



Title	Application of non-linear programming for large-scale AC-DC power flow analysis
Author(s)	Qin, Z; Hou, Y
Citation	The 2012 IEEE Power and Energy Society General Meeting, San Diego, CA., 22-26 July 2012, p. 1-8
Issued Date	2012
URL	http://hdl.handle.net/10722/160241
Rights	IEEE Power and Energy Society General Meeting Proceedings. Copyright © I E E E.

Application of Non-linear Programming for Large-Scale AC-DC Power Flow Analysis

Zhijun Qin, *Student Member, IEEE*, and Yunhe Hou, *Member, IEEE*

Abstract—This paper proposes a robust, non-divergent model for large-scale AC-DC power flow analysis. By introducing relaxation variables into power flow equations, the conventional power flow model is turned into a non-linear programming model to minimize the L_2 norm of the relaxation variables subject to operation constraints and DC station control modes. The interior point method (IPM) is used to solve the model. Due to the special structure of the proposed model, the size of coefficient of correction equations in the IPM can be reduced to the same as Jacobian in the conventional Newton method. Further, the coefficient is symmetrical positive-definite with a diagonal perturbation, which leads to a better convergence than Newton direction. The model and algorithm proposed also have the following features: (1) no special initial value is needed; (2) strong robustness for power grids with plenty of small/negative impedance; (3) applicable for searching a desirable operation point by appropriate settings of inequality constraints; (4) striving for ultimate close to power flow solution avoiding to stop too early at a local minimizer. Numerical simulations on a 6527 buses power grid in China with 8 HVDC lines show that the proposed model has robust convergent property in the feasible region, around the boundary of the feasible region and even beyond the feasible region of power flow.

Index Terms— AC-DC transmission system; power flow analysis, Newton-Raphson method, optimal multiplier, minimization technology, interior point method

I. INTRODUCTION

A. Motivation

The power flow analysis is a basic tool for operation and planning for power grids. Due to the good convergence property, high efficiency both in memory storage and calculation speed, the Newton-Raphson (N-R) method is popularly used in power flow study both in AC power grids [1] and AC-DC power grids [2-4]. However, N-R method will not converge in many practical situations due to the following reasons [5, 6]: a) special features of the network such as small/negative impedance branches, radial distribution network, etc., b) inappropriate initial voltage value, c) numerical difficulties when the operation point is close to

operation limit or voltage collapse point, d) no solution exists within the feasible region.

In this paper, we try to propose a solution for AC-DC power flow analysis with the following desirable properties:

- 1) Robust to cover all the divergent problems mentioned above. Provide information to identify the main reason for divergence.
- 2) Iterations will not diverge whether the power flow solution exists or not. Converge to the power flow solution, if exists, otherwise provide accurate measurement to explicitly locate the network bottleneck or the inappropriate load settings.
- 3) Efficiently implemented without notable drop in performance compared to N-R method.

B. Related Work

Much research has been done to solve the divergent problems based on N-R method. Methods in [7,8] were proposed to deal with problems a) and b).

For c) and d), conventional N-R method will diverge and less indication of the problem is provided after iterations fail. There are mainly three categories of approaches to make the iteration converge.

1) N-R method with optimal multiplier. The optimal multiplier is obtained by a subroutine to minimize a quadratic function based on the sum of squares of mismatch of power flow equation. The voltage correction is scaled by the optimal multiplier during iterations. N-R methods with optimal multiplier in rectangular and polar coordinates are proposed and discussed in [9-13]. Since these methods are based on N-R method, appropriate initial values are needed. Moreover, in the polar coordinate power flow, or in the AC-DC power flow, the object function is an approximation to quadratic. Hence the iterations may stop too early without helpful information. This phenomenon will be demonstrated later in section V.

2) Parameterized continuation power flow [14-18] for problem c) and especially for the computation of the points of collapse. These algorithms start from a feasible operation point to search for the maximum load level scalar. Parameterized algorithms are difficult to identify the feasibility for one certain operation point.

3) Non-linear programming model, which makes relaxation to power flow equations and converts power flow calculation into minimization of L_2 norm of relaxation variables [19,20], or minimization of L_1 norm of relaxation variables[21]. When the solution exists, the object function will drop close to zero, otherwise, the object function will stop at a positive value.

This work is partly supported by the Research Funding of EPRI (EP-P35571/C16133 , EP-P35424/C16059) , Research Grant Council, Hong Kong SAR (RGC-HKU 7124/10E, RGC-HKU 7124/11E) , National Key Research Program(973 Program, 2012CB215102)

Zhijun Qin and Yunhe Hou are with the Electrical and Electronic Engineering Department, University of Hong Kong, Hong Kong, China. (e-mail: zjqin@eee.hku.hk; yhhou@eee.hku.hk)

Since the non-linear models change the structure of power flow equations, the implementation is more complicated and less efficient compared to N-R methods. As a result, N-R methods have been prevailing for conventional power flow analysis. However, utilities' practice brings more challenges to power flow analysis. At present, China has been striving to construct a strong and extremely huge power grid to fulfill the bulk requirement of electric energy for sustainable development. North China Grid, East China Grid and Central China Grid will be inter-connected into a complicated AC-DC hybrid power grid with multi-infeed DC systems. Robust and efficient power flow analysis tool for large-scale AC-DC power grid has attracted more attention both in operation stage and planning stage.

C. Paper Work and Organization

In this paper, we propose a non-linear programming model for AC-DC power flow analysis with robustness and non-divergent property. This model is an extension of the models proposed in [19, 20] and accounts for the operating constraints by including inequality constraints. Interior point method (IPM) is used to solve this model.

Although a group of relaxation variables are explicitly introduced to the power flow equations, the L_2 norm in the objective function plays a significant role in reducing the size of correction equation and in changing the search direction. The proposed model and algorithm have the following features:

- 1) Strong convergence can be achieved around the boundary of feasible region and beyond the feasible region, within a stable number of iterations.
- 2) No special voltage initial value is needed.
- 3) For power grids with plenty of small or negative impedance branches, the convergent property of this algorithm is preponderate over the Newton direction.
- 4) The size of correction equation can be reduced to the same as Jacobian in conventional N-R method. The symmetrical positive-definite coefficient of the reduced correction equations has the same non-zeros as that of Jacobian in N-R method. This feature makes the correction equations be solved without notable drop in computational performance.

The remaining parts of the paper are organized as follows. With the notation in section II, the non-linear model of AC-DC power flow analysis is proposed in section III. IPM is used to solve the model in section IV with the deduction of the reduction process for the size of correction equations. Numerical results in section V demonstrate the effectiveness and efficiency of the proposed method. The conclusion is drawn in section VI.

