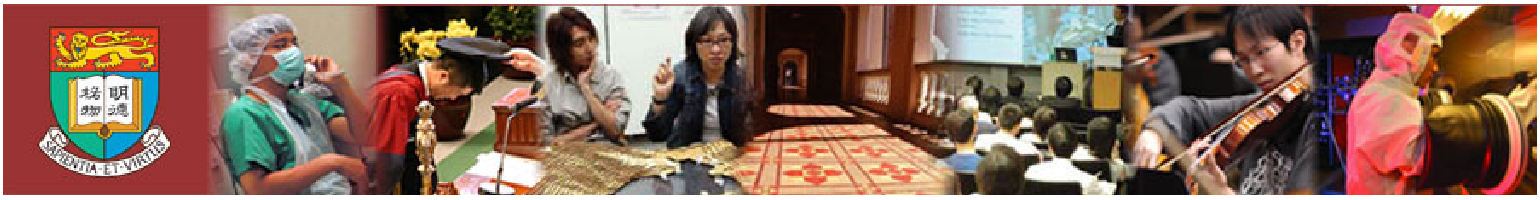
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Technical Correspondence

Decentralized Adaptive Pinning Control for Cluster Synchronization of Complex Dynamical Networks

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***Abstract*—In this brief, we investigate pinning control for cluster synchronization of undirected complex dynamical networks using a decentralized adaptive strategy. Unlike most existing pinning-control algorithms with or without an adaptive strategy, which require global information of the underlying network such as the eigenvalues of the coupling matrix of the whole network or a centralized adaptive control scheme, we propose a novel decentralized adaptive pinning-control scheme for cluster synchronization of undirected networks using a local adaptive strategy on both coupling strengths and feedback gains. By introducing this local adaptive strategy on each node, we show that the network can synchronize using weak coupling strengths and small feedback gains. Finally, we present some simulations to verify and illustrate the theoretical results.**

***Index Terms*—Cluster synchronization, complex dynamical network, local decentralized adaptive strategy, pinning control.**

I. Introduction

Various synchronization phenomena are ubiquitous in nature. Stimulated by two pioneering papers on synchronization in coupled systems [1] and in chaotic systems [2], there has been much interest in the study of synchronization of coupled complex dynamical systems. In the past few years, global synchronization [3] and local synchronization [4] of coupled complex dynamical systems have been studied intensively. In general, synchronization can be classified as complete synchronization [3], [4], phase synchronization [5], cluster synchronization [6]–[10], and so on. The first-order consensus of multiagent systems [11]–[13] can be regarded as a special case of the synchronization of coupled complex dynamical systems, in which the intrinsic dynamics of all agents are zero. In particular, cluster synchronization of complex dynamical networks aims to make nodes synchronize with each other in the same cluster but desynchronize with each other among different clusters, where clusters represent subgroups of the coupled oscillators in a network [6]–[10]. Due to the specific goals in practice, many biological, social, and technological networks functionally divide into communities. Therefore, cluster synchronization has many applications in practice. For example, two subgroups will be naturally formed in social networks when a crowd of people choose to accept or reject an opinion according to their preference. When a group of robots is to carry out a complex task, subtasks will divide the robot network into communities, and consensus should be achieved within each community.

Recently, some efforts have been devoted to the investigation in synchronization and control of complex dynamical networks [3]. In particular, pinning control is an effective method to control the collective dynamics of a complex network to a desired state such as an equilibrium point or a periodic orbit [14]–[21]. A common feature of the works presented in [14]–[21] is that there are certain convergence conditions that require global information on the underlying network. The convergence conditions given in [14]–[19] actually require the knowledge of eigenvalues of the coupling matrix of the network. The adaptive strategies developed in [20] and [21] are centralized, in which the adaptive parameter for each node contains the state information about all nodes and the information of the homogeneous stationary state or various heterogeneous stationary states of the whole network. If the size of the network is very large, the calculation of the eigenvalues of the coupling matrix of the network or the centralized adaptive strategies may be too difficult or costly to implement, even if it is possible at all. Recently, synchronization of complex networks is investigated without global information on the underlying network via decentralized adaptive coupling, suggesting two effective local adaptive strategies, i.e., vertex-based strategy and edge-based strategy [22]–[24].

