TVAR model in (1) can be approximated by

$$p(\mathbf{x}';\boldsymbol{\theta}) = \frac{1}{(2\pi\sigma_{\eta}^{2})^{(N-p)/2}} \exp\left(-\sum_{i=p}^{N} \frac{\left(\mathbf{x}(i) - \boldsymbol{a}^{T}(i)\mathbf{x}(i)\right)^{2}}{2\sigma_{\eta}^{2}}\right)$$
(3)

where $\mathbf{x'} = [x(p), \dots, x(N)]^T$ after excluding the first *p* samples and $\boldsymbol{\theta} = [\boldsymbol{a}^T(n), \sigma_\eta^2]^T$. Usually, the parameter vector can be solved by maximizing the LF which gives the ML estimator. For recursive estimation, less emphasis will be paid to distance samples and hence one can assume that the noise variance increases exponentially for samples with increasing delays from the current one. Consequently, one can define a "local" LF as

$$p(\mathbf{x}'; \boldsymbol{\theta}) = \frac{1}{(2\pi\sigma_{\eta}^{2})^{(n-p)/2}} \exp\left(-\sum_{i=p}^{n} \frac{\lambda_{n-i}(n)e^{2}(i)}{2\sigma_{\eta}^{2}}\right)$$
(4)

where $e(i)=x(i)-a^{T}(i)x(i)$, $\lambda_{n-i}(n)=\lambda(n)\lambda_{n-i-1}(n-1)$ with $\lambda_0(n)=1$ and $\lambda(n)$ is the FF used at the time index *n*, which satisfies $0 < \lambda(n) < 1$. Note, maximizing (3) gives the least squares estimator while maximizing (4) gives the recursive least squares estimator with an exponential window. To differentiate the two solutions to the problem, the latter will be denoted by *w* in the subsequent discussion. For the hypothesis test in (2), the Wald test statistic reads

$$T(\boldsymbol{x}, \boldsymbol{w}_0) = (\boldsymbol{w} - \boldsymbol{w}_0)^T \boldsymbol{B}^{-1}(\boldsymbol{w})(\boldsymbol{w} - \boldsymbol{w}_0)$$
(5)

where B(w) is the Cramer-Rao bound (CRB) at the true parameter w or some estimate of it, whereas w_0 is the candidate vector to be tested in the hypothesis. For notational convenience, we shall drop the dependence of T(.)on w_0 . To proceed further, we therefore need to determine the CRB and w. First, the CRB of any unbiased estimator has the simplified form of the inverse of the Fisher information matrix (FIM), whose (l,m)-th entry is given by

$$\begin{bmatrix} \boldsymbol{I}(\boldsymbol{w}) \end{bmatrix}_{lm} = -E \begin{bmatrix} \frac{\partial^2 \ln p(\boldsymbol{x}'; \boldsymbol{\theta})}{\partial w_l \partial w_m} \end{bmatrix}$$
$$= E \begin{bmatrix} \sum_{i=p}^n \lambda_{n-i}(n) x(i-l) x(i-m) / \sigma_{\eta}^2 \end{bmatrix}$$
$$= \left(\sigma_{\eta}^{-2} r_x(l-m) \right) \sum_{i=p}^n \lambda_{n-i}(n)$$
(6)

where $r_x(k)$ is the autocorrelation at lag k of $\{x(n)\}$.

To determine w, we need to maximize the local LF which can be obtained by finding the zero of the Fisher score vector

$$\mathbf{s}(\mathbf{w}) = \frac{\partial \ln p(\mathbf{x}'; \boldsymbol{\theta})}{\partial \mathbf{w}} = \left[\frac{\partial \ln p(\mathbf{x}'; \boldsymbol{\theta})}{\partial w_1} \cdots \frac{\partial \ln p(\mathbf{x}'; \boldsymbol{\theta})}{\partial w_p}\right]^{t} \qquad (7)$$
$$= \sigma_{\eta}^{-2} \sum_{i=p}^{n} \lambda_{n-i}(n) e(i) \mathbf{x}(i)$$

which gives the normal equation for RLS algorithm

$$\boldsymbol{R}_{XX}(n)\boldsymbol{w}_{opt}(n) = \boldsymbol{p}_{X}(n)$$
(8)