II. LIST OF SYMBOLS AND NOTATIONS

V : voltage magnitudes of AC buses.
 δ : voltage angles of AC buses.
 N : number of AC buses.
 N_d : number of DC stations.
 $\dot{V} = V \angle \delta$: complex voltages of AC buses.
 P_G, Q_G : nodal active and reactive injections respectively.

P_L, Q_L : nodal active and reactive power demands respectively.
 V_d : direct voltages of DC stations.
 I_d : direct currents of DC stations.
 K_d : tap positions of converter transformers in DC stations.
 $A_d = \cos(\theta_d)$: cosine value of the control angles (ignition angle for rectifier or extinction angle for inverter) of DC Stations.
 φ_d : power factor angles of DC stations.
 S_d : type of DC stations, "+1" stands for a rectifier while "-1" stands for an inverter.
 X_c : converters' equivalent commutation reactance.
 Ω_{PV} : AC buses set of PV control mode.
 Ω_{PQ} : AC buses set of PQ control mode.
 Ω_{Slack} : slack buses set.
 $\Omega_{destination}$: AC buses set with DC Station connected.
 Ω_{CP} : DC stations set of constant active power mode.
 Ω_{CV} : DC stations set of constant voltage mode.
 Ω_{CI} : DC stations set of constant current mode.
 Ω_{CA} : DC stations set of constant control angles mode.
 Ω_{CK} : DC stations set of constant tap position mode.
 $X_{ac} = (V^T, \delta^T, P_G^T, Q_G^T)^T$: variables of AC network.
 $X_{dc} = (V_d^T, I_d^T, K_d^T, \cos\theta_d^T, \varphi_d^T)^T$: variables of DC network.
 $\varepsilon = (\varepsilon_p^T, \varepsilon_Q^T, \varepsilon_{dc1}^T, \varepsilon_{dc2}^T, \varepsilon_{dc3}^T)^T$: relaxation variables of AC-DC power flow equations.
 $X = (X_{ac}^T, X_{dc}^T)^T$: total variables of power flow calculation.
 Y : admittance matrix of AC network.
 G_d : conductance matrix of DC network.
 $\text{Re}(\bullet)$: retrieve the real part of a complex vector.
 $\text{Im}(\bullet)$: retrieve the imaginary part of a complex vector.
 $[\bullet]$: converts vector into a diagonal matrix.
 $\text{sparse}(i, j, s, m, n)$: uses the rows of $[i, j, s]$ to generate an m-by-n sparse matrix. Any elements of s which have duplicate values of i and j are added together. It is an important function in Matlab although not defined in linear algebra. The semantics of this function is utilized to describe formulas in this article.
 $x(a)$: retrieve part of vector x assigned by integer vector a .
 $\cdot \times$: element-wise multiplication between vectors or between matrices.
 $\overline{(\bullet)}, \underline{(\bullet)}$: lower and upper bound of variables.

III. NON-LINEAR PROGRAM MODEL FOR AC-DC POWER FLOW

A. Model

$$\min f(X, \varepsilon) = \varepsilon^T \varepsilon \quad (1)$$

Subject to:

1) Equality constraints include AC power flow equations, balance equations of DC stations and nodal current equations of DC network as below:

$$\mathbf{P}_G - \mathbf{P}_L - \text{Re}(\dot{\mathbf{V}} \times (\hat{\mathbf{Y}}\hat{\mathbf{V}})) - \mathbf{P}_d - \boldsymbol{\varepsilon}_p = \mathbf{0} \quad (2)$$

$$\mathbf{Q}_G - \mathbf{Q}_L - \text{Im}(\dot{\mathbf{V}} \times (\hat{\mathbf{Y}}\hat{\mathbf{V}})) - \mathbf{Q}_d - \boldsymbol{\varepsilon}_q = \mathbf{0} \quad (3)$$

$$\mathbf{V}_d - \mathbf{K}_d \times \mathbf{V}(\boldsymbol{\Omega}_{\text{destination}}) \times \mathbf{A}_d + \mathbf{X}_c \times \mathbf{I}_d - \boldsymbol{\varepsilon}_{d1} = \mathbf{0} \quad (4)$$

$$\mathbf{V}_d - 0.995 \times \mathbf{K}_d \times \mathbf{V}(\boldsymbol{\Omega}_{\text{destination}}) \times \cos(\varphi_d) - \boldsymbol{\varepsilon}_{d2} = \mathbf{0} \quad (5)$$

$$\mathbf{S}_d \times \mathbf{I}_d - \mathbf{G}_d \mathbf{V}_d - \boldsymbol{\varepsilon}_{d3} = \mathbf{0} \quad (6)$$

\mathbf{P}_d and \mathbf{Q}_d in (2)~(3) are powers of DC stations with non-zeros in the position associated with the AC buses connected by DC stations. In a multi-infeed DC system, multiple DC stations may connect to one AC bus. For example, if i -th DC station and j -th DC station connect to k -th AC buses, we have:

$$\mathbf{P}_d[k] = \mathbf{P}_d[k] + \mathbf{S}_d[i] \times \mathbf{V}_d[i] \times \mathbf{I}_d[i] \quad \text{and}$$

$$\mathbf{P}_d[k] = \mathbf{P}_d[k] + \mathbf{S}_d[j] \times \mathbf{V}_d[j] \times \mathbf{I}_d[j], \text{ which can be denoted as:}$$

$$\mathbf{P}_d = \text{sparse}(\boldsymbol{\Omega}_{\text{destination}}, 1, \mathbf{S}_d \times \mathbf{V}_d \times \mathbf{I}_d, N, 1) \quad (7)$$

$$\mathbf{Q}_d = \text{sparse}(\boldsymbol{\Omega}_{\text{destination}}, 1, \mathbf{S}_d \times \mathbf{V}_d \times \mathbf{I}_d \times \tan(\varphi_d), N, 1) \quad (8)$$

2) Inequality constraints include bus voltage level, active power limits of generators, VAR limits of generators and shunts, and DC station operational constraints.

$$\underline{\mathbf{V}} \leq \mathbf{V} \leq \overline{\mathbf{V}} \quad (9)$$

$$\underline{\mathbf{P}}_G \leq \mathbf{P}_G \leq \overline{\mathbf{P}}_G \quad (10)$$

$$\underline{\mathbf{Q}}_G \leq \mathbf{Q}_G \leq \overline{\mathbf{Q}}_G \quad (11)$$

$$\underline{\mathbf{P}}_d \leq \mathbf{V}_d \times \mathbf{I}_d \leq \overline{\mathbf{P}}_d \quad (12)$$

$$\underline{\mathbf{V}}_d \leq \mathbf{V}_d \leq \overline{\mathbf{V}}_d \quad (13)$$

$$\underline{\mathbf{I}}_d \leq \mathbf{I}_d \leq \overline{\mathbf{I}}_d \quad (14)$$

$$\underline{\mathbf{K}}_d \leq \mathbf{K}_d \leq \overline{\mathbf{K}}_d \quad (15)$$

$$\underline{\mathbf{A}}_d \leq \mathbf{A}_d \leq \overline{\mathbf{A}}_d \quad (16)$$

$$\underline{\varphi}_d \leq \varphi_d \leq \overline{\varphi}_d \quad (17)$$

B. Upper and Lower Bounds for Power Flow Compatibility and Operation Constraints

To make the model compatible of power flow calculation, appropriate upper and lower bounds should be set to reflect the control mode of DC stations and generators. For those constant variables, the related upper bound and lower bound are set to the same value. For other variables, the related upper bound and lower bound are set to account for operational constraints.

