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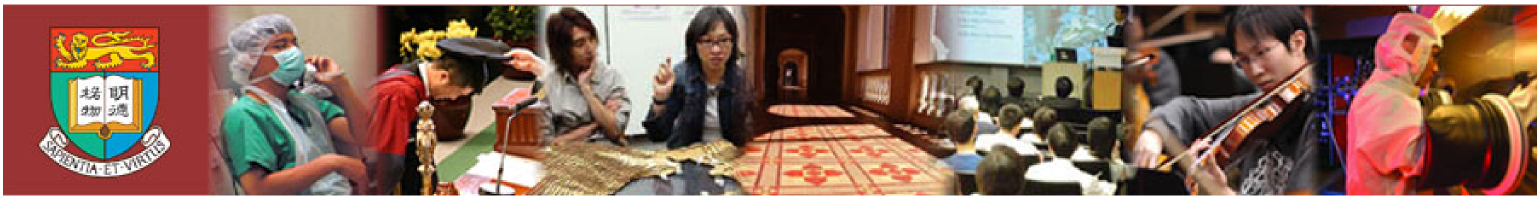

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Adaptive Beamforming for Uniform Linear Arrays With Unknown Mutual Coupling

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***Abstract*—This letter proposes a new adaptive beamforming algorithm for uniform linear arrays (ULAs) with unknown mutual coupling. It is based on the fact that the mutual coupling matrix (MCM) of a ULA can be approximated as a banded symmetric Toeplitz matrix as the mutual coupling between two sensor elements is inversely related to their separation, and hence it is negligible when they are separated by a few wavelengths. Taking advantage of this specific structure of the MCM, a new approach to calibrate the signal steering vector is proposed. By incorporating this improved steering vector estimate with a diagonally loaded robust beamformer, a new adaptive beamformer for ULA with unknown mutual coupling is obtained. Simulation results show that the proposed steering vector estimate considerably improves the robustness of the beamformer in the presence of unknown mutual coupling. Moreover, with appropriate diagonal loading, it is found that the proposed beamformer can achieve nearly optimal performance at all signal-to-noise ratio (SNR) levels.**

***Index Terms*—Adaptive beamforming, mutual coupling, uniform linear array (ULA).**

I. Introduction

TRADITIONALLY, methods for array signal processing were developed by assuming that the steering vector is exactly known to users, given the array geometry and the signal location. However, such an assumption is far from reality since the steering vector in real systems may be easily distorted by various impairments such as mutual coupling. It is known that mutual coupling is generally caused by the interaction between the sensor elements and can seriously degrade the performance of array signal processing methods, including direction finding and beamforming techniques. Therefore, the calibration of an array with unknown mutual coupling is of great importance. Among various calibration techniques, much effort has been devoted to the problem of direction finding, and interested readers are referred to [1]–[5] and references therein.

In this letter, we consider the problem of beamforming for uniform linear arrays (ULAs) in the presence of unknown mutual coupling. As mentioned earlier, mutual coupling would lead to the steering vector mismatches and hence significantly degrade the performance of adaptive beamforming. In order to develop robust methods for beamforming with steering vector mismatches, quadratic constraints on the Euclidean norm of the beamformer weight vector or the array steering vector mismatch are considered in [6] and [7], where the array covariance matrix is diagonally loaded with an appropriate multiple of the identity matrix in order to satisfy the imposed quadratic constraint. However, the loading level in these methods is not directly related to the uncertainty bounds of the steering vector. To overcome this problem, a series of diagonal loading-based robust Capon beamforming (RCB) algorithms has been proposed [8]–[10], where the loading level can be determined according to the uncertainty set of the steering vector.