where $w_{opt}(n)$ is estimate of w, $R_{XX}(n) = \sum_{i=0}^{n} \lambda_{n-i}(n) x(i) x^{T}(i)$ and $p_{X}(n) = \sum_{i=0}^{n} \lambda_{n-i}(n) x(i) x(i)$ are the autocorrelation matrix of x(n) and the crosscorrelation vector of x(n) and x(n), respectively. We will propose in Section 3 an adaptive filter algorithm to solve (8). Assuming w has been estimated, we will determine the difference between w and w_0 and further simplify the Wald test in (5). At $\theta = w$, we have from the mean value theorem that

$$\boldsymbol{\theta} \approx \boldsymbol{s}(\boldsymbol{w}) = \boldsymbol{s}(\boldsymbol{w}_0) + \boldsymbol{I}(\boldsymbol{\xi})(\boldsymbol{w} - \boldsymbol{w}_0) \tag{9}$$

for some point $\xi \in L$, the interval joining w and w_0 . If w_0 is sufficiently close to w, we can approximate $I(\xi)$ by I(w), due to the continuity property at I(w) under reasonable regularity assumption. Consequently, one gets $w - w_0 = -\Gamma^1(w)$ $s(w_0)$ and

$$T(\mathbf{x}) \approx \left(\sigma_{\eta}^{-2} \sum_{i=p}^{n} \lambda_{n-i}(n)\right)^{-1} [\mathbf{s}^{T}(\mathbf{w}_{0}) \mathbf{R}_{x}^{-1} \mathbf{s}(\mathbf{w}_{0})]$$
(10)

where \mathbf{R}_x is the autocorrelation matrix of $\{x(n)\}$, $s(w_0) = \sigma_{\eta}^{-2} \sum_{i=p}^{n} \lambda_{n-i}(n) e_0(i) \mathbf{x}(i)$ and $e_0(i) = x(i) - w_0^T \mathbf{x}(i)$. Eqn. (10) can further be simplified by using the normal equation to obtain

$$\sum_{i=p}^{n} \lambda^{n-i} \mathbf{x}(i) (\mathbf{x}(i) - \mathbf{x}^{T}(i) \mathbf{w}_{0}) = \hat{\mathbf{R}}_{\mathbf{x}} (\hat{\mathbf{w}} - \mathbf{w}_{0}),$$

where $\hat{\boldsymbol{R}}_{x} = \sum_{i=p}^{n} \lambda^{n-i} \boldsymbol{x}^{T}(i) \boldsymbol{x}(i) \approx \frac{1-\lambda^{n-p+1}}{1-\lambda} \boldsymbol{R}_{x} \approx \frac{1}{1-\lambda} \boldsymbol{R}_{x}$, and $\hat{\boldsymbol{w}} = \hat{\boldsymbol{R}}_{x}^{-1} \sum_{i=n}^{n} \lambda^{n-i} \boldsymbol{x}(i) \boldsymbol{x}(i)$ is the RLS estimate. Thus,

$$\boldsymbol{s}^{T}(\boldsymbol{w}_{0})\hat{\boldsymbol{R}}_{x}^{-1} = \sigma_{\eta}^{-2} \left(\sum_{i=p}^{n} \lambda^{n-i} e_{0}(i) \boldsymbol{x}^{T}(i) \right) \hat{\boldsymbol{R}}_{x}^{-1} = \sigma_{\eta}^{-2} (\hat{\boldsymbol{w}} - \boldsymbol{w}_{0})^{T}.$$

Substituting it into (10) gives the simplified test that we should reject the hypothesis of stationarity if

$$T(\boldsymbol{x}) \approx \sigma_{\eta}^{2} [\boldsymbol{s}^{T}(\boldsymbol{w}_{0}) \hat{\boldsymbol{R}}_{x}^{-1} \boldsymbol{s}(\boldsymbol{w}_{0})] = (\hat{\boldsymbol{w}} - \boldsymbol{w}_{0})^{T} \boldsymbol{s}(\boldsymbol{w}_{0}) > \gamma \qquad (11)$$

where γ is chosen to maintain a constant false alarm (CFAR). To make use of the above test for checking stationarity recursively, we can compute the TVAR estimate using a SR-VFF-QRRLS algorithm (to be described in Section 3) at the current time instant n, \hat{w} , and then apply the test (11) to a previous estimate w_0 , say at time instant $(n-N_0)$. Note, the original test assumes that the origin is at time n=0. To use it for the current setting, the Fisher score vector at w_0 will be modified to $s(w_0) = \sigma_\eta^{-2} \sum_{i=n-N_0}^n \lambda_{n-i}(n)e_0(i)\mathbf{x}(i)$, i.e. we only compare the two candidates using the data samples inside a window of size N_0 , whereas the two estimates are obtained by all the samples up to their respective time instants. The new $s(w_0)$ can then be computed recursively starting at time instant $(n-N_0)$ with O(p) complexity. The required exciting variance σ_η^2 can be approximated from the residual of the algorithm, i.e. $\sigma_\eta^2 \approx \sigma_\eta^2(n) = \lambda(n)\sigma_\eta^2(n) + (1-\lambda(n))e^2(n)$, which again can be computed recursively. The choice of N_0 depends on the scale of changes that we wish to detect. A

