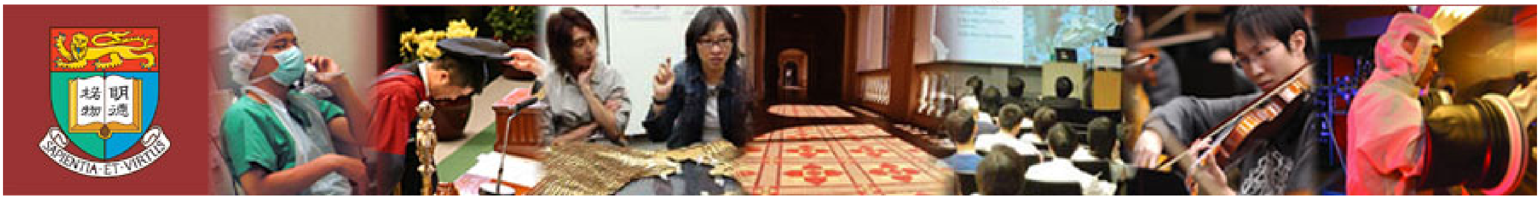

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A New Closed-Form Evaluation of Layered Medium Green's Function

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***Abstract*—A new closed-form evaluation of layered medium Green's function is proposed in this paper. The discrete complex image method (DCIM) is extended to sampling along the Sommerfeld branch cut, to capture the far field interaction. Contour deformation technique is applied to decompose the Green's function into radiation modes (branch cut integration) and guided modes (surface-wave poles). The matrix pencil method is implemented to get a closed-form solution, with the help of an alternative Sommerfeld identity. Numerical results are presented to demonstrate the accuracy of this method.**

I. Introduction

The integral equation method [1] based on the layered medium Green's function [2] is very important in modeling microstrip antennas and integrated circuits. However, the layered medium Green's function is very hard to calculate compared to its free space counterpart, due to the so-called Sommerfeld integral, which is oscillatory and slowly convergent. The discrete complex image method (DCIM) [3], [4] is one of the most popular methods to speed up the calculation. In this method, the integration kernel is first approximated by a set of complex exponential functions, by using either Prony's method [5] or matrix pencil method [6], and the Sommerfeld identity [7] is then applied to get the closed-form solution. In this way, the layered medium Green's function can be expressed by a summation of scalar free space Green's function, with complex locations and magnitudes, and the evaluation is thus very efficient.

However, the DCIM becomes inaccurate when the interaction is in the far field region ($\rho \gg 0$), since the original sampling path cannot effectively capture certain singularities, which correspond to the guided waves and lateral waves physically. To remedy this, several modifications have been made during the last two decade. A robust two-level approximation [8] was proposed to modify the sampling path or integration path. This path can separate the sharp-transition region from the smooth-varying region, and thus can carry more singularity information. In [9], a direct DCIM was developed to push the sampling path closer to the poles. Recently, a three-level DCIM [10] was developed to capture not only pole singularities, but also branch-point singularity, to make the DCIM more accurate for far field interaction.

In this paper, we propose an alternative way to capture the far field. Other than introducing extra segments of the sampling path, we simply deform the sampling path to the Sommerfeld branch cut (SBC) when ρ is relatively large. The matrix pencil method is then applied to approximate the function along the SBC, which can be mapped into the real axis in the k_z plane. The pole contributions are accounted for by applying a robust pole-searching algorithm [11]. Numerical results are shown to demonstrate the accuracy of this method.

II. Formulation

A. Layered Medium Green's Function

There are various methods to derive the layered medium Green's function, the one applied here follows [12],

$$\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') = (\nabla\times\hat{z})(\nabla'\times\hat{z})g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') + \frac{1}{k_{nm}^2}(\nabla\times\nabla\times\hat{z})(\nabla'\times\nabla'\times\hat{z})g^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') \quad (1)$$

where m denotes the layer where source point is, and n denotes the layer where observation point is. Here $k_{nm}^2 = \omega^2\epsilon_n\mu_m$, and $g^{\mathrm{TE/TM}}(\mathbf{r},\mathbf{r}')$ can be expressed as

$$g(\mathbf{r},\mathbf{r}') = \frac{i}{8\pi}\int_{-\infty}^{+\infty}\frac{dk_\rho}{k_{mz}k_\rho}H_0^{(1)}(k_\rho\rho)F(k_\rho,z,z') \quad (2)$$

where $F(k_\rho,z,z')$ is the propagation factor [7], $k_{mz} = \sqrt{k_m^2-k_\rho^2}$, and $H_0^{(1)}(k_\rho\rho)$ is the first kind Hankel function of order 0. By applying integration by part, the dyadic Green's function can be cast into several scalar Green's functions in the electric field integral equation (EFIE) formulation. In this formula, only the zeroth-order Hankel function is involved; hence the Green's functions have the lowest singularity. We only consider the following two Green's functions for simplicity, which is commonly encountered in the microstrip structures.

$$g_{ss}(\mathbf{r},\mathbf{r}') = k_\rho^2 g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') \quad (3)$$

$$g_\phi(\mathbf{r},\mathbf{r}') = \frac{\partial_z\partial_{z'}}{k_{nm}^2}g^{\mathrm{TM}}(\mathbf{r},\mathbf{r}') - g^{\mathrm{TE}}(\mathbf{r},\mathbf{r}') \quad (4)$$

where the partial derivative can be easily calculated in the spectral domain.

