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| Author（s） | Qiao，D；Pang，GKH |
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# Two-range Connectivity-based Sensor Network Localization 

Dapeng Qiao, Grantham K.H. Pang<br>Department of Electrical and Electronic Engineering<br>The University of Hong Kong<br>\{dpqiao, gpang\}@eee.hku.hk


#### Abstract

This paper presents a new connectivity-based sensor network localization method. The novelty is on a two-level range/indication of connectivity between each pair of nodes. The method relies on the information of the connectivity which is either strong, weak or zero. Research results in this paper have shown that such sensor node information can give better accuracy in localization. We have also obtained a suitable setting for the two-level ranges. Modified algorithms based on MDS, DV-hop and SDP have also been developed with the 2-level range. The simulation results show that the 2-level range-based MDS, DVhop and SDP are more accurate than the usual 1-level range connectivity-based method.


## 1. Introduction

Position estimation is necessary in many applications such as remote patient monitoring, package and personnel tracking, environment monitoring and wildlife habitat monitoring. In these systems, there could be hundreds or even thousands of low-cost sensor nodes, which can take some simple measurements. Based on either the signal strength or the connectivity among the nodes, we would like to estimate the location of these nodes in the sensor network. It is necessary to accurately localize the sensors in order to measure data which is geographically meaningful. This localization issue has been studied by many researchers and there are many different methods and algorithms [1-4] dealing with this situation.

In a typical sensor network, a few nodes have known positions, and they are called the anchors. However, the positions of the majority of the nodes need to be estimated using their relationships to the anchors and other unknown nodes. Based on whether the distances between nodes in a sensor network are known or not, the localization methods can be grouped
into two categories: range-based and range-free. Range-based methods can be applied to the situation in which the distances between each pair of nodes are estimated or measured. The information is then communicated to a centralized station in the sensor network and algorithms such as MDS[3] compute the location of each sensor in the network. Usually, the distance between each pair of nodes is estimated by the signal strength received between them, and this information is very noisy in practice. On the other hand, range-free methods, which can also be called connectivity-based methods, assume that the distances between any two nodes are unknown. However, connectivity information between them is known. If the distance between any two nodes in the network is within a range, connectivity between the two nodes is said to be established. Although the actual distance is not known, this would provide many connectionimposed proximity constraints to the problem. These connectivity-based methods only require very simple and low-cost hardware. Yet, they can give adequate position estimation based on just connectivity information among the nodes. Current connectivitybased methods assume only one communication range among all the nodes. In this paper, we modify the nature of the problem and allow the connectivity between two nodes to be at two levels. Hence, more information about connectivity can be obtained which is helpful to localize the nodes accurately.

The use of a two-level communication range is available in practice. The topic about controlling the range of transmission (including the transmission of omni-directional antenna) has been studied for many years [5] and there are techniques to change the range of transmission for two-level range nodes. An example of applying controllable transmission power on sensor network can be found in [6], which is a Mica2 sensor node developed by UC Berkeley. The radio part of this sensor node is a ChipCon CC1000 radio, which supports programmable transmission power levels ranging from -20 dBm to +10 dBm .

## 2. Problem definition

A formal definition of the two-range connectivitybased localization problem is given next. Let $G=(V, E)$ be a given network, where $V$ denotes the nodes of the network and $E$ denotes the edge of the network. Let $V$ be partitioned into two sets: $V_{a}=\{1, \ldots, m\}$ of anchors, and $V_{b}=\{m+1, \ldots, m+n\}$ of sensors. $E$ is also partitioned into two sets: $E_{a b}=\left\{(i, j) \in E: i \in V_{a}, j \in V_{b}\right\}$ which are the edges between a sensor and an anchor. $E_{b b}=\left\{(i, j) \in E: i, j \in V_{b}\right\}$ which are the edges between two sensors. For each anchor $i \in V_{a}$, the position $a_{i} \in \mathfrak{R}^{2}$ is assumed to be known. For each sensor $i \in V_{b}$, the position $b_{i} \in \mathfrak{R}^{2}$ is assumed to be unknown.