depends on the scale of changes that we wish to detect. A small N_0 means we wish to detect fast varying changes in the signal under consideration, such as abrupt jumps. On the other hand, a large N_0 tends to detect slow variations in the process.

The performance of the Wald test above can be found asymptotically to have a central chi-squared distribution of order p under H₀ or

$$T(\mathbf{x}) \sim \chi_p^2(0) \,. \tag{12}$$

Under H1, it has a noncentral chi-squared distribution or

$$T(\mathbf{x}) \sim \chi_p^2(\boldsymbol{\phi}) ,$$
 (13)

where the noncentrality parameter is given by

$$\phi \approx \frac{1}{\sigma_{\eta}^{2}(1-\lambda)} \left[\hat{\boldsymbol{w}} - \boldsymbol{w}_{0} \right]^{T} \boldsymbol{R}_{x} \left[\hat{\boldsymbol{w}} - \boldsymbol{w}_{0} \right].$$
(14)

3. THE SR-VFF-QRRLS ALGORITHM

In this section, we propose to solve (8) using the SR-VFF-QRRLS algorithm [10]. Let the weight vector of the adaptive filter be $w(n) = [w_1(n), ..., w_p(n)]^T$. Eqn. (8) can be efficiently implemented using a QRD-based algorithm as summarized in Table I with the first update only. The following method is used to update the FF

$$\lambda(n) = 1 - 1/L(n) \tag{15}$$

where $L(n) = round\{L_{L} + [1 - g(\overline{G}_{N}(n))](L_{U} - L_{L})\}$ is the exponential window size with a lower and upper bound of L_{L} and L_{U} , $g(x)=\min\{x,1\}$ is a clipping function which keeps its positive argument x within the interval [0, 1], and $\overline{G}_{N}(n) = \sigma_{xe}^{2}(n)/\overline{\sigma}_{0}^{2}$ with $\overline{\sigma}_{0}^{2}$ the average of the first T_{s0} -th estimates of $\sigma_{xe}^{2}(n)$ at the beginning of adaptation and

$$\sigma_{xe}^{2}(n) = \lambda_{e}\sigma_{xe}^{2}(n-1) + (1-\lambda_{e})\boldsymbol{x}_{e}^{T}(n)\boldsymbol{x}_{e}(n)$$
(16)

where λ_e is a FF and $x_e(n)$ is averaged from x(n)e(n) over a time window of length T_{xe} so as to suppress the effect of background noises on $\sigma_{xe}^2(n)$, which is a measure of the convergence status [10].

To address the ill-conditioned problem in $R_{XX}(n)$, a state



where $Q^{(1)}(n)$ and Q(n) are calculated by Givens rotation to obtain the left hand side of each equation above and $\mu(n) = \kappa(n)p$. For the QRRLS algorithm, $\mathbf{R}^{(1)}(n) = \mathbf{R}(n)$ (ii). $w(n) = \mathbf{R}^{-1}(n)u(n)$ (back-substitution).

regularization is imposed and the solution, instead of (8), will be modified to

$$(\boldsymbol{R}_{XX}(n) + \kappa(n)\boldsymbol{I})\boldsymbol{w}(n) = \boldsymbol{p}_{X}(n) + \kappa(n)\boldsymbol{w}(n)$$
(17)

where $\kappa(n)$ is a regularization parameter used to balance between bias and variance and I is a positive definite matrix. It can be seen that the optimal solution to (17) is identical to that of (8) and therefore, (17) is unbiased and depends on w(n). To iteratively solve (17), w(n) on the right hand side is approximated by w(n-1). Hence, the algorithm is asymptotically unbiased. According to [10] $\kappa(n)$ is given by

$$\kappa(n) = \overline{\sigma}_x^2 \lambda^{-1}(n) \sqrt{1 - \lambda(n)} \sqrt{\beta(\sigma_\eta^2 / \sigma_x^2(n))} / \| \boldsymbol{w}_r \|_2^2 \qquad (18)$$

