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Robust Beamforming With Magnitude Response Constraints Using Iterative Second-Order Cone Programming

B. Liao, K. M. Tsui, and S. C. Chan

Abstract—The problem of robust beamforming for antenna arrays with arbitrary geometry and magnitude response constraints is one of considerable importance. Due to the presence of the non-convex magnitude response constraints, conventional convex optimization techniques cannot be applied directly. A new approach based on iteratively linearizing the non-convex constraints is then proposed to reformulate the non-convex problem to a series of convex subproblems, each of which can be optimally solved using second-order cone programming (SOCP). Moreover, in order to obtain a more robust beamformer against array imperfections, the proposed method is further extended by optimizing its worst-case performance using again SOCP. Different from some conventional methods which are restricted to linear arrays, the proposed method is applicable to arbitrary array geometries since the weight vector, rather than its autocorrelation sequence, is used as the variable. Simulation results show that the performance of the proposed method is comparable to the optimal solution previously proposed for uniform linear arrays, and it also gives satisfactory results under different array specifications and geometries tested.

Index Terms—Adaptive beamforming, linear and arbitrary arrays, magnitude response, second-order cone programming (SOCP), worst-case optimization.

I. INTRODUCTION

Sensor array processing using antenna arrays have been successfully applied to many engineering fields including wireless communications,

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radar, radio astronomy, etc. In particular, the theoretical and applied aspects of beamforming have received great research interests during the last decades [1]. One of the most popular beamformers is the minimum variance distortionless response (MVDR) beamformer, which is developed based on an ideal antenna array with exactly known array manifold. However, antenna arrays in real systems may suffer from various types of uncertainties or mismatches, such as look direction mismatch, imperfectly known sensor positions and orientations, and the mismatch between the estimated array covariance matrix and theoretical one. It is known that the performance of the MVDR beamformer may considerably degrade due to the existence of these imperfections. Therefore, much effort has been spent on improving the robustness of the MVDR beamformer with appropriate additional linear or quadratic constraints [1]–[9].

However, most of the above mentioned robust methods lack the flexibility in controlling the beamwidth and response ripple in the look direction. As a result, they may not offer sufficient robustness against large look direction errors [10]. Recently, a number of advanced robust beamforming methods have been proposed to address this issue by imposing prescribed magnitude response constraints over a given beamwidth in the look direction [11], [12], where the magnitude constraints are non-convex. In particular, the approach in [12] simplified the problem by expressing the beamformer weight vector in terms of its autocorrelation sequence. Hence, similar to filter design method in [13], the autocorrelation sequence can be solved optimally using linear programming, and the beamformer weight vector can be determined using an additional step of spectral factorization. However, this method is derived based on the assumption that the array covariance matrix is a Hermitian Toeplitz matrix. Hence, its performance may degrade when the assumption is violated due to array imperfections. Also, it is found that this method may further be affected by the numerical error of the spectral factorization employed. More importantly, it is restricted to linear arrays with inter-element spacing being integer multiples of a base distance, because the conventional spectral factorization is only well developed for one-dimensional Laurent polynomials [13]. As a result, it may not be applicable directly to other array geometries.

In this communication, we propose a new method for addressing the robust beamforming problem with magnitude response constraints using iterative SOCP. The basic idea of the proposed approach is to linearize the non-convex magnitude squared response constraints in a neighborhood of the complex array weights in each iteration. For this linearization, it is shown that the problem of finding the optimal updates around the previous iterates is a convex SOCP problem that can be efficiently solved. It should be noted that, different from the outer product matrix used in [11] and autocorrelation sequence used in [12], the beamformer weight vector can be directly obtained in the proposed approach, and hence the extra spectral factorization is not required. Thus, the proposed approach is generally applicable to arrays with arbitrary geometries. Motivated by the conventional robust adaptive beamformers [14], we further extend the proposed approach to deal with the optimization of worst-case performance in order to obtain a more robust beamformer against possible array imperfections. This suggests that the proposed approach offers a general framework for the design of beamformers of arbitrary array geometries in satisfying different commonly used robustness requirements.