1) Constant current mode DC stations

$$\underline{\mathbf{I}}_d(\boldsymbol{\Omega}_{\text{CI}}) = \overline{\mathbf{I}}_d(\boldsymbol{\Omega}_{\text{CI}})$$

2) Constant voltage mode DC stations

$$\underline{\mathbf{V}}_d(\boldsymbol{\Omega}_{\text{CV}}) = \overline{\mathbf{V}}_d(\boldsymbol{\Omega}_{\text{CV}})$$

3) Constant active power mode DC stations

$$\underline{\mathbf{P}}_d(\boldsymbol{\Omega}_{\text{CP}}) = \overline{\mathbf{P}}_d(\boldsymbol{\Omega}_{\text{CP}})$$

4) Constant control angle mode DC stations

$$\underline{\mathbf{A}}_d(\boldsymbol{\Omega}_{\text{CA}}) = \overline{\mathbf{A}}_d(\boldsymbol{\Omega}_{\text{CA}})$$

5) Constant tap position mode DC stations

$$\underline{\mathbf{K}}_d(\boldsymbol{\Omega}_{\text{CK}}) = \overline{\mathbf{K}}_d(\boldsymbol{\Omega}_{\text{CK}})$$

6) PV buses

$$\underline{\mathbf{V}}(\boldsymbol{\Omega}_{\text{PV}}) = \overline{\mathbf{V}}(\boldsymbol{\Omega}_{\text{PV}})$$

7) PV generators

$$\underline{\mathbf{P}}_G(\boldsymbol{\Omega}_{\text{PV}}) = \overline{\mathbf{P}}_G(\boldsymbol{\Omega}_{\text{PV}})$$

IV. INTERIOR POINT METHOD FOR PROPOSED MODEL

The model in section III can be simplified and expressed in a general and concise form as following:

$$\begin{aligned} & \min \boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon} \\ & \text{s.t.} \quad \mathbf{h}(\mathbf{X}) - \boldsymbol{\varepsilon} = \mathbf{0} \\ & \quad \quad \bar{\mathbf{g}} \leq \mathbf{g}(\mathbf{X}) \leq \underline{\mathbf{g}} \end{aligned} \quad (18)$$

The primal-dual interior point method framework applied on model (18) can be described as:

1) Slack variable vectors $(\mathbf{l}, \mathbf{u}) \geq 0$ are introduced hence inequality constraints are converted into:

$$\begin{cases} \mathbf{g}(\mathbf{X}) - \underline{\mathbf{g}} - \mathbf{l} = \mathbf{0} \\ \mathbf{g}(\mathbf{X}) - \bar{\mathbf{g}} + \mathbf{u} = \mathbf{0} \end{cases} \quad (19)$$

2) Lagrangian function is defined as:

$$\begin{aligned} \mathbf{L}(\mathbf{X}, \mathbf{l}, \mathbf{u}, \mathbf{y}, \mathbf{z}, \mathbf{w}, \boldsymbol{\varepsilon}) \equiv & \boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon} - \mathbf{y}^T (\mathbf{h}(\mathbf{X}) - \boldsymbol{\varepsilon}) - \mathbf{z}^T (\mathbf{g}(\mathbf{X}) - \mathbf{l} - \underline{\mathbf{g}}) \\ & - \mathbf{w}^T (\mathbf{g}(\mathbf{X}) + \mathbf{u} - \bar{\mathbf{g}}) - \mu (\sum u_i + \sum l_i) \end{aligned} \quad (20)$$

where $(\mathbf{y}, \mathbf{z}, \mathbf{w})$ are Lagrangian multipliers.

3) KKT system with the Lagrangian function is:

$$\mathbf{L}_x = \nabla \mathbf{h}(\mathbf{X}) \mathbf{y} + \nabla \mathbf{g}(\mathbf{X}) (\mathbf{z} + \mathbf{w}) = \mathbf{0} \quad (21)$$

$$\mathbf{L}_z = \mathbf{g}(\mathbf{X}) - \mathbf{l} - \underline{\mathbf{g}} = \mathbf{0} \quad (22)$$

$$\mathbf{L}_w = \mathbf{g}(\mathbf{X}) + \mathbf{u} - \bar{\mathbf{g}} = \mathbf{0} \quad (23)$$

$$\mathbf{L}_l^\mu = [\mathbf{l}] [\mathbf{z}] \mathbf{e} - \mu \mathbf{e} = \mathbf{0} \quad (24)$$

$$\mathbf{L}_u^\mu = [\mathbf{u}] [\mathbf{w}] \mathbf{e} + \mu \mathbf{e} = \mathbf{0} \quad (25)$$

$$\mathbf{L}_\varepsilon = 2\boldsymbol{\varepsilon} + \mathbf{y} = \mathbf{0} \quad (26)$$

$$\mathbf{L}_y = \mathbf{h}(\mathbf{X}) - \boldsymbol{\varepsilon} = \mathbf{0} \quad (27)$$

Eliminate $\boldsymbol{\varepsilon}$ from (26) and (27), we can get:

$$\mathbf{L}_y = \mathbf{h}(\mathbf{X}) + 0.5 \mathbf{y} = \mathbf{0} \quad (28)$$

where $\mathbf{L}_l^\mu, \mathbf{L}_u^\mu$ are perturbed complementary conditions, μ is perturbed factor, $\mathbf{w} \leq 0$; $\mathbf{y} \neq 0$; $\mathbf{l} \geq 0$; $\mathbf{u} \geq 0$; $\mathbf{z} \geq 0$.