It is worth noting that the above-mentioned methods do not make use of the structure of the mismatched steering vector. On the other hand, some uncertainties such as the mutual coupling considered in this letter do possess certain structure. For instance, in the case of a ULA, the mutual coupling matrix (MCM) can be represented by a symmetric Toeplitz matrix [11]. Furthermore, it is known that the mutual coupling between two sensors is inversely related to their distance, and thus it can be ignored when these two sensors are separated by few wavelengths. Therefore, for a ULA with M sensor elements, the MCM can be sufficiently modeled as a banded symmetric Toeplitz matrix as follows [2]–[5], [11]:

$$\boldsymbol{C} = \text{Toeplitz}\left\{1, c_1, \cdots, c_{P-1}, \mathbf{0}^{1\times(M-P)}\right\}. \qquad (1)$$

As can be seen from (1), it is assumed that when the distance between two sensors is more than P intersensor spacing, the mutual coupling coefficients are assumed to be zero.

Given the above MCM model, we consider a ULA with M sensors. Ideally, the steering vector of the signal of interest (SOI) is given by

$$\boldsymbol{a}_0 = [1, \beta, \ldots, \beta^{M-1}]^T \qquad (2)$$

where $\beta = e^{j2\pi f c^{-1} d \sin\theta}$, d denotes the intersensor spacing, f is the frequency, and c is the wave propagation velocity, θ is the direction-of-arrival (DOA), and the superscript $(\cdot)^T$ denotes matrix transposition. Taking the mutual coupling into account, the true steering vector should be rewritten as

$$\boldsymbol{a} = \boldsymbol{C} a_0. \qquad (3)$$

In the following sections, we consider the problem of beamforming with known DOA of SOI but unknown mutual coupling as in [11]; the specific structure of the MCM will be utilized for the calibration of the steering vector, which is then employed to improve the performance of beamforming.

II. Robust Beamforming

A. RCB

Due to the unknown MCM in (1), the signal steering vector in (3) deviates from its nominal value and is in general unknown. If the nominal steering vector $\boldsymbol{a}_0$ is directly adopted for Capon

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beamforming, the signal may be wrongly suppressed as the interference leading to signal cancellation. To overcome this problem, it is assumed in [8] and [9] that the steering vector lies inside an uncertainty ellipsoid of certain size

$$\|\boldsymbol{a}-\boldsymbol{a}_0\|^2 \leq \varepsilon. \tag{4}$$

Consequently, the problem of steering vector estimation for robust beamforming can be formulated as

$$\min_{\boldsymbol{a}} \boldsymbol{a}^H \hat{\boldsymbol{R}}^{-1} \boldsymbol{a} \text{ suject to } \|\boldsymbol{a}-\boldsymbol{a}_0\|^2 \leq \varepsilon \tag{5}$$

where the superscript $(\cdot)^H$ denotes the complex conjugate transpose operator. $\hat{\boldsymbol{R}}$ is the array covariance matrix, which is usually estimated from N snapshots as $\hat{\boldsymbol{R}} = N^{-1}\boldsymbol{X}X^H$, where $\boldsymbol{X} = [\boldsymbol{x}(1), \cdots, \boldsymbol{x}(N)]$. It has been shown in [9] that (5) can be solved using the *Lagrange multiplier method*, with the solution given by

$$\hat{\boldsymbol{a}}_{\mathrm{RCB}} = \boldsymbol{a}_0 - (\boldsymbol{I} + \mu\hat{\boldsymbol{R}})^{-1}\boldsymbol{a}_0 \tag{6}$$

where $\boldsymbol{I}$ is an $M \times M$ identity matrix, and μ can be calculated from the constraint equation $\|(\boldsymbol{I}+\mu\hat{\boldsymbol{R}})^{-1}\boldsymbol{a}_0\|^2 = \varepsilon$ using Newton's method. The estimated steering vector $\hat{\boldsymbol{a}}_{\mathrm{RCB}}$ is then used to obtain the robust Capon beamformer as

$$\boldsymbol{w}_{\mathrm{RCB}} = \frac{\hat{\boldsymbol{R}}^{-1}\hat{\boldsymbol{a}}_{\mathrm{RCB}}}{\hat{\boldsymbol{a}}_{\mathrm{RCB}}^H \hat{\boldsymbol{R}}^{-1}\hat{\boldsymbol{a}}_{\mathrm{RCB}}}. \tag{7}$$