II. ROBUST BEAMFORMER DESIGN

A. MVDR Beamformer

Consider an arbitrary antenna array with M sensors, it is well known that the conventional MVDR beamformer is chosen by minimizing the

output power subject to a constraint of unity array response at the direction of arrival (DOA) of the signal of interest (SOI). That is

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R}_x \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{a}(\theta_0, \phi_0) = 1 \end{aligned} \quad (1)$$

where \mathbf{w} is the $M \times 1$ complex array beamformer weight vector and the superscript H denotes the Hermitian transpose. $\mathbf{a}(\theta_0, \phi_0)$ is the $M \times 1$ steering vector corresponding to the DOA of the SOI, i.e., (θ_0, ϕ_0) . \mathbf{R}_x is the $M \times M$ array covariance matrix, which can be estimated using N received samples $\{\mathbf{x}(1), \dots, \mathbf{x}(N)\}$ as $\hat{\mathbf{R}}_x = N^{-1} \sum_{t=1}^N \mathbf{x}(t) \mathbf{x}^H(t)$.

The constraint $\mathbf{w}^H \mathbf{a}(\theta_0, \phi_0) = 1$ prevents the gain in the DOA of SOI from being reduced, and the solution of (1) can be easily determined using Lagrange multiplier method as:

$$\mathbf{w}_{MVDR} = \frac{\mathbf{R}_x^{-1} \mathbf{a}(\theta_0, \phi_0)}{\mathbf{a}^H(\theta_0, \phi_0) \mathbf{R}_x^{-1} \mathbf{a}(\theta_0, \phi_0)}. \quad (2)$$

However, it is known that the performance of the MVDR beamformer in (2) is sensitive to the mismatch between the nominal and true steering vectors due to the uncertainty in the DOA of the SOI as well as other array imperfections [1]–[12].

B. Robust Beamforming Using Iterative SOCP (RB-ISOCP)

A possible way to improve the robustness of the MVDR beamformer is to impose additional linear equality constraints. However, this approach may lead to a decrease in degrees of freedom in interference rejection. Recently, much effort has been made to overcome this problem [11], [12]. More precisely, instead of equality constraints, inequality constraints are used to control the array response in the region of interest (ROI), where the SOI comes with a high probability. To maintain a fairly stable gain in the ROI, the following inequality constraints on the magnitude response are imposed [12]:

$$L(\theta, \phi) \leq |G(\theta, \phi)| \leq U(\theta, \phi), \quad (\theta, \phi) \in \Omega \quad (3)$$

where $G(\theta, \phi) = \mathbf{w}^H \mathbf{a}(\theta, \phi)$ denotes the array response, $L(\theta, \phi)$ and $U(\theta, \phi)$ are respectively the prescribed lower and upper bounds of the magnitude response, and Ω denotes the ROI. Consequently, the robust beamforming with the magnitude constraints in (3) can be written as:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R}_x \mathbf{w} \\ \text{s.t.} \quad & L(\theta, \phi) \leq \left| \mathbf{w}^H \mathbf{a}(\theta, \phi) \right| \leq U(\theta, \phi), \quad (\theta, \phi) \in \Omega. \end{aligned} \quad (4)$$

It can be seen that (4) is not a convex optimization problem due to the presence of the non-convex constraint $L(\theta, \phi) \leq |\mathbf{w}^H \mathbf{a}(\theta, \phi)|$. As a result, conventional convex programming techniques are not directly applicable. Recently, a number of studies have been devoted to solving this problem [11], [12]. However, these approaches are only suitable for some specific antenna arrays, such as uniform linear arrays (ULAs). We now propose to solve the non-convex optimization problem (4) with an iterative SOCP technique, which has been successfully applied to power pattern synthesis [15]. An important advantage of the proposed method is that it can be applied to arbitrary array geometries.