4) By applying Newton's method, the correction equations of the reduced system can be written as follows:

$$\begin{pmatrix} \mathbf{F} & \nabla \mathbf{h}(\mathbf{X}) & \nabla \mathbf{g}(\mathbf{X}) & \nabla \mathbf{g}(\mathbf{X}) & \mathbf{0} & \mathbf{0} \\ \nabla^T \mathbf{h}(\mathbf{X}) & 0.5[\mathbf{e}] & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \nabla^T \mathbf{g}(\mathbf{X}) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\ \nabla^T \mathbf{g}(\mathbf{X}) & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & [\mathbf{l}] & \mathbf{0} & [\mathbf{z}] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & [\mathbf{u}] & \mathbf{0} & [\mathbf{w}] \end{pmatrix} \begin{pmatrix} \Delta \mathbf{X} \\ \Delta \mathbf{y} \\ \Delta \mathbf{z} \\ \Delta \mathbf{w} \\ \Delta \mathbf{l} \\ \Delta \mathbf{u} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_{x0} \\ -\mathbf{L}_{y0} \\ -\mathbf{L}_{z0} \\ -\mathbf{L}_{w0} \\ -\mathbf{L}_{l0}^\mu \\ -\mathbf{L}_{u0}^\mu \end{pmatrix} \quad (29)$$

where:

$$\mathbf{F} = \nabla^2 \mathbf{h}(\mathbf{X}) \mathbf{y} + \nabla^2 \mathbf{g}(\mathbf{X}) (\mathbf{z} + \mathbf{w}) \quad (30)$$

and $(\mathbf{L}_{x0}, \mathbf{L}_{y0}, \mathbf{L}_{z0}, \mathbf{L}_{w0}, \mathbf{L}_{l0}^\mu, \mathbf{L}_{u0}^\mu)$ denotes the residuals of the perturbed KKT systems (21), (28), (22)~(25), respectively. As absolutely the same approach as in conventional IPM, the reduced form of (29) can be written in the following form as:

$$\begin{pmatrix} \mathbf{M} & \mathbf{J}^T(\mathbf{X}) \\ \mathbf{J}(\mathbf{X}) & 0.5[\mathbf{e}] \end{pmatrix} \begin{pmatrix} \Delta\mathbf{X} \\ \Delta\mathbf{y} \end{pmatrix} = - \begin{pmatrix} \boldsymbol{\psi} \\ \mathbf{L}_{y_0} \end{pmatrix} \quad (31)$$

where:

$$\mathbf{M} = \mathbf{F} + \nabla g(\mathbf{X})([\mathbf{u}]^{-1}[\mathbf{w}] - [\mathbf{I}]^{-1}[\mathbf{z}])\nabla^T g(\mathbf{X}) \quad (32)$$

$$\mathbf{J}(\mathbf{X}) = \nabla^T h(\mathbf{X}) \quad (33)$$

$$\boldsymbol{\psi} = \nabla h(\mathbf{X})\mathbf{y} + \nabla g(\mathbf{X})([\mathbf{u}]^{-1}[\mathbf{w}]\mathbf{L}_{w_0} - [\mathbf{I}]^{-1}[\mathbf{z}]\mathbf{L}_{z_0} - \mu([\mathbf{u}]^{-1} - [\mathbf{I}]^{-1})\mathbf{e}) \quad (34)$$

By investigating the structure of (31), we find that the coefficient matrix of (31) becomes symmetrical and positive-definite, which is different from conventional IPM with a constant diagonal perturbation. Further, from (31) we can have:

$$\Delta\mathbf{y} = -2\mathbf{L}_{y_0} - 2\mathbf{J}(\mathbf{X})\Delta\mathbf{X} \quad (35)$$

By substitution (35) into (31), we can have:

$$\mathbf{M}\Delta\mathbf{X} + \mathbf{J}^T(\mathbf{X})(-2\mathbf{L}_{y_0} - 2\mathbf{J}(\mathbf{X})\Delta\mathbf{X}) = -\boldsymbol{\psi} \quad (36)$$

So the reduced equation of the proposed model is:

$$(\mathbf{M} - 2\mathbf{J}^T(\mathbf{X})\mathbf{J}(\mathbf{X}))\Delta\mathbf{X} = -\boldsymbol{\psi} + 2\mathbf{J}^T(\mathbf{X})\mathbf{L}_{y_0} \quad (37)$$

After solving (37) and calculation on (35), get other variables' correction by:

$$\begin{cases} \Delta\mathbf{l} = \nabla^T g(\mathbf{X})\Delta\mathbf{X} + \mathbf{L}_{z_0} \\ \Delta\mathbf{u} = -(\nabla^T g(\mathbf{X})\Delta\mathbf{X} + \mathbf{L}_{w_0}) \\ \Delta\mathbf{z} = -[\mathbf{I}]^{-1}[\mathbf{z}]\nabla^T g(\mathbf{X})\Delta\mathbf{X} - [\mathbf{I}]^{-1}([\mathbf{z}]\mathbf{L}_{z_0} + \mathbf{L}_{z_0}^{\mu}) \\ \Delta\mathbf{w} = [\mathbf{u}]^{-1}[\mathbf{w}]\nabla^T g(\mathbf{X})\Delta\mathbf{X} + [\mathbf{u}]^{-1}([\mathbf{w}]\mathbf{L}_{w_0} - \mathbf{L}_{w_0}^{\mu}) \end{cases} \quad (38)$$

5) Calculate the step length in the search direction and update primal and dual variables ($\mathbf{X}, \mathbf{l}, \mathbf{u}, \mathbf{y}, \mathbf{z}, \mathbf{w}$), finally update $\boldsymbol{\varepsilon}$ by (26).

In the framework mentioned above, the main computational burden is to construct and solve (37), which has the same size of the correction equation in N-R method. Although, the linear combination of Hessian matrix of power flow equations and inequality constraints have to be figured out during iterations, its AC parts can be calculated efficiently using complex matrix operation as proposed in [22]. Since the DC equations and inequality constraints are much simpler than AC equations with more sparser feature in second derivatives, the number of total non-zeros in the coefficient of (37) is almost the same as that of AC-DC power flow equations. Due to the positive-definite property of coefficient matrix, (37) can be solved with a fast direct solve based on Cholesky factorization as in [23].

V. NUMERICAL RESULTS

A. Computational Efforts

The authors develop a C++ vectorization calculation library based on STL (Standard Template Library) to implement the basic linear algebra. The algorithm for Hessian computation proposed in [22] is implemented in C++ with high efficiency.

LU and Cholesky factorization modules are implemented using parts of UMFPAK in [24]. The efficiency of these modules is well confirmed in [19, 20, 25, 26].

All programs are compiled with Microsoft Visual Studio V.6.0. Tests were run on a PC (Intel-Pentium4 2.0GHz, 1GB RAM).

B. Introduction to Test System

The test system is part of a power grid in China. The basic active power level is 109.2GW and reactive load power 40.15GVar. The details of the system are shown in Table I.