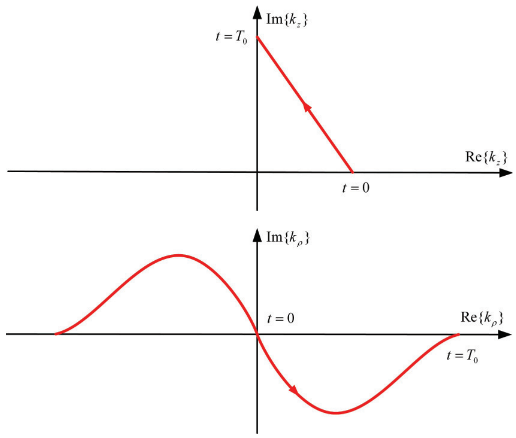

Fig. 1. The sampling path in the k_z plane and k_ρ plane in the traditional DCIM.

B. Traditional DCIM

For the Green's function shown in (3) and (4), a general expression can be written down as

$$g(\rho) = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dk_\rho \frac{k_\rho}{k_z} H_0^{(1)}(k_\rho \rho)\tilde{g}(k_\rho) \tag{5}$$

where the z and z' dependence is not explicitly shown. The basic idea of the DCIM is to approximate the integration kernel as a superposition of weighted complex exponentials,

$$\tilde{g}(k_\rho) = \sum_{i=1}^{M} a_i e^{ik_z b_i} \tag{6}$$

In order to apply the matrix pencil method to obtain a_i and b_i, the sampling path in the k_z plane of the traditional DCIM is set to be

$$k_z = k\left[it + (1 - \frac{t}{T_0})\right], \quad 0 \le t \le T_0 \tag{7}$$

where k is the wavenumber in the layer to be analyzed, and t is a real variable. The sampling path in the k_z plane and corresponding k_ρ plane are shown in Fig. 1. The closed-form solution of the Green's function shown in (5) can be finally obtained

$$g(\rho) = \sum_{i=1}^{M} a_i \frac{e^{ikr_i}}{4\pi r_i}, \;\; r_i = \sqrt{\rho^2 + b_i^2} \tag{8}$$

with the help of the Sommerfeld identity [7]

$$\frac{e^{ikr}}{r} = \frac{i}{2}\int_{-\infty}^{+\infty} dk_\rho \frac{k_\rho}{k_z} H_0^{(1)}(k_\rho \rho) e^{ik_z z} \tag{9}$$

The far field prediction of the traditional DCIM is poor, and various remedies have been proposed [8], [9], [10]. In the following, we propose an new way to solve this problem.

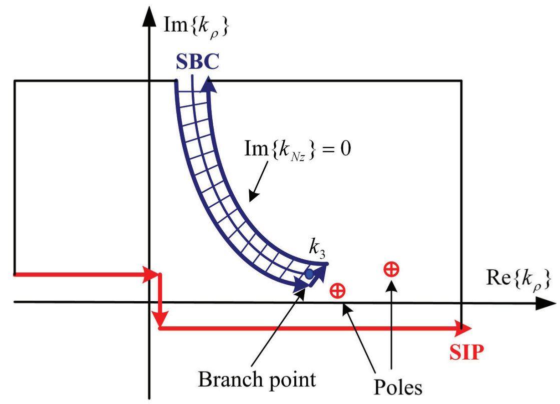

Fig. 2. The Sommerfeld integration path (SIP), Sommerfeld branch cut (SBC) and poles in the complex k_ρ plane.

C. DCIM Based on the Sommerfeld Branch Cut

When the transverse distance is large, we can deform the integration contour to the Sommerfeld branch cut (SBC). Here we assume that the layered medium is backed by a PEC ground plane. In this case, there is only one branch cut associated with the top layer. The integral defined along the Sommerfeld integration path (SIP) is equivalent to the the path integral along the SBC and some pole contributions, shown in Fig. 2. Mathematically,

$$g = g_{\text{branch}} + g_{\text{pole}} \tag{10}$$

where

$$g_{\text{branch}} = \frac{i}{8\pi}\int_{\text{SBC}} dk_\rho \frac{k_\rho}{k_{Nz}} H_0^{(1)}(k_\rho \rho)\tilde{g}(k_\rho) \tag{11}$$

$$g_{\text{pole}} = -\frac{1}{4}\sum_q \frac{k_{\rho,q}}{k_{Nz,q}} H_0^{(1)}(k_{\rho,q}\rho)\text{Res}\left[\tilde{g}(k_{\rho,q})\right] \tag{12}$$

The poles and residues can be obtained by a pole-searching algorithm [11]; they correspond to the guided modes. For the branch cut integration, we can perform a variable transform. So (11) becomes

$$g_{\text{branch}} = \frac{i}{8\pi}\int_{-\infty}^{+\infty} dk_{Nz} H_0^{(1)}(k_\rho \rho)\tilde{g}(k_\rho) \tag{13}$$