To start with, we first rewrite the problem in (4) as (e.g. [7])

$$\begin{aligned} \min_{\mathbf{z}} \quad & \|\mathbf{C}\mathbf{z}\| \\ \text{s.t.} \quad & H(\mathbf{z}) \leq U^2(\theta, \phi), \quad (\theta, \phi) \in \Omega \\ & H(\mathbf{z}) \geq L^2(\theta, \phi), \quad (\theta, \phi) \in \Omega \end{aligned} \quad (5)$$

where $H(\mathbf{z}) = \mathbf{z}^T \mathbf{Q} \mathbf{z} = \mathbf{w}^H \mathbf{R}_x \mathbf{w}$, $\mathbf{z} = [\text{Re}\{\mathbf{w}^T\}, \text{Im}\{\mathbf{w}^T\}]^T$, $\mathbf{Q} = \begin{bmatrix} \text{Re}\{\mathbf{R}_x\} & -\text{Im}\{\mathbf{R}_x\} \\ \text{Im}\{\mathbf{R}_x\} & \text{Re}\{\mathbf{R}_x\} \end{bmatrix}$, $\mathbf{R}_x = \mathbf{a}(\theta, \phi) \mathbf{a}^H(\theta, \phi)$, and \mathbf{C} is the Cholesky factor of $\mathbf{R} = \begin{bmatrix} \text{Re}\{\mathbf{R}_x\} & -\text{Im}\{\mathbf{R}_x\} \\ \text{Im}\{\mathbf{R}_x\} & \text{Re}\{\mathbf{R}_x\} \end{bmatrix} = \mathbf{C}^T \mathbf{C}$.

In what follows, we shall describe the proposed algorithm for solving the non-convex problem in (5). Suppose that our algorithm starts with a reasonably feasible initial guess \mathbf{z}_0 and arrives at a point \mathbf{z}_k after k iterations. At a sufficiently small neighborhood of \mathbf{z}_k , the magnitude squared response of the array, $H(\mathbf{z})$, which is smooth, can be approximated by the following linear approximation:

$$H(\mathbf{z}_k + \boldsymbol{\delta}) \approx H(\mathbf{z}_k) + \mathbf{g}^T(\mathbf{z}_k) \boldsymbol{\delta} \quad (6)$$

where $\mathbf{g}(\mathbf{z})$ is the gradient of $H(\mathbf{z})$ with respect to \mathbf{z} and $\boldsymbol{\delta}$ is the linear update vector to be determined to satisfy (3) under the approximation in (6). As $\mathbf{Q} = \mathbf{Q}^T$, we have $\mathbf{g}(\mathbf{z}_k) = 2\mathbf{Q}\mathbf{z}_k$. Once $\boldsymbol{\delta}$ is available, the new solution can be updated as $\mathbf{z}_{k+1} = \mathbf{z}_k + \boldsymbol{\delta}$. This process is repeated until the relative change of two successive solutions is sufficiently small or the maximum number of iterations is reached. To determine $\boldsymbol{\delta}$, we substitute $\mathbf{z} = \mathbf{z}_k + \boldsymbol{\delta}$ into (5) and obtain

$$\begin{aligned} \min_{\boldsymbol{\delta}} \quad & \|\mathbf{C}\mathbf{z}_k + \mathbf{C}\boldsymbol{\delta}\| \\ \text{s.t.} \quad & H(\mathbf{z}_k) + \mathbf{g}^T(\mathbf{z}_k) \boldsymbol{\delta} \leq U^2(\theta, \phi), \quad (\theta, \phi) \in \Omega \\ & H(\mathbf{z}_k) + \mathbf{g}^T(\mathbf{z}_k) \boldsymbol{\delta} \geq L^2(\theta, \phi), \quad (\theta, \phi) \in \Omega \\ & \|\boldsymbol{\delta}\| \leq \boldsymbol{\delta}_{\max} \end{aligned} \quad (7)$$