Two sets of data of test system are provided to test the convergent property of the proposed algorithm. Data (A) is the original data, which keeps all small impedance and negative impedance branches. Data (B) uses bus merging to eliminate part of small/negative impedance branches. The number of small/negative impedance branches of both sets of data is shown in Table II.

TABLE I
NUMBER OF COMPONENTS IN MARCH 2010

Power System resources	Number
AC buses	6527
AC lines	2688
transformers	5416
compensators	1816
synchronous machines	635
energy consumers	1137
converters	16
DC lines	8

TABLE II
NUMBER OF SMALL/NEGATIVE IMPEDANCE BRANCHES

Impedance Range	Data (A)	Data (B)
(1E-4~1E-3)	287	102
(1E-4~1E-5)	349	0
(0~1E-5)	143	0
(-1E-3~0)	1315	921
(-1E-3~-1E-4)	4	0

C. Case Studies

Various algorithms are imposed on data (A), with a known feasible load level, to test convergent property on ill-condition network parameters. Algorithms used are: (I) N-R algorithm with flat start; (II) fast decouple algorithm with DC system; (III) N-R algorithm with optimal multiplier. Algorithm (I) and (III) solve the AC-DC power flow equations simultaneously. The convergent criterion is that the maximum power mismatch is less than 1E-5. The results are listed in Table III. It is shown that the proposed algorithm has the best convergent property among the tested algorithms. The iteration processes of the algorithms are shown in Fig. 1 to Fig. 4, respectively.

TABLE III
COMPARISON OF DIFFERENT ALGORITHMS ON DATA (A)

Algorithm	Convergent Property	Maximum Power Mismatch
Newton-Raphson with Flat Start	Diverge	N/A
Fast Decouple	Diverge	N/A
Newton-Raphson with Optimal Multiplier	Converge	4.361 pu
Proposed Algorithm	Converge	0.0669 pu

Moreover, the proposed algorithm helps to locate the real bottleneck of the network, with the maximum power

mismatch lying in a PQ node connecting with a positive small impedance branch and a negative small impedance branch. It means that a subroutine eliminating these branches will mitigate divergence. However, N-R algorithm with optimal multiplier points out the maximum power mismatch bus to a PQ bus near slack generator, which is helpless for analysts.

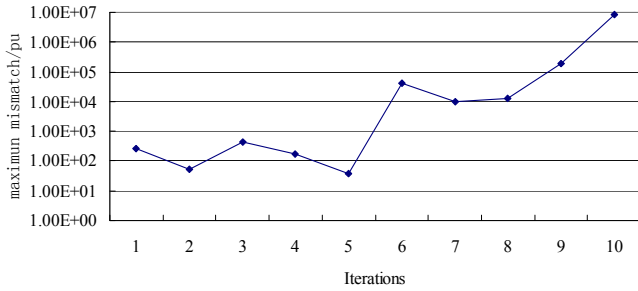


Fig. 1 Iteration process of Newton-Raphson method with flat start on data (A)

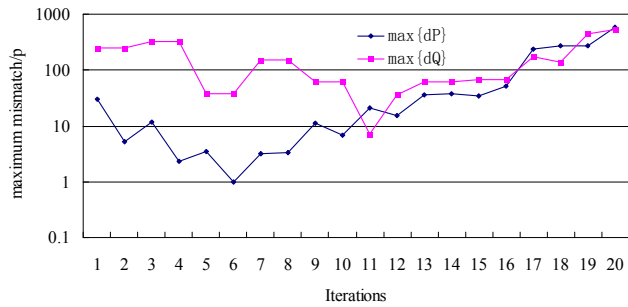


Fig. 2 Iteration process of fast decoupled method on data (A)

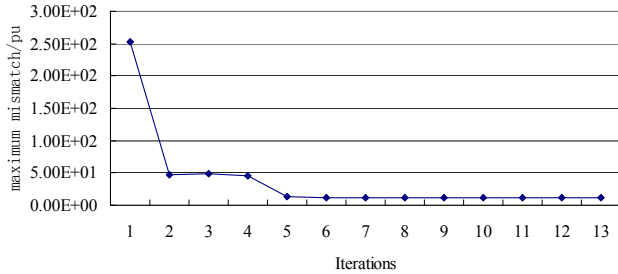


Fig. 3 Iteration process of Newton-Raphson method with optimal multiplier on data (A)

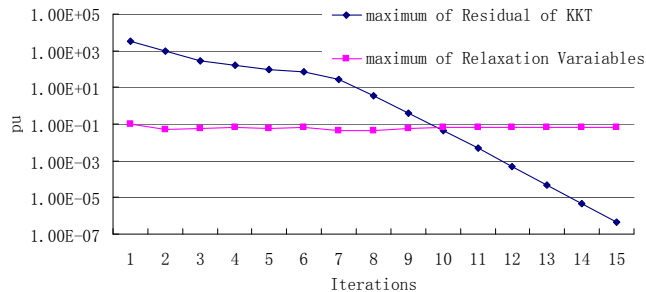


Fig. 4 Iteration process of proposed method on data (A)

Various algorithms are imposed on data (B) to test convergent property in the feasible region and beyond the feasible region with different load levels. The results are shown in Table IV. It is shown that the proposed algorithm can converge with different load levels, whether solutions exist or not. If the maximum of relaxation variables is less

than $1E-5$ after iteration converges, the power flow solution can be found; if not, there is no solution exists. The iterations need in the feasible region and beyond the feasible region are almost the same, as in Fig.5 and Fig.6.

The proposed algorithm helps to locate with the maximum power mismatch in some load bus. It means that cutting down this load by the power mismatch will mitigate divergence.

TABLE IV
COMPARISON OF DIFFERENT ALGORITHMS ON DATA (B)

Algorithm	Convergent Property in Feasible Region	Convergent Property beyond Feasible Region
Newton-Raphson with Flat Start	Diverge	Diverge
Fast Decouple Initiated Newton-Raphson	Converge	Diverge
Fast Decouple	Diverge	Diverge
Newton-Raphson with Optimal Multiplier	Converge	Diverge
Proposed Algorithm	Converge	Converge

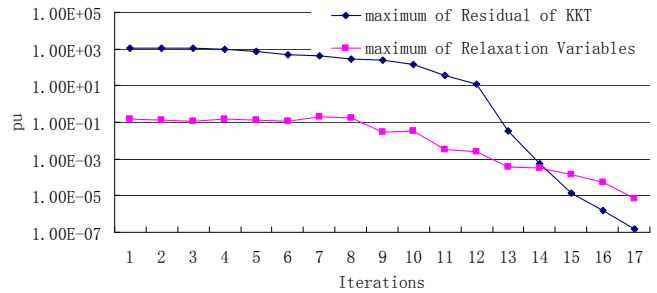


Fig. 5 Iteration process of proposed method when power flow solution exists on Data (B)