In this brief, by introducing local adaptive strategies to both coupling strengths and feedback gains, we develop a decentralized adaptive pinning-control scheme for cluster synchronization of undirected complex dynamical networks without using any global information on the underlying network. Generally speaking, this brief extends the centralized adaptive strategies in [20] and [21] to the decentralized case and extends the decentralized adaptive complete synchronization in [22]–[24] to the decentralized adaptive cluster synchronization. In contrast to the centralized adaptive strategies developed in [20] and [21], each node only acquires the state information from its neighbors and only selects those few nodes that have the information of their desired heterogeneous stationary states. This is a significant improvement upon the existing centralized algorithms. Different from the known results on adaptive complete synchronization in [22]–[24], we investigate the adaptive cluster-synchronization problem here. The cluster-synchronization problem cannot be viewed as a simple collective form for the independent complete synchronization problems. This is because, unlike complete synchronization, each node is affected not only by the nodes in the same cluster but also by the nodes in different clusters, which have different synchronized states; therefore, there is no guarantee that a node will definitely synchronize with the nodes in the same cluster. In order to achieve cluster synchronization, unlike the traditional definition of the coupling matrix of the network, inhibitory coupling is introduced to the coupling matrix in

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our network. Thus, the decentralized adaptive algorithms derived in [22]–[24] cannot solve the cluster-synchronization problems formulated here. To overcome this difficulty, unlike the results presented in [22]–[24], pinning control for cluster synchronization of complex dynamical networks is investigated here, and local adaptive strategies are introduced to only parts of edges in the graph, which calls for a more delicate treatment.

II. Problem Statement

Consider a network G consisting of N linearly and diffusively coupled identical nodes described by an n-dimensional dynamical system

$$\dot{x}_i(t) = f\left(x_i(t), t\right) + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} \left(x_j(t) - x_i(t)\right), \quad i = 1, \dots, N \tag{1}$$

where $x_i = [x_i^1, \dots, x_i^n]^{\mathrm{T}} \in \mathbf{R}^n$ is the state vector of the ith node, $f : \mathbf{R}^n \times [0, +\infty) \to \mathbf{R}^n$ is a continuous map, and $c_{ij} > 0$ denotes the coupling strength between node i and node $j, i, j = 1, \dots, N$. With $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$, the matrix $A = (a_{ij}) \in \mathbf{R}^{N\times N}$ represents the coupling configuration of this network. Define the matrix of the weighted coupling configuration of the network as follows:

$$B = \begin{bmatrix} c_{11}a_{11} & c_{12}a_{12} & \cdots & c_{1n}a_{1n} \\ c_{21}a_{21} & c_{22}a_{22} & \cdots & c_{2n}a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1}a_{n1} & c_{d2}a_{d2} & \cdots & c_{nd}a_{nd} \end{bmatrix} \in \mathbf{R}^{N\times N}$$

where $c_{ii}a_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}a_{ij}, i, j = 1, \dots, N$.

Suppose that d nonempty subsets (clusters) $\{G_1, \dots, G_d\}$ is a partition of the index set $\{1, 2, \dots, N\}$, where $\cup_{l=1}^{d} G_l = \{1, 2, \dots, N\}$ and $G_l \neq \emptyset$. A network with N nodes is said to realize d-cluster synchronization if $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$ for all i and j in the same cluster and $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| > 0$ for all i and j in different clusters.

The problem of pinning control for d-cluster synchronization is to directly control a small fraction of nodes in network (1) to achieve $\lim_{t\to\infty} \sum_{l=1}^{d} \sum_{i\in G_l} \|x_i(t) - \overline{x}_l(t)\| = 0$, where $\overline{x}_l(t)$ is the desired state of the lth cluster G_l, which satisfies

$$\dot{\overline{x}}_l(t) = f\left(\overline{x}_l(t), t\right), \qquad l = 1, \dots, d \tag{2}$$

where $\overline{x}_l(t)$ can be an equilibrium, a limit cycle, or even a chaotic attractor. The controlled network is described as follows:

$$\begin{aligned} \dot{x}_i(t) = f\left(x_i(t), t\right) &+ \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} \left(x_j(t) - x_i(t)\right) \\ &+ h_i c_i \left(\overline{x}_{\hat{i}}(t) - x_i(t)\right), \qquad i = 1, \dots, N \end{aligned} \tag{3}$$

where $\hat{i}$ is the subscript of the subset for which $i \in G_{\hat{i}}$. If node i is selected to be pinned, then $h_i = 1$; otherwise, $h_i = 0$.

In this brief, an adaptive pinning-control scheme is developed for cluster synchronization of network (3). The approach is to use a decentralized method, i.e., each node only needs the state information of its neighbors and only selects those few nodes that have the information of their desired states. The main advantage of the proposed scheme over the existing centralized algorithms is that it does not need to acquire or use state information from all nodes in the network.