where an additional quadratic constraint $\|\boldsymbol{\delta}\| \leq \boldsymbol{\delta}_{\max}$ is imposed to ensure that the linear approximation in (6) is sufficiently accurate. Obviously, we can see that the optimization problem in (7) is convex, and it can be solved using SOCP by discretizing the ROI as in [12] and [15]. Hence, the new iteration can be updated using the optimal $\boldsymbol{\delta}$ as $\mathbf{z}_{k+1} = \mathbf{z}_k + \boldsymbol{\delta}$. For the sake of presentation, the proposed method described above is referred to as robust beamforming using iterative SOCP (RB-ISOCP). With appropriate choice of initial guess to be presented in Section II-D, the proposed algorithm converges quickly to a satisfactory solution as we shall elaborate further by the simulation results in Section III.

C. RB-ISOCP via Worst-Case Performance Optimization (RB-ISOCP-WC)

It is worth noting that the robust beamformer derived in the previous sub-section is based on the assumption that the covariance matrix \mathbf{R}_x is known exactly or well-estimated. However, certain mismatches between the nominal covariance matrix $\check{\mathbf{R}}_x$ and the actual one \mathbf{R}_x always exist in practice due to various kinds of array imperfections as mentioned in Section I. To address this uncertainty in the true covariance matrix, the following mismatch model is adopted in this communication:

$$\check{\mathbf{R}}_x = \mathbf{R}_x + \Delta \quad (8)$$

where Δ is an unknown Hermitian error matrix of \mathbf{R}_x , and its Frobenius norm is bounded by a certain known constant $\gamma > 0$ as $\|\Delta\| \leq \gamma$. Using a similar idea as in the conventional method [14], the worst-case performance of the robust beamformer in (4) can be optimized by solving the following problem

$$\begin{aligned} \min_{\mathbf{w}} \quad & \max_{\Delta} \mathbf{w}^H (\mathbf{R}_x + \Delta) \mathbf{w} \\ \text{s.t.} \quad & \|\Delta\| \leq \gamma \\ & L(\theta, \phi) \leq \left| \mathbf{w}^H \mathbf{a}(\theta, \phi) \right| \leq U(\theta, \phi), \quad (\theta, \phi) \in \Omega \end{aligned} \quad (9)$$

which is again a non-convex problem. Following the descriptions in [14], we first solve the following problem

$$\max_{\Delta} \mathbf{w}^H (\mathbf{R}_x + \Delta) \mathbf{w} \quad \text{s.t.} \quad \|\Delta\| \leq \gamma \quad (10)$$

with respect to Δ . It can be shown that the solution of (10) is given by $\Delta = \gamma \mathbf{w} \mathbf{w}^H / \|\mathbf{w}\|^2$. Substituting this solution back to (10) yields the maximum value $\mathbf{w}^H (\mathbf{R}_x + \gamma \mathbf{I}) \mathbf{w}$, where \mathbf{I} is an $M \times M$ identity matrix. Hence, the problem in (9) can be rewritten as

$$\min_{\mathbf{w}} \mathbf{w}^H (\mathbf{R}_x + \gamma \mathbf{I}) \mathbf{w} \\ \text{s.t.} \quad L(\theta, \phi) \leq \left| \mathbf{w}^H \mathbf{a}(\theta, \phi) \right| \leq U(\theta, \phi), \quad (\theta, \phi) \in \Omega. \quad (11)$$

It can be seen that the problems in (4) and (11) are identical except for the covariance matrices in the respective objective function. Therefore, the iterative SOCP algorithm in the previous sub-section can be similarly employed to solve (11). More precisely, at the k th iteration, the problem in (11) can be approximated as the following convex problem

$$\min_{\delta} \|\check{\mathbf{C}} \mathbf{z}_k + \check{\mathbf{C}} \delta\| \\ \text{s.t.} \quad H(\mathbf{z}_k) + \mathbf{g}^T(\mathbf{z}_k) \delta \leq U^2(\theta, \phi), \quad (\theta, \phi) \in \Omega \\ H(\mathbf{z}_k) + \mathbf{g}^T(\mathbf{z}_k) \delta \geq L^2(\theta, \phi), \quad (\theta, \phi) \in \Omega \\ \|\delta\| \leq \delta_{\max} \quad (12)$$