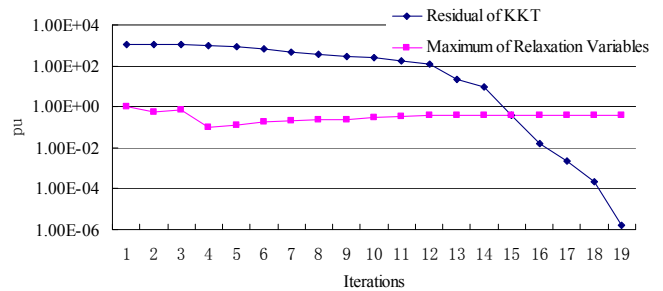


Fig. 6 Iteration process of proposed method when no power flow solution exists on Data (B)

D. Performance Statistics of Calculation

CPU time for different modules in each iteration of the proposed algorithm is listed in Table V. It shows that the computational burdens of first derivatives and second derivatives are quite equivalent with vectorization technology proposed in [22]. Cholesky factorization algorithm proposed in [23] is quite efficient for solving the correction equations. For the test system mention above, each iteration needs 0.33 second. The total CPU time needed (18 iterations or so) is 5-6 seconds, which is promising for operation/dispatch applications.

The conventional N-R algorithm on the same test systems is accomplished in 2 seconds with 6-7 iterations using LU

factorization proposed in [24]. LU factorization in [24] uses 0.14 second to solve N-R correction equations, which is nearly 10 times as that of Cholesky factorization. In other words, the computational burden to calculate Hessian is partially compensated by the symmetrical positive-definite property in correction equations.

TABLE V
CPU TIME FOR DIFFERENT MODULES

Module	CPU Time(s)
Calculation for (32)	0.158
Calculation for (33)	0.128
Construction of (37)	0.015
Solving for (37)	0.015
Correction of Variables	0.013

VI. CONCLUSIONS

This paper proposes a robust non-divergent power flow calculation algorithm based on the non-linear programming. The proposed method is applicable for large-scale AC-DC power grid. The algorithm has the following features: 1) the size of the correction equations is the same as that in conventional N-R method; 2) the number of total non-zeros in the coefficient of correction equations is almost the same as Jacobian of power flow equations; 3) the coefficient of correction equations is symmetrical positive-definite.

With vectorization implementation of construction of the correction equations and appropriate direct solver for symmetrical positive-definite linear systems, the algorithm can be accomplished within 3-4 times of CPU time as the conventional Newton-Raphson algorithm for the large-scale power flow analysis.

VII. APPENDIX A

Case data with a 14 buses AC/DC system is given to validate the correctness of the proposed algorithm.

TABLE VI
BUSES DATA OF 14 BUSES AC/DC TEST SYSTEM

	P_G	Q_G	P_D	Q_D
1	0.60	0.	0.	0.
2	0.65	0.424	0.217	0.127
3	0.	0.2339	0.942	0.190
4	0.	0.	0.478	-0.039
5	0.	0.	0.076	0.016
6	0.85	0.1224	0.112	0.075
7	0.	0.	0.	0.
8	0.	0.1736	0.	0.
9	0.	0.	0.295	0.166
10	0.	0.	0.09	0.058
11	0.	0.	0.035	0.018
12	0.	0.	0.061	0.016
13	0.	0.	0.135	0.058
14	0.	0.	0.149	0.05

TABLE VII
AC LINES DATA OF 14 BUSES AC/DC TEST SYSTEM

Start	End	R	X	B/2
1	2	0.01938	0.05917	0.0264
1	5	0.05403	0.22304	0.0246
2	3	0.04699	0.19797	0.0219
2	4	0.05811	0.17632	0.0170
2	5	0.05695	0.17388	0.0173
3	4	0.06701	0.17103	0.0064
6	11	0.09498	0.19890	0.0

6	12	0.12291	0.15581	0.0
6	13	0.06615	0.13027	0.0
7	8	0.0	0.17615	0.0
7	9	0.0	0.11001	0.0
9	10	0.03181	0.0845	0.0
12	13	0.22092	0.19988	0.0
13	14	0.17093	0.34802	0.0
14	9	0.12711	0.27038	0.0
10	11	0.08205	0.19207	0.0

TABLE VIII
TRANSFORMERS DATA OF 14 BUSES AC/DC TEST SYSTEM

Start	End	R	X	k
4	7	0.0	0.20912	0.978
4	9	0.0	0.55618	0.969
5	6	0.0	0.25202	0.932

TABLE IX
EARTH BRANCH DATA OF 14 BUSES AC/DC TEST SYSTEM

Bus	X
9	0.19

TABLE X
LOAD DATA OF 14 BUSES AC/DC TEST SYSTEM

Bus	P_D	Q_D
2	0.217	0.127
3	0.942	0.19
4	0.478	-0.039
5	0.076	0.016
6	0.112	0.075
9	0.295	0.166
10	0.09	0.058
11	0.035	0.018
12	0.061	0.016
13	0.135	0.058
14	0.149	0.05

TABLE XI
PV BUS OF 14 BUSES AC/DC TEST SYSTEM

Bus	V_{SET}
2	1.045
3	1.01
6	1.07
8	1.09

TABLE XII
BALANCE BUS OF 14 BUSES AC/DC TEST SYSTEM

Bus	V_{SET}
1	1.06

TABLE XIII
DC LINK OF 14 BUSES AC/DC TEST SYSTEM

Parameters	Converter Type	Bus Number	Leakage Reactance	DC Line Resistance
C1	Rectifier	5	0.126	0.00334
C2	Inverter	4	0.0728	

TABLE XIV
DC LINES CONTROL MODE DATA OF 14 AC/DC BUSES TEST SYSTEM

Parameters	Control Type	Vmax	Vmin	Pmax	Pmin
C1	Constant P	1.0	0.9	0.7	0.7
C2	Constant V	0.9384	0.9384	2	0

The output of the active and reactive sources is as follows.

TABLE XV
OUTPUT OF GENERATORS

Bus	P_G	Q_G
1	1.30821	-0.20386
2	0.65	0.51943

3		0.59129
6	0.85	0.02177
8		0.52973

The output of the active and reactive sources is as follows.

TABLE XVII
OUTPUT OF BUS VOLTAGE

BUS	MAGNITUDE	ANGLE (RAD)
1	1.06	0
2	1.045	-0.07085
3	1.01	-0.26210
4	0.96039	-0.24911
5	1.06744	-0.05054
6	1.07	-0.05012
7	1.00439	-0.23179
8	1.09	-0.23178
9	0.96283	-0.22250
10	0.96976	-0.19504
11	1.01105	-0.12254
12	1.05055	-0.06879
13	1.03304	-0.08468
14	0.97125	-0.17962

The residual of KKT and maximum power mismatch are showed in Fig. 7.