III. Decentralized Adaptive Pinning-Control Scheme

A. Algorithm Description

The proposed adaptive strategy for node i is designed as

$$\begin{aligned} \dot{x}_i(t) &= f\left(x_i(t), t\right) + \sum_{j=1, j\neq i}^{N} c_{ij}(t) a_{ij} \left(x_j(t) - x_i(t)\right) \\ &\quad + h_i c_i(t) \left(\overline{x}_{\hat{i}}(t) - x_i(t)\right) \\ \dot{c}_{ij}(t) &= h_{ij} a_{ij} k_{ij} \left(x_i(t) - x_j(t)\right)^{\mathrm{T}} P \left(x_i(t) - x_j(t)\right) \\ \dot{c}_i(t) &= h_i k_i \left(x_i(t) - \overline{x}_{\hat{i}}(t)\right)^{\mathrm{T}} P \left(x_i(t) - \overline{x}_{\hat{i}}(t)\right) \end{aligned} \tag{4}$$

where $c_{ij}(0) = c > 0$, $c_i(0) \geq 0$, and a positive-definite diagonal matrix $P = \mathrm{diag}\{p_1, \dots, p_n\}$. The positive constants $k_{ij} = k_{ji}$ and k_i are the weights of the adaptive laws for parameters $c_{ij}(t)$ and $c_i(t)$, respectively. If nodes i and j are in the same cluster, then $h_{ij} = 1$; otherwise, $h_{ij} = 0$. Clearly, the adaptive parameters $c_{ij}(t)$ for node i only contain the state information of its neighbors.

B. Main Results and Theoretical Analysis

The following assumption and lemmas are needed for our main result.

Assumption 1 [20], [25], [26]: For any $x, y \in \mathbf{R}^n$, $t \in [0, +\infty)$, the nonlinear map $f(x, t) : \mathbf{R}^n \times [0, +\infty) \to \mathbf{R}^n$ is uniformly continuous in t and satisfies

$$(x-y)^{\mathrm{T}} P\{[f(x,t) - f(y,t)] - \Delta(x-y)\} \leq -\omega(x-y)^{\mathrm{T}}(x-y) \tag{5}$$

for a diagonal matrix $\Delta = \mathrm{diag}\{\delta_1, \dots, \delta_n\}$ and a positive constant $\omega > 0$.

It is assumed in [21] that, for an $N \times N$ symmetric matrix

$$A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1d} \\ A_{21} & A_{22} & \cdots & A_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ A_{d1} & A_{d2} & \cdots & A_{dd} \end{bmatrix} \tag{6}$$

where $A = (a_{ij}) \in \mathbf{R}^{N\times N}$, each block $A_{uv} = (z_{ij}) \in \mathbf{R}^{k_u \times k_v} (u, v = 1, \dots, d)$ is a zero-row-sum matrix, i.e., $\sum_{j=1}^{k_v} z_{ij} = 0$, and each block $A_{uu} = (s_{ij}) \in \mathbf{R}^{k_u \times k_u}$ satisfies $s_{ii} = -\sum_{j=1, j\neq i}^{k_u} s_{ij}$, where $s_{ij} = s_{ji} \geq 0 (i \neq j)$ and k_u and k_v are the numbers of nodes in the subsets u and v, respectively.

Note also that, unlike the traditional definition of the coupling matrix of the network, the element a_{ij} with $i \in G_u$ and $j \in G_v$ in (6) may be negative here, which is called an inhibitory coupling [21]. This provides a mechanism to desynchronize two nodes belonging to two different clusters.

Lemma 1 [20]: If $L = (l_{ij}) \in \mathbf{R}^{N\times N}$ is a symmetric irreducible matrix with $l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}$, $l_{ij} = l_{ji} \geq 0 (i \neq j)$, then, for any matrix $E = \mathrm{diag}(e, 0, \dots, 0)$ with $e > 0$, all eigenvalues of the matrix $(L - E)$ are negative.

Lemma 2 [21]: For any $x \in \mathbf{R}^p$, $y \in \mathbf{R}^q$, and matrix $M = (m_{ij}) \in \mathbf{R}^{p\times q}$, $x^{\mathrm{T}} M y \leq (1/2) \max[p, q] \cdot \max_{i,j} |m_{ij}| (x^{\mathrm{T}} x + y^{\mathrm{T}} y)$.