where $\check{\mathbf{C}}$ is obtained by the Cholesky factorization of the regularized array covariance matrix $\mathbf{R}_x + \gamma \mathbf{I}$ as

$$\check{\mathbf{R}} = \begin{bmatrix} \text{Re}\{\mathbf{R}_x + \gamma \mathbf{I}\} & -\text{Im}\{\mathbf{R}_x + \gamma \mathbf{I}\} \\ \text{Im}\{\mathbf{R}_x + \gamma \mathbf{I}\} & \text{Re}\{\mathbf{R}_x + \gamma \mathbf{I}\} \end{bmatrix} = \check{\mathbf{C}}^T \check{\mathbf{C}}. \quad (13)$$

Consequently, the problem in (12) can be solved using SOCP by discretizing the ROI.

D. Choice of the Initial Guess \mathbf{z}_0

As described earlier, the proposed robust beamformers are obtained through an iterative procedure. Hence, it is important to choose a reasonably good initial guess \mathbf{w}_0 for the problem in (4) or $\mathbf{z}_0 = [\text{Re}\{\mathbf{w}_0^T\}, \text{Im}\{\mathbf{w}_0^T\}]^T$ for the problem in (7) to obtain a satisfactory solution. Following the recommendations in [15], the non-convex constraint in (3) is first rewritten as

$$\left| \mathbf{w}^H \mathbf{a}(\theta, \phi) \right| - F(\theta, \phi) \leq E(\theta, \phi), \quad (\theta, \phi) \in \Omega \quad (14)$$

where $E(\theta, \phi) = (U(\theta, \phi) - L(\theta, \phi))/2$ and $F(\theta, \phi) = (U(\theta, \phi) + L(\theta, \phi))/2$. Then, it is approximated as the following convex constraint:

$$\left| \mathbf{w}^H \mathbf{a}(\theta, \phi) - F(\theta, \phi) \right| \leq E(\theta, \phi), \quad (\theta, \phi) \in \Omega. \quad (15)$$

Hence, the initial guess can be obtained by solving the following SOCP problem

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_x \mathbf{w} \\ \text{s.t.} \quad \left| \mathbf{w}^H \mathbf{a}(\theta, \phi) - F(\theta, \phi) \right| \leq E(\theta, \phi), \quad (\theta, \phi) \in \Omega \quad (16)$$

for the problem in (7), and

$$\min_{\mathbf{w}} \mathbf{w}^H (\mathbf{R}_x + \gamma \mathbf{I}) \mathbf{w} \\ \text{s.t.} \quad \left| \mathbf{w}^H \mathbf{a}(\theta, \phi) - F(\theta, \phi) \right| \leq E(\theta, \phi), \quad (\theta, \phi) \in \Omega \quad (17)$$

for the worst-case optimization problem in (12). It should be noted that there may be other approaches to find such feasible initial guess. Nevertheless, given the initial guess designed above, the proposed algorithm converges quite quickly to a satisfactory result. For instance in Example 1, the initial guess converges to a nearly optimal solution in only five iterations for a ULA with a beamwidth of 8° and a ripple of 0.4 dB.

E. Convergence Behavior and Complexity

It should be noted that the algorithm presented above converges to a local solution due to the linear approximation. Fortunately, as illustrated subsequently in a representative example for the design of robust beamformers in ULAs (Example 1 in Section III-A), the proposed algorithm is capable of finding solutions that are very close to the optimal ones obtained using the method in [12]. In fact, when applying the proposed algorithm to this problem, we did not fail in finding a nearly optimal solution for every specification we have tried. The good convergence performance of the proposed algorithm is largely attributed to the global convergence of individual subproblem, as suggested in [15]. On the other hand, the proposed algorithm might also be viewed as a SOCP-based trust region method with simplified update steps [16]. Thanks to the efficient interior-point method, the step size and step direction characterized by the norm bound constraint in each convex subproblem can be optimally solved. Through extensive computer simulations, it is also interesting to note that the norm of δ tends to converge to zero as the iteration increases regardless of the value of δ_{\max} . Therefore, the choice of δ_{\max} becomes less critical. This allows us to set a larger norm bound initially to speed up the convergence and hence significantly reduce the computational complexity.