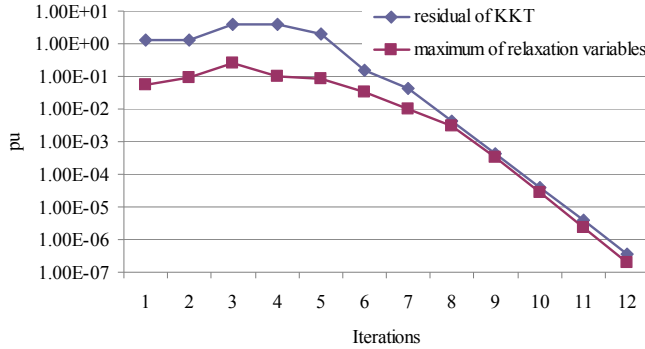


Fig. 7 Residual of KKT and Power Mismatch with Iterations

VIII. APPENDIX B

The first derivatives and second partial derivatives of AC power flow equation to AC variables are deduced in reference [19]. In the section, the gradient and hessian of AC-DC power flow equation using vector/matrix manipulation are deduced.

The power flow equation can be expressed as:

$$\Delta P = P_G - P_L - \text{Re}(\dot{V} \times (\hat{Y}\hat{V})) - P_d - \varepsilon_p = 0$$

$$\Delta Q = Q_G - Q_L - \text{Im}(\dot{V} \times (\hat{Y}\hat{V})) - Q_d - \varepsilon_q = 0$$

The gradient of the power flow equation has the following structure, which focuses on the DC variables.

$$\nabla h(X) = \frac{1}{V_d} \begin{pmatrix} \Delta P & \Delta Q \\ \times & \times \\ \delta & \times \\ P_G & \times \\ Q_G & \times \\ B_1 & B_3 \\ I_d & B_2 & B_4 \\ K_d & & \\ A_d & & \\ \varphi_d & & B_5 \end{pmatrix}$$

$$B_1 = \left(\frac{\partial \Delta P}{\partial V_d} \right)^T = -\text{sparse}(1 : N_{dc}, \Omega_{\text{destination}}, S_d \times I_d, N_{dc}, N)$$

$$B_2 = \left(\frac{\partial \Delta P}{\partial I_d} \right)^T = -\text{sparse}(1 : N_{dc}, \Omega_{\text{destination}}, S_d \times V_d, N_{dc}, N)$$

$$B_3 = \left(\frac{\partial \Delta Q}{\partial V_d} \right)^T = -\text{sparse}(1 : N_{dc}, \Omega_{\text{destination}}, S_d \times I_d \times \tan(\varphi_d), N_{dc}, N)$$

$$B_4 = \left(\frac{\partial \Delta Q}{\partial I_d} \right)^T = -\text{sparse}(1 : N_{dc}, \Omega_{\text{destination}}, S_d \times V_d \times \tan(\varphi_d), N_{dc}, N)$$

$$B_5 = \left(\frac{\partial \Delta Q}{\partial \varphi_d} \right)^T = -\text{sparse}(1 : N_{dc}, \Omega_{\text{destination}},$$

$$S_d \times V_d \times I_d \times (1 + \tan(\varphi_d) \times \tan(\varphi_d)), N_{dc}, N)$$

The hessian of power flow equations includes 2 parts as:

$$\nabla^2(\Delta P)y_p = \frac{1}{V_d} \begin{pmatrix} V & \delta & P_G & Q_G & V_d & I_d & K_d & A_d & \varphi_d \\ \times & \times & & & & & & & \\ \delta & \times & & & & & & & \\ P_G & & & & & & & & \\ Q_G & & & & & & & & \\ B_6 & & & & & & & & \\ I_d & & & & & & & & \\ K_d & & & & & & & & \\ A_d & & & & & & & & \\ \varphi_d & & & & & & & & \\ B_6 & & & & & & & & \\ V & \delta & P_G & Q_G & V_d & I_d & K_d & A_d & \varphi_d \\ \times & \times & & & & & & & \\ \delta & \times & & & & & & & \\ P_G & & & & & & & & \\ Q_G & & & & & & & & \\ B_7 & & & & & & & & \\ I_d & & & & & & & & \\ K_d & & & & & & & & \\ A_d & & & & & & & & \\ \varphi_d & & & & & & & & \\ B_7 & & & & & & & & \\ B_8 & & & & & & & & \\ B_8 & B_9 & & & & & & & \\ B_9 & & & & & & & & \\ B_9 & & & & & & & & \\ B_{10} & & & & & & & & \end{pmatrix}$$

where:

$$B_6 = -\text{sparse}(1 : N_{dc}, 1 : N_{dc}, S_d, N_{dc}, N_{dc})[y_p(\Omega_{\text{destination}})]$$

$$\begin{aligned}
\mathbf{B}_7 &= -\text{sparse}(1 : N_{dc}, 1 : N_{dc}, \mathbf{S}_d \times \tan(\boldsymbol{\varphi}_d), N_{dc}, N_{dc}) [\mathbf{y}_q(\boldsymbol{\Omega}_{\text{destination}})] \\
\mathbf{B}_8 &= -\text{sparse}(1 : N_{dc}, 1 : N_{dc}, \mathbf{S}_d \times \mathbf{I}_d \times \sec(\boldsymbol{\varphi}_d)^2, N_{dc}, N_{dc}) \\
&\quad \times [\mathbf{y}_q(\boldsymbol{\Omega}_{\text{destination}})] \\
\mathbf{B}_9 &= -\text{sparse}(1 : N_{dc}, 1 : N_{dc}, \mathbf{S}_d \times \mathbf{V}_d \times \sec(\boldsymbol{\varphi}_d)^2, N_{dc}, N_{dc}) \\
&\quad \times [\mathbf{y}_q(\boldsymbol{\Omega}_{\text{destination}})] \\
\mathbf{B}_{10} &= -\text{sparse}(1 : N_{dc}, 1 : N_{dc}, \mathbf{S}_d \times \mathbf{V}_d \times \mathbf{I}_d \times \sec(\boldsymbol{\varphi}_d)^2 \times \tan(\boldsymbol{\varphi}_d), \\
&\quad N_{dc}, N_{dc}) \times [2 \times \mathbf{y}_q(\boldsymbol{\Omega}_{\text{destination}})]
\end{aligned}$$