B. Robust Beamforming Using Middle Subarray

As earlier discussed, the MCM can be modeled as a banded symmetric Toeplitz matrix, which has been adopted for robust beamforming against unknown mutual coupling in [11], where the authors showed that the steering vector of the middle subarray with $M-2P+2$ elements can be written as

$$\tilde{\boldsymbol{a}} = g\boldsymbol{a}_{\mathrm{MS}} \tag{8}$$

where

$$g = 1 + \sum_{i=1}^{P-1} c_i(\beta^i + \beta^{-i}) \tag{9}$$

and $\boldsymbol{a}_{\mathrm{MS}}$ is the ideal steering vector of the middle subarray, which is denoted as

$$\boldsymbol{a}_{\mathrm{MS}} = [1, \beta, \ldots, \beta^{M-2P+1}]^T. \tag{10}$$

Letting $\hat{\boldsymbol{R}}_{\mathrm{MS}}$ be the array covariance matrix estimate of the middle subarray, one can get the robust beamformer as

$$\boldsymbol{w}_{\mathrm{MS}} = \frac{\hat{\boldsymbol{R}}_{\mathrm{MS}}^{-1}\tilde{\boldsymbol{a}}}{\tilde{\boldsymbol{a}}^H\hat{\boldsymbol{R}}_{\mathrm{MS}}^{-1}\tilde{\boldsymbol{a}}} = \frac{\hat{\boldsymbol{R}}_{\mathrm{MS}}^{-1}\boldsymbol{a}_{\mathrm{MS}}}{g^*\boldsymbol{a}_{\mathrm{MS}}^H\hat{\boldsymbol{R}}_{\mathrm{MS}}^{-1}\boldsymbol{a}_{\mathrm{MS}}} \tag{11}$$

where the superscript $*$ denotes complex conjugate. The array covariance matrix of the middle subarray can be obtained as $\hat{\boldsymbol{R}}_{\mathrm{MS}} = N^{-1}\boldsymbol{X}_{\mathrm{MS}}\boldsymbol{X}_{\mathrm{MS}}^H$, where $\boldsymbol{X}_{\mathrm{MS}} = [\boldsymbol{x}_{\mathrm{MS}}(1), \cdots, \boldsymbol{x}_{\mathrm{MS}}(N)]$ represents N snapshots of the middle subarray. Moreover, a diagonal loaded (DL) form of the beamformer in (11) can be given by

$$\boldsymbol{w}_{\mathrm{DL-MS}} = \frac{(\hat{\boldsymbol{R}}_{\mathrm{MS}} + \lambda\tilde{\boldsymbol{I}})^{-1}\boldsymbol{a}_{\mathrm{MS}}}{g^*\boldsymbol{a}_{\mathrm{MS}}^H(\hat{\boldsymbol{R}}_{\mathrm{MS}} + \lambda\tilde{\boldsymbol{I}})^{-1}\boldsymbol{a}_{\mathrm{MS}}} \tag{12}$$

where λ is the diagonal loading factor, and $\tilde{\boldsymbol{I}}$ is an $(M-2P+2) \times (M-2P+2)$ identity matrix.

It is worth noting that only the middle subarray is used for beamforming. This implies that only a part of the array aperture is effective, and in general this will lead to performance degradation compared to other robust beamforming methods based on the whole array. Motivated by the shortcoming, we now derive a new robust beamforming method in unknown mutual coupling, which makes full use of the whole array to improve the performance.