For this integration kernel, similar technique can be applied to approximate it with complex exponentials, and the alternative Sommerfeld identity can be implemented to get a closed-form solution,

$$\frac{e^{ikr}}{r} = \frac{i}{2}\int_{-\infty}^{+\infty} dk_z H_0^{(1)}(k_\rho \rho) e^{ik_z z} \tag{14}$$

III. Numerical Results

A four-layer model with a PEC bottom layer is shown in Fig. 3. The working frequency is set to be $f = 2.6$ GHz, and the source and observation points are at the interface of the air layer (top layer) and the dielectric layer. We apply the traditional DCIM for small ρ, and switch it to the new DCIM when ρ is large. The transition region can be set in

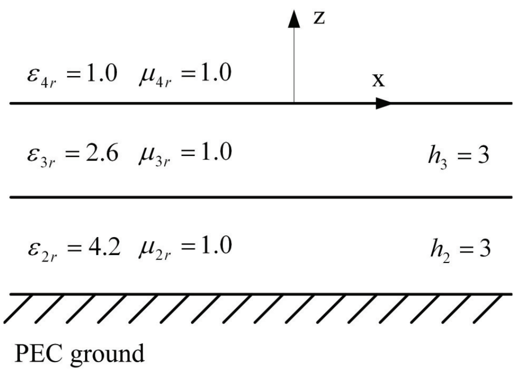

Fig. 3. The four-layer model with a PEC bottom layer, unit: mm.

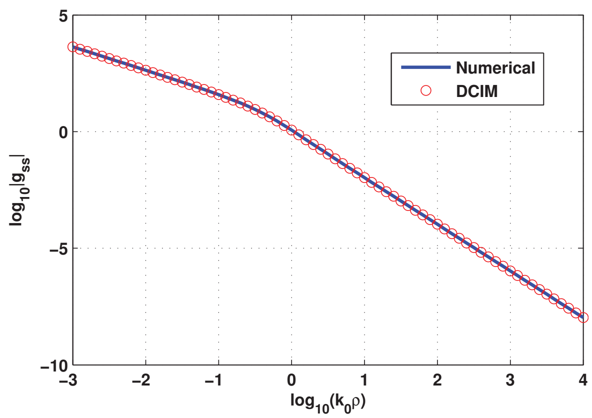

Fig. 4. The magnitude of g_{ss} versus $k_0\rho$ in log scale for the four-layer model. The working frequency is $f = 2.6$ GHz, and the source and observation points are at the air-substrate interface. The asymptotic behavior is $1/\rho^2$.

$10^0 < k_0\rho < 10^1$, where k_0 is the free space wavenumber. We here set the transition point at $k_0\rho = 10^{0.5}$. The g_{ss} shown in (3) is first calculated by the DCIM, with magnitude versus $k_0\rho$ shown in Fig. 4. The results agree very well with that of numerical integration. Next the g_ϕ in (4) is calculated and shown in Fig. 5. Again good agreement between the DCIM and the numerical integration can be observed. Since there is only one real TM pole found for this layered medium at this frequency, the far field of g_{ss} is dominated by the lateral wave ($\sim 1/\rho^2$) and that of g_ϕ is dominated by the guided wave ($\sim 1/\rho^{\frac{1}{2}}$), as are shown in Fig. 4 and Fig. 5.

IV. Conclusion

A new discrete complex image method (DCIM) is developed to accurately evaluate the far field interaction for the layered medium Green's function, as a complement to the traditional DCIM. By contour deformation, the Green's function can be naturally decomposed into the radiation modes and guided modes. The guided modes can be obtained by a robust pole-searching algorithm and the radiation modes can be calculated in a closed form, so the evaluation can be made efficient compared to the direct numerical integration. For near field, we can switch back to the traditional DCIM, since the latter

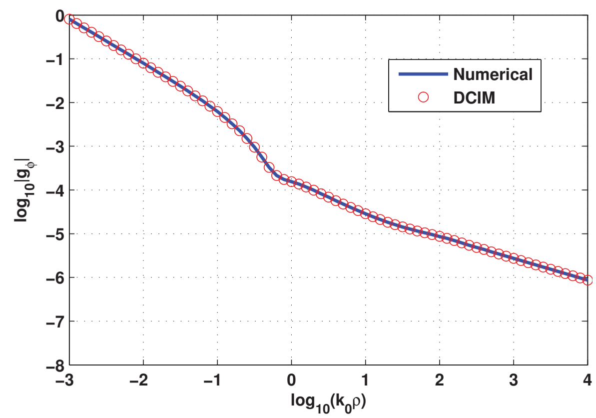

Fig. 5. The magnitude of g_ϕ versus $k_0\rho$ in log scale for the four-layer model. The working frequency is $f = 2.6$ GHz, and the source and observation points are at the air-substrate interface. The asymptotic behavior is $1/\rho^{\frac{1}{2}}$.

one is well-developed and has good performance for the near field interaction.

Acknowledgment

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