Lemma 3: Let matrix A be given as in (6). Then, under the adaptive strategy (4)

$$B = (a_{ij} c_{ij}(t)) = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1d} \\ B_{21} & B_{22} & \cdots & B_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ B_{d1} & B_{d2} & \cdots & B_{dd} \end{bmatrix} \in \mathbf{R}^{N\times N}$$

is a symmetric matrix, in which each block $B_{uv}(u, v = 1, \ldots, d)$ is a zero-row-sum matrix and each block B_{uu} is with $a_{ii}c_{ii}(t) = -\sum_{j=1,j\neq i}^{k_u} a_{ij}c_{ij}(t)$, $a_{ij}c_{ij}(t) = a_{ji}c_{ji}(t) \geq 0 (i \neq j)$, where k_u is the number of nodes in the subset u.

Proof: If nodes i and j are not in the same cluster, then, by the adaptive strategy (4), one has $\dot{c}_{ij}(t) = 0$. If nodes i and j are in the same cluster, then, by the adaptive strategy (4), one has $\dot{c}_{ij}(t) = \dot{c}_{ji}(t)$. Therefore, B is a symmetric irreducible matrix. Similar to (6), each block $B_{uv}(u, v = 1, \ldots, d)$ is a zero-row-sum matrix, and each block B_{uu} is with $a_{ii}c_{ii}(t) = -\sum_{j=1,j\neq i}^{k_u} a_{ij}c_{ij}(t)$, $a_{ij}c_{ij}(t) = a_{ji}c_{ji}(t) \geq 0 (i \neq j)$, where k_u is the number of nodes in the subset u.

Lemma 4: Let matrix A be given as in (6). Then

$$\sum_{i=1}^{N}(x_i - \overline{x}_{\hat{i}})^{\mathrm{T}} P \sum_{j=1,\hat{i}=\hat{j},j\neq i}^{N} a_{ij}c_{ij}(x_i - x_j) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} h_{ij}a_{ij}c_{ij}(x_i - x_j)^{\mathrm{T}} P(x_i - x_j).$$

Proof: The proof is similar to that of Lemma 2 in [27].

Lemma 5 [28]: If a scalar function $V(x, t)$ satisfies the following conditions, then $\dot{V}(x, t) \to 0$, as $t \to \infty$.

a) $V(x, t)$ is lower bounded.
b) $\dot{V}(x, t)$ is negative semidefinite.
c) $\dot{V}(x, t)$ is uniformly continuous in t.

Note that Lemma 5 is, in fact, an extension of Barbalat's Lemma. More details about Lemma 5 can be seen in , Lemma 4.3 (Lyapunov-like Lemma)[28].

Theorem 1: Consider network (1), where each node is steered by the adaptive strategy (4). Suppose that Assumption 1 holds and at least one node in each cluster is selected to be controlled. Let matrix A be given as in (6). Then, all clusters asymptotically synchronize to their given heterogeneous stationary states, namely

$$\lim_{t\to\infty}\sum_{l=1}^{d}\sum_{i\in G_l}\|x_i(t) - \overline{x}_l(t)\| = 0.$$

Proof: Define the Lyapunov functional candidate as

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}(x_i - \overline{x}_{\hat{i}})^{\mathrm{T}} P(x_i - \overline{x}_{\hat{i}}) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij} - m)^2}{2k_{ij}} + \frac{(c_i - m)^2}{k_i} \tag{7}$$

where the positive constant $mI_{k_u} > (\delta_r + (d-1)\max_{u,v}[k_u, k_v] \max_{i,j}|a_{ij}c_{ij}|)I_{k_u}(H_{uu} - B_{uu})^{-1}$.