C. Proposed Robust Beamforming

Instead of the uncertainty ellipsoid in Section II-A and middle subarray in Section II-B, we reexploit the specific structure of the MCM and derive a method for calibration the steering vector, which is then utilized for improving the performance of adaptive beamforming. According to the MCM and signal models above, the steering vector of the array can be rewritten as [4], [5]

$$\boldsymbol{a} = [1 + \sum_{i=1}^{P-1} c_i(\beta^i + \beta^{-i})]\Gamma\boldsymbol{a}_0 = g\Gamma\boldsymbol{a}_0 \tag{13}$$

where Γ is an $M \times M$ diagonal matrix given by

$$\Gamma = \mathrm{diag}[\mu_1 \quad \cdots \quad \mu_{P-1} \quad 1 \quad \cdots \quad 1 \quad \alpha_1 \quad \cdots \quad \alpha_{P-1}]. \tag{14}$$

In (14), there are $M-2P+2$ ones between the entry μ_{P-1} and α_1. Moreover, μ_k and α_k, $k = 1, \cdots, P-1$, are given by

$$\mu_k = \frac{\beta^{P-1} + \sum_{i=1}^{k-1} c_i\beta^{P-1-i} + \sum_{i=1}^{P-1-k} c_i\beta^{P-1+i}}{\beta^{P-1} + \sum_{i=1}^{P-1} c_i\beta^{P-1-i} + \sum_{i=1}^{P-1} c_i\beta^{P-1+i}} \tag{15a}$$

and

$$\alpha_k = \frac{\beta^{P-1} + \sum_{i=1}^{P-1} c_i\beta^{P-1-i} + \sum_{i=1}^{P-1-k} c_i\beta^{P-1+i}}{\beta^{P-1} + \sum_{i=1}^{P-1} c_i\beta^{P-1-i} + \sum_{i=1}^{P-1} c_i\beta^{P-1+i}}. \tag{15b}$$

From (14), it can be noted that angularly independent mutual coupling can be viewed as angularly dependent array gain and phase uncertainties. Since Γ is a diagonal matrix with $M-2P+2$ ones and $\boldsymbol{a}$ is a column vector, (13) can be rewritten as the following parameterization:

$$\boldsymbol{a} = g\Gamma\boldsymbol{a}_0 = g\boldsymbol{T}v \tag{16}$$

where $\boldsymbol{T}$ is an $M \times (2P-1)$ block diagonal matrix given by

$$\boldsymbol{T} = \mathrm{blkdiag}\{\boldsymbol{T}_1, \boldsymbol{T}_2, \boldsymbol{T}_3\} \tag{17}$$

where blkdiag constructs from input arguments, $\boldsymbol{T}_1 = \mathrm{diag}\{1, \beta, \cdots, \beta^{P-2}\}$, $\boldsymbol{T}_2 = [\beta^{P-1}, \cdots, \beta^{M-P}]^T$, and

$\boldsymbol{T}_3 = \operatorname{diag}\{\beta^{M-P+1}, \cdots, \beta^{M-1}\}$. In (16), $\boldsymbol{v}$ is a $(2P-1) \times 1$ vector given by

$$\boldsymbol{v} = [\mu_1 \quad \cdots \quad \mu_{P-1} \quad 1 \quad \alpha_1 \quad \cdots \quad \alpha_{P-1}]^T. \tag{18}$$

It can be seen that the Pth entry of $\boldsymbol{v}$ is equal to 1, and it can be written in a compact form as

$$\boldsymbol{v}^H \boldsymbol{e} = 1 \tag{19}$$

where $\boldsymbol{e}$ is a $(2P-1) \times 1$ vector with the Pth entry being 1, and 0 elsewhere.

We now derive the robust beamformer based on the above properties. First, we assume that there is also bounded array covariance matrix mismatch, which is denoted by Δ and a known constant bound γ, i.e.,

$$\|\Delta\|_{\mathrm{F}} \leq \gamma \tag{20}$$

where $\|\cdot\|_{\mathrm{F}}$ represents the Frobenius norm. As a result, we first have the following robust beamforming problem:

$$\min_{\boldsymbol{w}} \max_{\|\Delta\|_{\mathrm{F}} \leq \gamma} \boldsymbol{w}^H (\hat{\boldsymbol{R}} + \Delta) \boldsymbol{w} \text{ subject to } \boldsymbol{w}^H \boldsymbol{a} = 1 \tag{21}$$