Let $C_{a b}=\left\{(i, j, k): i \in V_{a}, j \in V_{b}, k \in\{0,0.5,1\}\right\}$ be the connectivity information between a sensor and an anchor. Also let $C_{b b}=\left\{(i, j, k): i, j \in V_{b}, k \in\{0,0.5,1\}\right\}$ be the connectivity information between two sensors. The value $k$ in $C_{a b}$ or $C_{b b}$ has the value $0,0.5$ or 1 :
$k=0$ if there is no connection between node $i$ and $j$.
$k=0.5$ for a strong connection between node $i$ and $j$.
$k=1$ for a weak connection between node $i$ and $j$.
Let $a$ be a vector containing the positions of the anchors $a=\left(a_{i}\right)_{i \in V_{a}} \in \mathfrak{R}^{2 m}$. The goal of the two-range connectivity-based network localization problem is to determine the coordinates of all the sensors (unknown nodes) $b=\left(b_{i}\right)_{i \in V_{b}} \in \mathfrak{R}^{2 n}$ such that $b$ satisfies the following constraints:

$$
\begin{aligned}
& \text { If } k=1\left\{\begin{array}{l}
\left\|a_{i}-b_{j}\right\|_{2}^{2} \leq R_{B}{ }^{2} \text { for }(i, j) \in E_{a b} \\
\left\|b_{i}-b_{j}\right\|_{2}{ }^{2} \leq R_{B}{ }^{2} \text { for }(i, j) \in E_{b b}
\end{array}\right. \\
& \text { elseif } k=0.5\left\{\begin{array}{l}
R_{A}^{2} \geq\left\|a_{i}-b_{j}\right\|_{2}^{2}>R_{B}{ }^{2} \text { for }(i, j) \in E_{a b} \\
R_{A}^{2} \geq\left\|b_{i}-b_{j}\right\|_{2}{ }^{2}>R_{B}{ }^{2} \text { for }(i, j) \in E_{b b}
\end{array}\right. \\
& \text { else } k=0 \begin{cases}\left\|a_{i}-b_{j}\right\|_{2}^{2}>R_{A}{ }^{2} \text { for }(i, j) \in E_{a b} . \\
\left\|b_{i}-b_{j}\right\|_{2}^{2}>R_{A}{ }^{2} \text { for }(i, j) \in E_{b b}\end{cases}
\end{aligned}
$$

where $R_{B}$ is the maximum distance (called the rangeB) within which strong connectivity can be established. $R_{A}$ is the maximum distance (called the range A ) within which weak connectivity can be established.

## 3. Related work

Current connectivity-based localization algorithms on sensor networks are all based on only one
communication range, and they include the multidimensional scaling-MAP (MDS-MAP) [3], DVhop [11] and the convex position estimation (CPE) [2].

### 3.1. MDS [3, 9]

The basic MDS method [3, 9] can estimate the positions of all the unknown nodes by using the distance information between any two nodes. An extension of MDS [3, 14] for the connectivity-based localization problem has also been developed. First, a rough estimate of the relative node distance is made. Then, the relative positions are obtained by using a Singular Value Decomposition on the estimated distance information matrix. Finally, absolute positions of the unknown nodes are estimated based on the relative positions and the positions of the anchors. The computation complexity of this method is about $O\left(n^{3}\right)$ time for a sensor network of $n$ nodes.

### 3.2. DV-Hop [11]

Another well-known localization algorithm is DVhop (distance vector-hop). One hop is one direct connection between two nodes. Hop count between two nodes roughly represents the distance. DV-hop applies this idea and estimates the distance using the hop count. It counts the hop between the anchors, and estimates the average distance per hop, then this information is used to estimate the distance between anchor and normal nodes with the hop count between them. At last, with distances between a node and the nearest 3 anchors, the position is obtained by triangulation.