Differentiating $V(t)$ gives

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N}\tilde{x}_i^{\mathrm{T}} P\left[f(x_i, t) - f(\overline{x}_{\hat{i}}, t)\right] + \sum_{i=1}^{N} h_i(c_i - m)\tilde{x}_i^{\mathrm{T}} P\tilde{x}_i \\
&\quad - \sum_{i=1}^{N}\tilde{x}_i^{\mathrm{T}} P\left[\sum_{j=1,j\neq i}^{N} a_{ij}c_{ij}(\tilde{x}_i - \tilde{x}_j) + h_i c_i\tilde{x}_i\right] \\
&\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} h_{ij}a_{ij}(c_{ij} - m)(\tilde{x}_i - \tilde{x}_j)^{\mathrm{T}} P(\tilde{x}_i - \tilde{x}_j) \\
&\leq \sum_{i=1}^{N}\tilde{x}_i^{\mathrm{T}} P\left[\Delta\tilde{x}_i - \sum_{j=1,j\neq i}^{N} a_{ij}c_{ij}(\tilde{x}_i - \tilde{x}_j) - h_i c_i\tilde{x}_i\right] \\
&\quad - \sum_{i=1}^{N}\omega\tilde{x}_i^{\mathrm{T}}\tilde{x}_i + \sum_{i=1}^{N} h_i(c_i - m)\tilde{x}_i^{\mathrm{T}} P\tilde{x}_i \\
&\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} h_{ij}a_{ij}(c_{ij} - m)(\tilde{x}_i - \tilde{x}_j)^{\mathrm{T}} P(\tilde{x}_i - \tilde{x}_j) \\
&= -\sum_{i=1}^{N}\omega\tilde{x}_i^{\mathrm{T}}\tilde{x}_i + \sum_{r=1}^{n} p_r\left[m\sum_{u=1}^{d} x^{ur\mathrm{T}}(B_{uu} - H_{uu})x^{ur}\right] \\
&\quad + \sum_{r=1}^{n} p_r\left[\delta_r\sum_{u=1}^{d} x^{ur\mathrm{T}} x^{ur} + \sum_{u=1}^{d}\sum_{v\neq u} x^{ur\mathrm{T}} B_{uv}x^{vr}\right] \\
&\leq \sum_{r=1}^{n} p_r\sum_{u=1}^{d} x^{ur\mathrm{T}}(mB_{uu} - mH_{uu} + \delta_r I_{k_u})x^{ur} \\
&\quad - \sum_{i=1}^{N}\omega\tilde{x}_i^{\mathrm{T}}\tilde{x}_i + \sum_{r=1}^{n} p_r(d-1) \\
&\quad \times\left[\max_{u,v}[k_u, k_v]\max_{i,j}|a_{ij}c_{ij}|\sum_{u=1}^{d} x^{ur\mathrm{T}} x^{ur}\right]
\end{aligned} \tag{8}$$

where $x^r = [x_1^r, \ldots, x_N^r]^{\mathrm{T}} = [x^{1r}, \ldots, x^{ur}, \ldots, x^{dr}]^{\mathrm{T}} \in \mathbf{R}^N$, $x^{ur} \in \mathbf{R}^{k_u}$ is the vector in the uth cluster, and $H = \mathrm{diag}\{H_{11}, H_{22}, \ldots, H_{dd}\}$. According to (6), at least one node in each cluster is selected to be controlled, and from Lemma 1 and Lemma 3, it can be verified that $(B_{uu} - H_{uu})$ is negative definite. Since the positive constant $mI_{k_u} > (\delta_r + (d-1)\max_{u,v}[k_u, k_v]\max_{i,j}|a_{ij}c_{ij}|)I_{k_u}(H_{uu} - B_{uu})^{-1}$, one has $\dot{V}(t) \leq -\sum_{i=1}^{N}\omega\tilde{x}_i^{\mathrm{T}}\tilde{x}_i$. Thus, $\dot{V}(t)$ is negative semidefinite. From the definition of $V(t)$, it is obvious that $V(t)$ is lower bounded. According to Assumption 1 and (8), $\dot{V}(t)$ is uniformly continuous in t. Therefore, Lemma 5 can be applied here. It is obvious that $\dot{V}(t) = 0$ if and only if $\tilde{x}_i = 0$, $i = 1, 2, \ldots, n$. The set $S = \{\tilde{x} \in \mathbf{R}^{Nn}, c \in \mathbf{R}^N, \tilde{c} \in \mathbf{R}^{N^2} : \tilde{x} = 0, c = c_0, \tilde{c} = \tilde{c}_0\}$, where $c = [c_1, c_2, \ldots, c_N]^{\mathrm{T}}$, $\tilde{c} = [\tilde{c}_{11}, \tilde{c}_{12}, \ldots, \tilde{c}_{NN}]^{\mathrm{T}}$, and c_0 and $\tilde{c}_0$ are positive constant vectors, is the largest invariant set contained in $M = \{\dot{V}(t) = 0\}$ for system (4). According to Lemma 5, the trajectories of the system (4) converge asymptotically to the set S, i.e., $\tilde{x} \to 0$, $c \to c_0$, and $\tilde{c} \to \tilde{c}_0$ as $t \to \infty$.

Thus, one has the result of Theorem 1. This completes the proof of Theorem 1.