whose cost function is equivalent to $\min_{\boldsymbol{w}} \boldsymbol{w}^H (\hat{\boldsymbol{R}} + \gamma \boldsymbol{I}) \boldsymbol{w}$, and the resultant solution is given by the following diagonal loaded beamformer [12]:

$$\boldsymbol{w}_{\mathrm{DL}} = \frac{(\hat{\boldsymbol{R}} + \gamma \boldsymbol{I})^{-1} \boldsymbol{a}}{\boldsymbol{a}^H (\hat{\boldsymbol{R}} + \gamma \boldsymbol{I})^{-1} \boldsymbol{a}}. \tag{22}$$

Note that the $\boldsymbol{a}$ above in (22) is still unknown. We propose to maximize the output power subject to the constraint imposed by the MCM structure in (19). This yields the following problem for estimating the steering vector:

$$\begin{aligned} &\min_{\boldsymbol{a}} \boldsymbol{a}^H (\hat{\boldsymbol{R}} + \gamma \boldsymbol{I})^{-1} \boldsymbol{a} \\ &\text{subject to } \boldsymbol{a} = g \boldsymbol{T} \boldsymbol{v} \text{ and } \boldsymbol{v}^H \boldsymbol{e} = 1 \end{aligned} \tag{23}$$

which is equivalent to the optimization problem with respect to $\boldsymbol{v}$ as

$$\min_{\boldsymbol{v}} |g|^2 \boldsymbol{v}^H \boldsymbol{T}^H (\hat{\boldsymbol{R}} + \gamma \boldsymbol{I})^{-1} \boldsymbol{T} \boldsymbol{v} \text{ subject to } \boldsymbol{v}^H \boldsymbol{e} = 1. \tag{24}$$

First, we assume that the constant $|g|^2$ is nonzero, so that it can be ignored and the optimization problem can be solved using the *Lagrange multiplier method*. After some manipulation, it can be shown that the solution is given by

$$\hat{\boldsymbol{v}} = \frac{(\boldsymbol{T}^H (\hat{\boldsymbol{R}} + \gamma \boldsymbol{I})^{-1} \boldsymbol{T})^{-1} \boldsymbol{e}}{\boldsymbol{e}^H (\boldsymbol{T}^H (\hat{\boldsymbol{R}} + \gamma \boldsymbol{I})^{-1} \boldsymbol{T})^{-1} \boldsymbol{e}}. \tag{25}$$

According to (16), the steering vector can be estimated as

$$\hat{\boldsymbol{a}}_{\mathrm{PRO}} = g \boldsymbol{T} \hat{\boldsymbol{v}} \tag{26}$$

and the proposed robust beamformer is given by

$$\boldsymbol{w}_{\mathrm{PRO}} = \frac{(\hat{\boldsymbol{R}}+\gamma \boldsymbol{I})^{-1} \hat{\boldsymbol{a}}_{\mathrm{PRO}}}{\hat{\boldsymbol{a}}_{\mathrm{PRO}}^H (\hat{\boldsymbol{R}}+\gamma \boldsymbol{I})^{-1} \hat{\boldsymbol{a}}_{\mathrm{PRO}}} = \frac{(\hat{\boldsymbol{R}}+\gamma \boldsymbol{I})^{-1} \boldsymbol{T} \hat{\boldsymbol{v}}}{g^* \hat{\boldsymbol{v}}^H \boldsymbol{T}^H (\hat{\boldsymbol{R}}+\gamma \boldsymbol{I})^{-1} \boldsymbol{T} \hat{\boldsymbol{v}}}. \tag{27}$$

It can be seen that different from RCB and middle subarray methods, we make use of the specific structure of the MCM as well as the whole array for steering vector estimation. It should be noted that the above approach is derived by assuming that g is nonzero, whereas this constant may be zero for some peculiar angles (blind angles) under some special mutual coupling coefficients $c_i, i = 1, \cdots, P-1$. In this case, the steering vector $\boldsymbol{a}$ in (3) and (13) is zero, i.e., $\boldsymbol{a} = 0$. Hence, if the signal impinges on the array from a blind angle, it will be inherently canceled by the array. This implies that a ULA with such MCM cannot be used to receive signals from these blind angles. To deal with this problem, one possible and simple way is rotating the array to a certain angle, which is equivalent to changing the DOA of the SOI. For simplicity, we only focus on the case of $g \neq 0$ in this letter.