where $\beta = 1/(\frac{1}{p}(2 + \frac{(1-\lambda(n))p}{\lambda(n)}) + \frac{(1-\lambda(n))p}{\lambda^2(n)})$, $\overline{\sigma}_x^2$ is the averaged input power over the whole duration while $\sigma_x^2(n)$ is the short term averaged input power estimated by using a FF, and $|| \mathbf{w}_r ||_2^2$ is the norm of the system channel which is usually assumed to be known a priori. The update of the regularization in (17) can be realized successively using another QRD. In particular, the QRD is executed once for the data vector $[\mathbf{x}^T(n), \mathbf{x}(n)]$ and once for the regularization vector $[\sqrt{\mu(n)}\mathbf{d}_1, \sqrt{\mu(n)}w_1(n-1)]$ at each time instant, where \mathbf{d}_l is the *l*-th row of \mathbf{I} and $\mu(n) = \kappa(n)p$. If the vector is sequentially applied, $l = (n \mod p) + 1$. This yields the SR-VFF-QRRLS algorithm, as shown in Table I.

4. SIMULATION RESULTS

We now apply the proposed recursive Wald test to the real speech data and evaluate its performance in detecting vocal tract variation. In the experiments, the waveform used is a vowel [a] (as in "father") followed by [ai] (as in "life"). It was downsampled to 1 kHz in order to focus on lower frequency formants. In the first experiment, we would like to detect the vocal tract change once it happens. In this case, w is the TVAR estimate at current time n whereas w_0 is the TVAR estimate at the last detected change point, say at n_0 and (11) will be used to test from the current estimate w to a point that corresponds to a new change. If so, n_0 will be set to the current time instant n and repeat the above process,

otherwise, it will proceed to the next time instant. Thus, N_0 is variable in this case. The proposed method is compared with the GLRT in [8]. According to the rule-of-thumb that "2 coefficients per kHz", a 2-order model is applied to both algorithms. According to the parameter selection in [8], a power series with q=4 is used for the TVAR modeling in the GLRT and the length of the basic segment is 16ms. For the proposed Wald test, the following parameters are suggested so as to achieve satisfactory performance under a wide range of conditions: a short window length $T_{xe}=20$ and $\lambda_e=0.99$ are used to achieve a quick response for FFs when the signal changes rapidly; a longer window $T_{s0}=50$ is used to estimate a reliable reference for the convergence status $\overline{\sigma}_0^2$. L_L and L_U are chosen as 3 and 10, respectively, so that the minimum and maximum FFs are around 0.7 and 0.9.

The performance of the two detection methods is compared in Fig. 1 with a CFAR of 1%. In contrast to the satisfactory segmentation results obtained by the Wald test, the GLRT results indicate that there are certain detection latencies especially at the beginning of each vowel and the segment duration at around 1.2 s is too short for further analysis. All of these are mainly due to the need of batch processing. In addition, large variation of T(x) is observed in GLRT results due to insufficient excitation at three different occasions (marked as squares in Fig. 1), which leads to FAs. On the other hand, the proposed method can combat these ill-conditioned problems by employing state regularization. Also shown as dashed lines in Fig. 1 are the continuous frequency tracks obtained from (upper) the segmented data using GLRT and (lower) SR-VFF-QRRLS. It shows the flexibility of the proposed approach in recursively tracking the desired parameters as well as providing a nonstationarity detection.

In the second experiment, we would like to detect the nonstationary at larger scales, which may help in segmenting large scale features in the signal. This is achieved by using a constant N_0 . Other settings are the same as above. Failure to reject (11) implies that the signal is stationary under a certain scale defined by N_0 . The black solid lines in Fig. 2 show the nonstationarity detected by the proposed approach, respectively, for window sizes of $N_0=10$ and 20. In order to indicate the changes of original spectrum on the same frequency scale, these lines are obtained by setting 300 Hz (200 Hz) when $T(\mathbf{x})$ is larger (smaller) than the threshold γ , and then smoothing with a median filter of window size N_0 . We can see that different N_0 's help to isolate changes at different scales, which are useful for automatic segmentation.

5. CONCLUSION

A new recursive Wald test has been presented for detecting the nonstationarity of signals. It is based on the TVAR model and a LLF, which can be implemented recursively in a QRRLS-type estimation algorithm with efficiency and low computational cost.

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Fig. 1. Change detection results for GLRT (16ms segment duration) and the proposed recursive Wald test.



Fig. 2. Nonstationarity duration at different scales: N₀=10 and 20.