Remark 1: Under the assumption that there is at least one single controller in each cluster, Assumption 1 and (6) hold, and d-cluster synchronization of complex networks was established in [21], where, however, the adaptive parameter at each node requires the state information from all nodes and the information on the heterogeneous stationary states over the whole network. Unlike the protocol in [21], each node in protocol (4) only has the state information of its neighbors and only selects those few nodes that have the information on the heterogeneous stationary states.

C. Complete Synchronization Case

When there is only one cluster in the network, the protocol (4) can also be used to realize complete synchronization of the network. The adaptive strategy for node i in this case can be rewritten as

$$\begin{aligned}\dot{x}_i(t) &= f\left(x_i(t), t\right) + \sum_{j=1, j\neq i}^{N} c_{ij}(t) a_{ij} \left(x_j(t) - x_i(t)\right) \\ &\quad + h_i c_i(t) \left(\overline{x}(t) - x_i(t)\right) \\ \dot{c}_{ij}(t) &= a_{ij} k_{ij} \left(x_i(t) - x_j(t)\right)^{\mathrm{T}} P \left(x_i(t) - x_j(t)\right) \\ \dot{c}_i(t) &= h_i k_i \left(x_i(t) - \overline{x}(t)\right)^{\mathrm{T}} P \left(x_i(t) - \overline{x}(t)\right). \end{aligned} \tag{9}$$

The graph Laplacian of the connected network naturally satisfies (6).

Corollary 1: Consider network (1) with a single cluster, where each node is steered by the adaptive strategy (9). Suppose that the whole network is connected, Assumption 1 holds, and at least one node is selected to be controlled. Then, all nodes asymptotically synchronize to the given homogeneous stationary state, namely

$$\lim_{t\to\infty} \|x_i(t) - \overline{x}(t)\| = 0.$$

Proof: This is a special case of Theorem 1.

IV. Simulation Results

In this section, numerical examples are given to verify and illustrate the theoretical results. In the following simulations, each node is a chaotic Chua circuit

$$\begin{cases} \dot{x}^1 = 10\left(x^2 - x^1 + f(x^1)\right) \\ \dot{x}^2 = x^1 - x^2 + x^3 \\ \dot{x}^3 = -15x^2 - 0.0385x^3 \end{cases} \tag{10}$$

where

$$f(x^1) = \begin{cases} -bx^1 - a + b, & x^1 > 1 \\ -ax^1, & |x^1| \leq 1 \\ -bx^1 + a - b, & x^1 < 1 \end{cases}$$

where $a = -1.27$ and $b = -0.68$. The Chua circuit (10) has three unstable equilibrium points: $[1.8586, 0.0048, -1.8539]^{\mathrm{T}}$, $[0, 0, 0]^{\mathrm{T}}$, and $[-1.8586, -0.0048, 1.8539]^{\mathrm{T}}$.

A. Example 1: Three Clusters

Simulations were performed with protocol (4) applied to 90 nodes grouped in three clusters. Each cluster is a homogenous small-world network, which starts from a nearest neighbor lattice with 30 nodes and an average degree $k = 10$. Two edges are randomly selected and then swapped, forbidding duplicate connections, until p percent of edges have been swapped. Through this reconnection, one can obtain a homogeneous small-world network with a high clustering coefficient and a short average path length without altering the connection distribution of the original network [29]. In this example, three homogeneous small-world clusters are generated randomly with swapping probability $p = 0.1$, and they are linked by a few random edges. The coupling matrix of the network satisfies (6). Initial states of the 90 nodes were chosen randomly from the cube $[-1, 1] \times [-1, 1] \times [-1, 1]$. The heterogeneous stationary states are the three unstable equilibrium points of the Lorenz system (10). There is only one node in each cluster that is chosen randomly to be controlled. The adaptive parameters are set as $c_{ij}(0) = 1$ and $c_i(0) = 0$, the weights are $k_{ij} = 1$ and $k_i = 1$, and the matrix is $P = \mathrm{diag}\{1, 1, \ldots, 1\}$.

In Fig. 1, plots (a)–(c) show the convergence of states on the x^1-, x^2-, and x^3-axes, respectively; plot (d) shows the mean square errors σ_i for the x^i-axis, where $\sigma_i = \sqrt{(1/(N-1)) \sum_{j=1}^{N} \left(x_j^i - (1/N) \sum_{j=1}^{N} x_j^i\right)^2}$; plot (e) shows the adaptive coupling strengths which eventually approach constants; and plot (f) shows the adaptive feedback gains which eventually approach constants.