It is worth noting that the proposed method can be applied to uniform circular array (UCA) due to its circularly symmetric geometry. However, details are omitted due to the page limitation. Moreover, it can be noticed that the complexity of the proposed algorithm and RCB is $O(M^3)$, whereas the middle subarray-based method has a complexity of $O((M-2P+2)^3)$.

III. Simulation Results

To investigate the performance of the proposed robust beamformer, a ULA with $M = 10$ sensors separated by half a wavelength is considered. One SOI and two interferences impinge on the array from the far field at angles $\theta_0 = 0°, \theta_1 = -20°$, and $\theta_2 = 30°$, respectively. The noise power is 0 dB, and the interference-to-noise ratios (INRs) of the two interferences are assumed to be 30 and 20 dB, respectively. First, we assume that $P = 2, \boldsymbol{C} = \operatorname{Toeplitz}\{1, 0.75e^{-j\pi/3}, \boldsymbol{0}^{1\times 8}\}, \gamma = 0$, and the signal-to-noise ratio (SNR) is 0 dB. Based on the theoretical array covariance matrix, we depict the beampatterns of various beamformers. It can be seen from Fig. 1(a) that both the RCB and proposed beamformer perform well. However, the conventional MVDR beamformer cannot achieve satisfactory sidelobe suppression. Note that, for a fair comparison, the required parameters for different approaches, such as ε for RCB, are chosen according to the suggestions provided in the respective references. Hence, in all experiments, the bound value ε of RCB is chosen as $\varepsilon = \|(\sqrt{M}\boldsymbol{a}/\|\boldsymbol{a}\|) - \boldsymbol{a}_0\|^2$, so that the steering vector, i.e., $\sqrt{M}\boldsymbol{a}/\|\boldsymbol{a}\|$, has the same norm of the nominal one. We now vary the SNR from -10 to 30 dB, 1000 snapshots are collected to compute the array covariance matrix, and the signal-to-interference-plus-noise ratio (SINR) at each SNR is averaged from 100 Monte Carlo experiments. Fig. 1(b) shows the output SINRs versus SNR of various beamformers, including MVDR, RCB, and diagonal loading (DL) with the loading level being 10 times of the noise power. We can notice that when the mutual coupling between elements is slight, the proposed method outperforms the conventional methods both at low SNRs and high SNRs. We now compare the output SINR of the proposed method with the robust method in (11) using the middle subarray [11]. It can be seen from Fig. 1(c) that due to the reduction of the array aperture of [11], there is a significant performance loss of this method.

Following the above settings, we assume that $P = 4$ and $\boldsymbol{C} = \operatorname{Toeplitz}\{1, 0.75e^{-j\pi/3}, 0.45e^{j\pi/3}, 0.15e^{j\pi/10}, \boldsymbol{0}^{1\times 6}\}$; the output SINRs of various beamformers versus SNR are