DV-hop (distance vector - hop) is first proposed by Niculescu [12,13], and has been improved by many researchers. It needs any anchor to broadcast their position information to other anchors, and such information will be flooded with the hop count increment. Every anchor knows the hop count from any other anchors. The information of this hop count and the anchors' position will be used in further estimation of unknown nodes.

### 3.3. Convex Constraints in Localization[2]

The connectivity between two nodes would tell whether the distance between these two nodes is less than a certain communication range [2]. The convex position estimation (CPE) uses this information in convex optimization and narrows the possible area by the solutions of the optimization.

Many researchers have formulated the connectivitybased localization problem as an optimization problem with some convex constraints. When two nodes are connected, the distance between them must be within a range distance $(R)$. All the connections are then expressed by semi-definite inequalities. As all the constraints are convex, this method is called semi-
definite programming (SDP) or convex programming. For example, the convex constraints that represent the connectivity among nodes are ( $k=1$ ):
$\sqrt{\left(x_{i}-x_{j}\right)^{2}+\left(y_{i}-y_{j}\right)^{2}} \leq R$
for node $i\left(x_{i}, y_{i}\right)$ in connection with node $j\left(x_{j}, y_{j}\right)$.

## 4. Two-level range

The connectivity between nodes in a sensor network is dependent on the range of the nodes, which means the maximum distance within which the two nodes can communicate. If we change the range, information on the connectivity would also be changed. Following this idea, a situation is shown below. Suppose a node can operate on a two-level range, such as Figure 1. The range of A is assumed to be a circle with radius of $R_{A}$, and the range of B is assumed to be a circle with radius of $R_{B}$.


Figure 1 A node with two different ranges in a node $N$
An example of part of a network is given in Figure 2. The range of $B$ for every node is expressed by dotted line. From the ranges of A and B , we know that the distance between node 1 and node 2 is less than $R_{B}$. As another example, the distance between node 1 and node 3 is less than $R_{A}$, but more than $R_{B}$. Compared with the previous connectivity-based sensor network with just one range $R_{A}$, this kind of two-range network can give us more useful information on node connectivity and provide more accurate localization results.


Figure 2 An example of part of a sensor network

Since the transmit power of the sensors can be controlled and changed (an example can be found in [5]), we can arrange one device to operate in two transmit power states. On the receiver side, the device is always listening for incoming messages, regardless of transmission activity. Therefore, the range of the device only relies on the transmit power, and changes when transmit power is changing.

For our device, the transmit power can have two states: one state for a longer range, and the other for a shorter range. Transmit power changes between these two states to get two different ranges at different time instants. Certainly, the communication information between nodes should include which range the nodes are using.

Therefore, the situation can be as follows.

1. There are some nodes in a sensor network. Some of the nodes are with known positions. Others' positions are unknown.
2. In each node, one device is attached. The device can transmit information using two transmit power states. One transmit power state makes the communication range longer. Another state makes the range shorter. The longer range is called range A , with radius $R_{A}$, and the shorter range is called range B with radius $R_{B}$.
3. The connectivity between any two devices in the sensor network is the information for estimating the location of all the unknown nodes.

This two-level range configuration gives more information to sensor node localization.

## 5. How to set the 2nd-level range?

First, we want to compute the average distance between the node $N$ at the center and the nodes which are located in the smaller circle (which are within Range B in Figure 1).

Assumption 1: The locations of the nodes in the network are random.

Since the locations of the nodes in the network are random, the nodes locations are also random if they are in range B. Therefore, we assume the probability of the nodes locating at any point in the area of the small circle should be even. Based on this assumption, we can get this conclusion: the probability of the nodes located in a particle area can be represented by its area.

The smaller circle (range B) is constructed by infinite number of thin rings with different radiuses. The black ring in Figure 3 shows a particular ring, whose radius is $r$, and thickness is $\Delta r$. Their radiuses $r$ should be from 0 to $R_{B}$, and their thickness $\Delta r$ should
be all infinite small.