IX. REFERENCES

- [1] W. F. Tinney and C. E. Hart, "Power flow solution by Newton's method," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-86, pp.1449-1460, Nov.1967.
- [2] D. A. Braunagel, L. A. Kraft, and J. L. Whyson, "Inclusion of DC converter and transmission equation directly in a Newton power flow," *IEEE Transactions on Power Apparatus and System*, vol. 95, No. 1, pp. 76-88, 1976.
- [3] J. R. D. Silva and C. P. Arnold, "A simple improvement to sequential AC/DC power flow algorithms," *Electrical Power & Energy Systems*, vol. 12, no. 3, pp. 219-221, Jul. 1990.
- [4] T. Smed, A. G. Andersson, G. B. Sheble, and L. L. Grigsby, "A new approach to AC/DC power flow," *IEEE Transactions on Power Systems*, vol 6, No.3, pp. 1238-1244, 1991.
- [5] L. M. C. Braz, C. A. Castro, and C. A. F. Murari, "A critical Evaluation of step size optimization based load flow methods," *IEEE Transactions on Power System*, vol. 15, No.1, pp.202-207, Feb. 2000.
- [6] Y. Wang, L. C. P. da Silva, W. Xu, and Y. Zhang, "Analysis of ill-conditioned power-flow problems using voltage stability methodology," *IEE Proceedings on Generation, Transmission and Distribution*, vol.148, No.5, pp. 384-390, Sep. 2001.
- [7] B. Scott, "Effective starting process for Newton-Raphson load flows," *Proceedings of IEE*, vol. 118, No.8, pp. 983-987, Aug. 1971.
- [8] Y. Yao, D. Wang, Y. Chen and Z. Wu, "The effect of small impedance branches on the convergence of the Newton Raphson power flow," in *Proc. Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies, 2008(DRPT2008)*, 6-9 April 2008, pp.1141-1146.
- [9] S. Iwamoto and Y. Tamura, "A load flow method for ill-conditioned power systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, pp. 1736-1743, Apr. 1981.
- [10] M. D. Schaffer and D. J. Tylavsky, "A nondiverging polar-form Newton-based power flow," *IEEE Transactions on Industry Applications*, vol. 24, No.5, pp.870-877, Oct. 1988.
- [11] L. M. C. Braz, C. A. Castro, and C. A. F. Murari, "A critical Evaluation of step size optimization based load flow methods," *IEEE Transactions on Power System*, vol. 15, No.1, pp.202-207, Feb. 2000.
- [12] J. E. Tate and T. J. Overbye, "A comparison of the optimal multiplier in polar and rectangular coordinates," *IEEE Transactions on Power System*, vol. 20, No.4, pp.1667-1674, Nov. 2005.
- [13] T.J. Overbye, "Computation of a practical method to restore power flow solvability," *IEEE Transactions on Power System*, vol.10, no.1, pp.280-287, Feb. 1995.
- [14] Jean-Jemeau R and Chiang H D, "Parameterizations of the load flow equations for eliminating ill-conditioning load flow solutions," *IEEE Transactions on Power System*, vol.8, No.3, pp. 1004-1012, Aug. 1993.
- [15] VAjjarapu and C Christy, "The continuation power flow: a tool for steady state voltage stability analysis," *IEEE Transactions on Power System*, vol.7, No.1, pp. 416-423, Feb. 1992.
- [16] C. A. Canizares and F. L. Alvalado, "Point of collapse and continuation methods for large AC/DC systems," *IEEE Transactions on Power System*, vol.8, No.1, pp. 1-8, Feb. 1993.
- [17] A. Semlyen, B. Gao, and W. Janischewskyj, "Calculation of the extreme loading condition of a power system for the assessment of voltage stability," *IEEE Transactions on Power System*, vol.6, no.1, pp.307-315, Feb. 1991.
- [18] H. Mori and K. Seki, "Non-linear-predictor-based continuation power flow for unbalanced distribution systems," in *Proc. Transmission &*

Distribution Conference & Exposition: Asia and Pacific, 2009, 26-30 Oct. 2009, pp.1-4.

- [19] Y. Yang and Z. Qin, "Vectorial power flow calculation based on nonlinear programming," in *Proc. 2008 IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, 20-24 July 2008, pp.1-6.
- [20] Y. Yang, Z. Qin and H. Wei, "Power flow calculation based on nonlinear programming and vectorization mode," in *Proc. International Conference on Mechatronics and Automation (ICMA 2007)*, 5-8 Aug. 2007, pp.1729-1733.
- [21] R. Singh, B.C. Pal, R.A. Jabr, and P.D. Lang, "Distribution System Load Flow Using Primal Dual Interior Point Method," in *Proc. Joint International Conference on Power System Technology and IEEE Power India Conference, 2008(POWERCON 2008)*, 12-15 Oct. 2008, pp.1-5.
- [22] R.D Zimmerman. AC power flows, generalized OPF costs and their derivatives using complex matrix notation. [Online].Available: <http://www.pserc.cornell.edu/matpower>.
- [23] T.A. Davis, "Algorithm 849: A concise sparse Cholesky factorization package," *ACM Transactions on Mathematical Software*, vol. 31, no. 4, pp. 587-591, 2005.
- [24] T.A.Davis, "Algorithm 832: UMFPACK, an unsymmetric-pattern multifrontal method," *ACM Transactions on Mathematical Software*, vol. 30, no. 2, pp. 196-199, 2004.
- [25] H. Wang H, C. E. Murillo-Sánchez, R. D. Zimmerman and R. J. Thomas, "On computation issue of market-based optimal power flow," *IEEE Transactions on Power Systems*, vol.22, No. 3, pp. 1185-1193, Aug. 2007.
- [26] Z. Qin and Y. Yang, "Vectorization implementation of optimal power flow in rectangular form based on interior point method," in *Proc. 2008 IEEE Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century*, 20-24 July 2008, pp.1-8.

X. BIOGRAPHIES

Zhijun Qin (S' 12) received BS and MS degree in automation of power system from Huazhong University of Science and Technology, P.R. China in 2000 and 2003, respectively. He worked with Guangxi University from 2004 to 2010. He worked with The University of Hong Kong as a research assistant from 2010 to 2011. Now he is a fulltime Ph.D student in The University of Hong Kong. His research interest includes system restoration, optimal dispatch and high performance computing.

Yunhe Hou (M' 06) received his B.E. (1999) and Ph.D. (2005) degrees from Huazhong University of Science and Technology, China. He worked as a postdoctoral research fellow at Tsinghua University from 2005 to 2007. He was a visiting scholar at Iowa State University, Ames, USA, and a research scientist at University College Dublin, Ireland, from 2008 to 2009. He was also a visiting scientist with Laboratory for Information and Decision Systems, Massachusetts Institute of Technology (MIT), Cambridge, MA, USA in 2010. He is currently with the University of Hong Kong, Hong Kong, as an Assistant Professor.