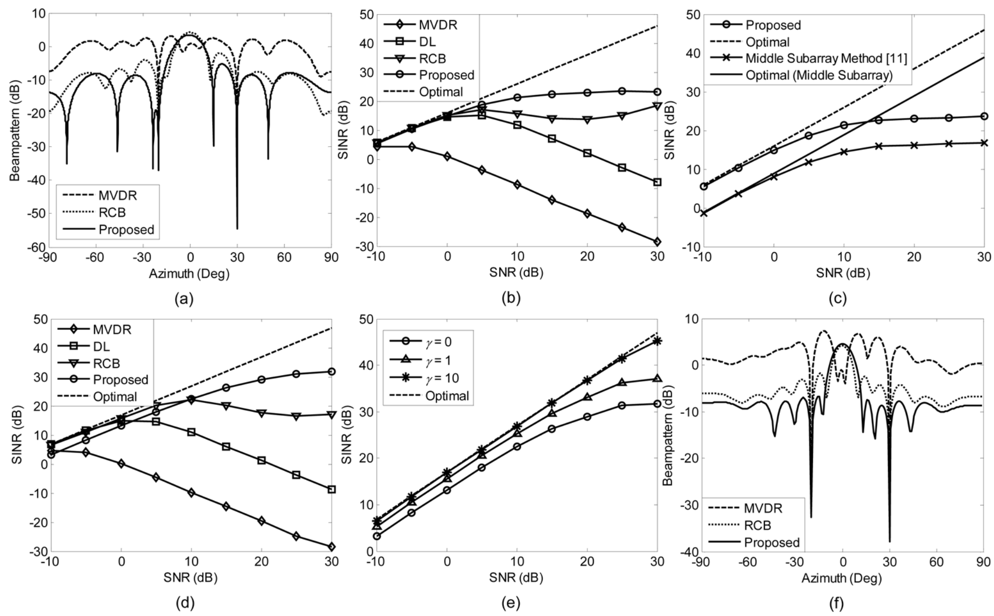

Fig. 1. (a) Beampatterns of various beamformers, $\gamma = 0$. (b) Output SINRs versus SNR with $P = 2$ and $\gamma = 0$. (c) Output SINRs versus SNR with $P = 2, \gamma = 0$, and $\lambda = 0$. (d) Output SINRs versus SNR with $P = 4$. (e) Output SINRs versus SNR with $P = 4$ and different γ's. (f) Beampatterns of various beamformers based on CST EM simulation using a ULA with 12 vertical dipoles, $f = 3$ GHz, $d = \lambda/4$, $P = 4$, and $\gamma = 10$.

shown in Fig. 1(d). It can be found that at low SNRs, the diagonal loading-based methods (i.e., DL and RCB) are slightly more robust to the unknown mutual coupling. On the other hand, at high SNRs, the proposed beamformer achieves much better performance. In all experiments above, we assume that the diagonal loading factor $\gamma = 0$. We now examine the influence of γ to our proposed method. Fig. 1(e) shows the SINRs versus SNR of our proposed method with different diagonal loading factors. Obviously, we notice that the performance of our proposed method can be significantly improved. Furthermore, it is found that when the factor is chosen to be 10, i.e., 10 times of the noise power as the traditional DL beamformer, our proposed method can even achieve nearly optimal performance both at low and high SNRs since it takes the uncertainty of the covariance matrix into account. Moreover, simulation results show that the suggested factor works well for different INR levels. Due to the page limitation, related results are omitted here.

Finally, electromagnetic (EM) simulation based on the CST Microwave Studio software is used to verify the effectiveness of the proposed method. A ULA with 12 vertical dipoles is constructed. The frequency is 3 GHz, and hence $\lambda = 10$ cm. The length of each dipole is $\lambda/2$, and the interelement spacing is $\lambda/4$. The three signals used in the first example are employed here, and the measurements across the antenna terminals were collected and processed by the proposed method with $P = 4$ and $\gamma = 10$, as well as conventional methods. Fig. 1(f) shows the resulting beampatterns. It can be noted that the interferences can be well rejected by the proposed method and RCB, but our method can generally achieve lower sidelobe.

IV. Conclusion

A new robust beamforming algorithm for ULAs with unknown mutual coupling is presented. It makes use of the banded symmetric Toeplitz matrix structure of the MCM to encapsulate the structure of the steering vector. By maximizing the output power, the steering vector and hence the robust beamformer can be estimated analytically. Simulation results demonstrate the improved performance of the proposed beamformer over conventional methods.

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