Figure 3 Black ring is the basic element of the smaller circle (shaped area)

Based on Assumption 1, the probability of the nodes in the black area is $\frac{2 \pi r \Delta r}{\pi R_{B}{ }^{2}}$. To integrate the probability, the average distance away the node $N$ can be obtained with $r$ being from 0 to $R_{B}$.

$$
\begin{equation*}
d_{1}=\int_{0}^{R_{B}} \frac{2 \pi r^{2}}{\pi R_{B}^{2}} d r=\frac{1}{\pi R_{B}^{2}} \int_{0}^{R_{B}} 2 \pi r^{2} d r=\frac{2}{3} R_{B} \tag{1}
\end{equation*}
$$

$d_{1}$ is the average of the distance between the center (position of node $N$ ) and the nodes which are within range $B$.

Next, we calculate the average distance between the center and the nodes which are outside circle $R_{B}$, but within circle $R_{A}$ by the similar integration. Since $\Delta r$ is infinite small, we assume the nodes, which are in the belt of the ring (the black area in Figure 4), to be with the same distance to the center (node $N$ ).


Figure 4 The area between range $A$ and range $B$ are also decomposed into many rings (the black area)

The average distance of the nodes located between $N$ and the nodes which are not in the circle $R_{B}$, but are in the circle $R_{A}$ should be

$$
\begin{align*}
& d_{2}=\int_{R_{B}}^{R_{A}} \frac{2 \pi r^{2}}{\pi R_{A}{ }^{2}-\pi R_{B}{ }^{2}} d r=\frac{2}{3} \frac{R_{A}{ }^{3}-R_{B}{ }^{3}}{R_{A}{ }^{2}-R_{B}{ }^{2}}  \tag{2}\\
& =\frac{2}{3} \frac{R_{A}{ }^{2}+R_{A} R_{B}+R_{B}{ }^{2}}{R_{A}+R_{B}}
\end{align*}
$$

We let $R_{A}=\alpha R_{B}$, then the above equation becomes

$$
\begin{equation*}
d_{2}=\frac{2}{3} \frac{\alpha^{2}+\alpha+1}{\alpha+1} R_{B} \quad \text { where } \alpha=R_{A} / R_{B}, \alpha>1 \tag{3}
\end{equation*}
$$

Next, we would like to find the best ratio $\alpha=R_{A} / R_{B}$ which would provide more accurate estimation of the node location.

The presentation of one hop in range $B$ should be a number from $0-1$, which can be added to the hop count in range $A$, which uses 1 to represent one hop. Among the integers of hop count in range $A$, setting 0.5 to each hop in range $B$ will make the difference between the range A and range B most obvious. In that case,

$$
d_{1}=0.5 * d_{2}
$$

With equations (1) and (3), we get

$$
\alpha^{2}-\alpha-1=0 \rightarrow \alpha=(\sqrt{5}+1) / 2=1.618
$$

Therefore, if we want the average distance of the node within Range $B$ to be half of the average distance of the node beyond Range B but within Range A (i.e. $\left.d_{1}=0.5^{*} d_{2}\right), R_{A} / R_{B}$ should be 1.618. That means $R_{B} / R_{A}$ is $(\sqrt{5}-1) / 2$ or 0.618 , which is the golden ratio.

## 6. Two-level Range-based MDS, DV-Hop \& SDP and Simulation results

An example is given in this section (Figure 5). When calculating the node location, if any two nodes are within Range $A$ (but beyond Range $B$ ), a value/weight of $1\left(d_{2}\right)$ would be assigned. If the two nodes are within Range B , then a value/weight of 0.5 $\left(d_{1}\right)$ would be assigned. With $d_{1}=0.5, d_{2}=1$, $\alpha=1.618$, we can calculate $R_{B}$ to be 0.75 from equation (3). Hence, $R_{A}$ is 1.2135 . The weights ( $d_{1} \&$ $d_{2}$ ) are also used for computing the hop count. For example, in Figure 5, the hop count between 2 and 5 is 1 , the hop count between 1 and 2 is 0.5 , the hop count between 1 and 5 is 1.5 .