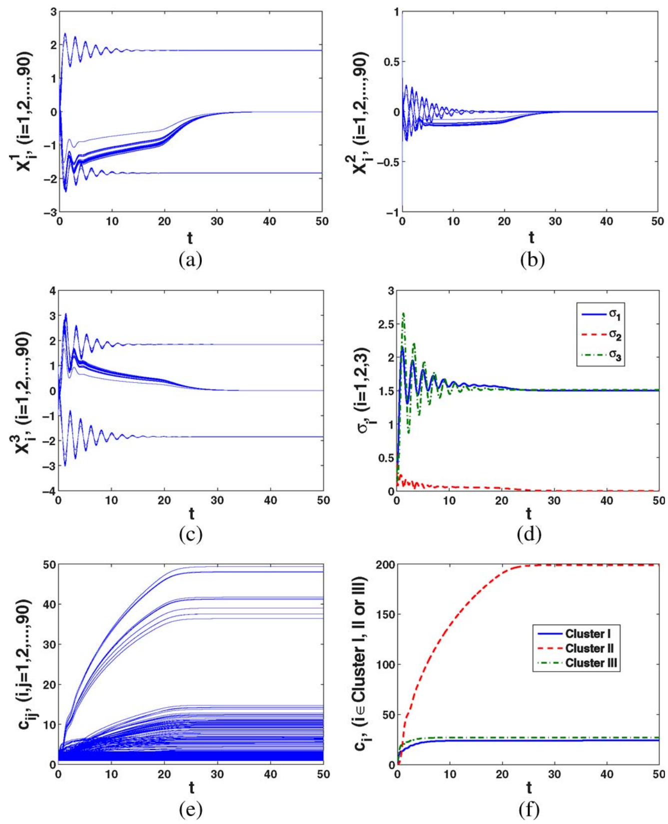

Fig. 1. Pinning control of 90 nodes under protocol (4).

B. Example 2: One Cluster

Simulations were performed by applying protocol (9) to a homogeneous network of 500 nodes with swapping probability $p = 0.1$ and an average degree $k = 10$[29]. The clustering coefficient of this small-world network $C = 0.4857$. The average path length of this small-world network $L = 4.0652$. Initial states of the 500 nodes were chosen randomly from the cube $[-1, 1] \times [-1, 1] \times [-1, 1]$. The homogeneous stationary state $\bar{x} = [1.8586, 0.0048, -1.8539]$. There is only one node which is chosen randomly from the network to be controlled. The adaptive parameters are set as $c_{ij}(0) = 0.01$ and $c_i(0) = 0$, the weights are $k_{ij} = 1$ and $k_i = 1$, and the matrix is $P = \mathrm{diag}\{1, 1, \ldots, 1\}$.

In Fig. 2, plots (a)–(c) show the convergence of states on the x^1-, x^2-, and x^3-axes, respectively; plot (d) shows the mean square errors σ_i for the x^i-axis; plot (e) shows the maximum adaptive coupling strength which eventually tends to a constant; and plot (f) shows the adaptive feedback gain which eventually tends to a constant.

For the same initial conditions of the states, we now make the comparison of the control gains among the nonadaptive pinning-control algorithm (3), the centralized pinning-control algorithm in [20], and the decentralized pinning-control algorithm (9). To ensure the fairness of the comparison, we select the same nodes to apply the pinning control. We use a more reasonable index, i.e., the average value of all the coupling strengths and feedback gains

$$Ave(c) = \frac{\sum_{i=1}^{N}\left(\left(\sum_{j=1, j\neq i}^{N} a_{ij} c_{ij}\right) + h_i c_i\right)}{\sum_{i=1}^{N}\left(\left(\sum_{j=1, j\neq i}^{N} a_{ij}\right) + h_i\right)}$$