Figure 5 An example: two nodes in range A is connected by line, two node in range B is connected by bold line.
Based on above configuration, the hop count between any two nodes in the network can be calculated. If there are more than one path between two nodes, the smallest hop count would be used. The
search for the shortest path can be carried out by the Dijkstra's algorithm.

MDS can give the relative coordinates of the unknown nodes. The relative coordinates can be shifted, rotated and reflected to give the absolute coordinates based on position of 3 anchors.


Figure 6 A sensor network example for DV-hop, strong connections are expressed by bold lines
In the above example, anchors broadcast their position information to other anchors. Hop counting will be done in this process. However, the two-level range can also give a weight into the hop count. i.e. hops in range B will have the weight of 0.5 , while hops in range $A$ with weight of 1 . Detailed description of DV-hop's procedures can be found in [6].

As an example, node 1 is found using DV-hop. Minimal hop count from $A 3$ to $A 1$ is 4.5 . Minimal hop count from $A 3$ to $A 2$ is 5.5 . Then the average hop distance (AHD) of anchor $A 3$ is $A H D_{3}=(15+18) /(4.5+5.5)=3.3$. There are many methods to find AHD of unknown node. Here, the closet anchor's AHD is used. A3 is the closet anchor to node 1 , so we use AHD of $A 3$ to estimate distance between node 1 and all the anchors.

$$
\begin{aligned}
& d_{1, A 1}=2.5 \times A H D_{3}=8.25 \\
& d_{1, A 2}=3.5 \times A H D_{3}=11.55 \\
& d_{1, A 3}=2 \times A H D_{3}=6.6
\end{aligned}
$$

At last, triangulation will be used to localize node 1 .
For SDP, the two-range connectivity-based convex constraints which form a semi-definite programming problem are solved using the Mosek Optimization Toolbox [15].

The simulation for comparing 1 -level range and 2 level range MDS has been carried out as follows. There are 104 nodes in a square of $[0,10]$ by $[0,10]$, in which, 24 nodes are anchors. Any two nodes are in contact with each other if and only if the distance between the two nodes is within Range. The hop counts between any two nodes are known. For the two- range case, any two nodes are in strong contact with each other if the distance between the two nodes is within Range*Ratio. Ratio has different values from 0.3 to 0.8 as the
abscissa in Figure 7-12.
To find the improvement of 2-level range on DV hop compared with 1 -level range on DV-hop, the similar simulation settings are conducted. 2-level range gives additional constraints for SDP, the result of 2level SDP is guaranteed to be more accurate. Simulation on SDP is also given below.

With Range is set as 2 , Figures 7,8 and 9 show that better accuracy is obtained using the 2-level range in which MDS, DV-hop and SDP are used for sensor node localization. While the error of 1-level range of the method is set as $100 \%$, the percentage of the error of 2-level range with the same method is drawn as small triangles in the figures.

For another Range, which is 1.5 , the result is shown in Figure 10-12 of the three methods. From Figure 712 , the 2 -level range can improve the accuracy by $20 \%-40 \%$ as most. Since the best accuracy is obtained when Ratio is $0.5,0.6$, or 0.7 , it is proved that the preferred ratio between the two ranges can be set as Golden ratio.


Figure 7 Average error of MDS for different range ratio


Figure 8 Average error of DV-hop for different range ratio


Figure 9 Average error of SDP for different range ratio


Figure 10 Average error of MDS for different range ratio


Figure 11 Average error of DV-hop for different range ratio


Figure 12 Average error of SDP for different range ratio

## 7. Conclusion

To improve the accuracy of sensor network localization based on connectivity, current methods mostly focus on algorithm. In this paper, a new twolevel range of connectivity between sensor nodes is proposed. As a result, more detailed connectivity information can be obtained, and hence the localization accuracy can be improved. We have also obtained a suitable setting for the two ranges. Modified algorithms based on MDS, DV-Hop and SDP are developed with the 2-level range. The simulation results show that the 2-level range-based MDS, DV-Hop and SDP are more accurate than normal MDS, DV-Hop and SDP respectively.

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