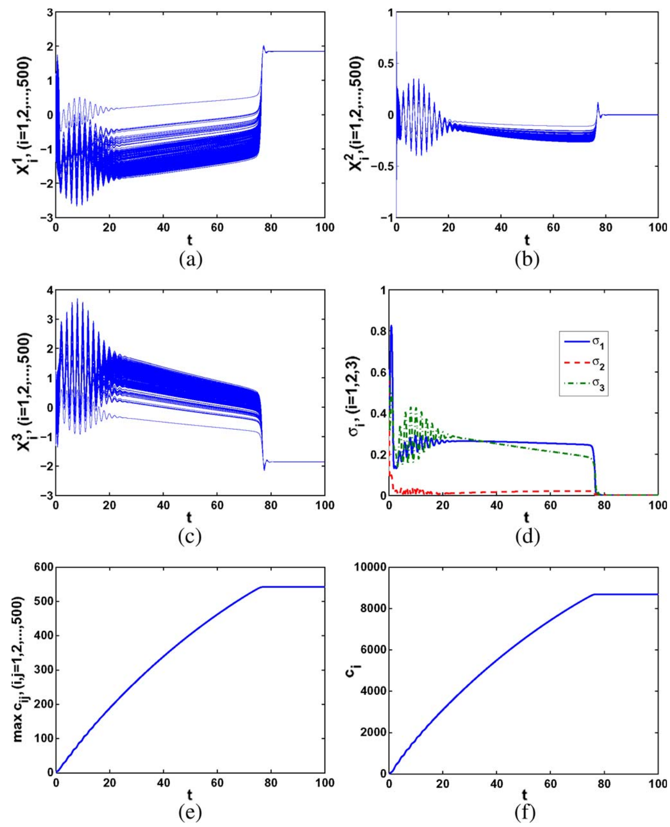

Fig. 2. Pinning control of 500 nodes under protocol (9).

to measure the control gains. Since the size of the network is very large, we cannot assign different least coupling strengths or feedback gains for different edges artificially for the nonadaptive pinning-control algorithm (3). Therefore, we choose the same value for all the coupling strengths and feedback gains when the nonadaptive pinning-control algorithm (3) is applied, i.e., all the coupling strengths and feedback gains are equal to the average value of all the coupling strengths and feedback gains. We choose the control gain $Ave(c) = 20$ and 30 for the nonadaptive pinning-control algorithm (3). The network cannot be synchronized for the case of the control gain $Ave(c) = 20$ but can be synchronized for the case of $Ave(c) = 30$. We then compare the decentralized pinning-control algorithm (9) against the nonadaptive pinning-control algorithm (3) with $Ave(c) = 30$ and the centralized pinning-control algorithm in [20] in Fig. 3. It is demonstrated by Fig. 3 that the practical average value of all the coupling strengths and feedback gains of the decentralized adaptive case is much less than the other two cases. The reason can be analyzed as follows. For a given large complex network, it is difficult or impossible to know which edges need larger coupling or control gains even if the initial states of all nodes are known. Therefore, we cannot assign different least coupling strengths or feedback gains for different edges in order to satisfy the theoretical least coupling and control gains for the synchronization of the network. For the nonadaptive algorithm, one optional method is to assign a uniform coupling and control gain, which must be larger than or equal to the largest value in the theoretical least coupling and control gains for the synchronization of the network, to all the coupling strengths or feedback gains so as to satisfy theoretical least coupling and control gains for the synchronization of the network. The centralized adaptive algorithm also has a common coupling and control gain for all edges, which should be larger than or equal to the largest value in the theoretical least coupling and control gains for the synchronization of the network. However, the decentralized adaptive

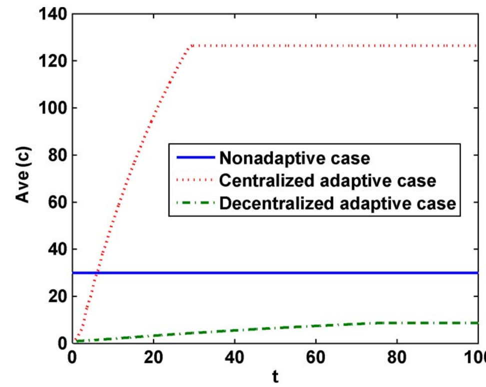

Fig. 3. Comparison of the control gains.

algorithm can adjust each coupling strength or feedback gain to its proper value through local interactions with neighbors, in which the gains for some edges may be smaller than the largest value in the theoretical least coupling and control gains for the synchronization of the network. Therefore, it is reasonable that the practical average value of all the coupling strengths and feedback gains of the decentralized adaptive case is much less than the other two cases.

V. Conclusion

In this brief, we have investigated the pinning-control problem for cluster synchronization of complex dynamical networks. By introducing local adaptive strategies for both coupling strengths and feedback gains, it was shown that the collective dynamics of the underlying complex network can be controlled to its heterogeneous stationary states without requiring global information of the network. This is superior to the existing protocols for the same problem since global information of the underlying network is not acquired or used. Simulations have shown that, by employing the proposed decentralized adaptive pinning-control scheme, it is possible to control the collective dynamics of a complex network to the desired heterogeneous stationary states in different clusters with only weak coupling strengths and small feedback gains, demonstrating a clear advantage of the approach developed in this brief.

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