Similar to the conventional methods in [8], [12], the total complexity of the proposed algorithm mainly depends on the complexity of solving each subproblem using convex optimization, which is $O(M^{3.5} + (2J + 1)M^{2.5})$, where J is the number of sampled points in the ROI [8]. With the dramatic increase in computing power and advanced coding techniques, it is suggested in [17] that nowadays convex optimization can almost be carried out in real-time for a modest-size problem. Nevertheless, taking the number of iterations into account, the proposed algorithm should in general have higher computational complexity than the conventional methods in [12]. Fortunately, since the number of iterations is usually small as mentioned earlier, the increase in computational complexity is still affordable by the virtue of the efficient convex optimization solver.

III. SIMULATION RESULTS

In our simulations, robust beamformer designs for ULAs and uniform circular arrays (UCAs) are considered to evaluate the performance of the proposed methods. In all experiments, the CVX Matlab Toolbox [18] is employed to solve the SOCP optimization problems. Also, we let $L(\theta, \varphi) = 10^{-r_{ab}/20}$ and $U(\theta, \varphi) = 10^{r_{ab}/20}$ for a given ripple r_{ab} in decibel scale.

A. Example-I: ULA

In this example, a ULA with $M = 10$ sensor elements separated by half wavelength is considered. Let the ROI be $\Omega = [\theta_L, \theta_U]$ so that the beamwidth of the ROI is $\theta_U - \theta_L$. The ROI is discretized with a step size of 0.1° . Two equal power interferences with an interference-to-noise

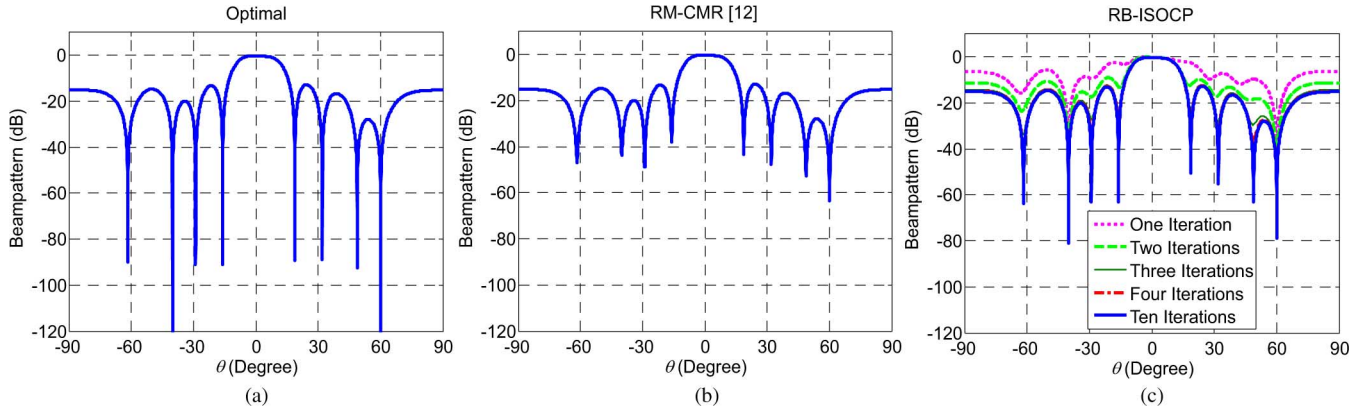


Fig. 1. Beam patterns of various beamformers with beamwidth of 8° and ripple of $r_{db} = 0.4$ dB. (a) Optimal beamformer. (b) RB-CMR. (c) RB-ISOCP with different numbers of iteration.

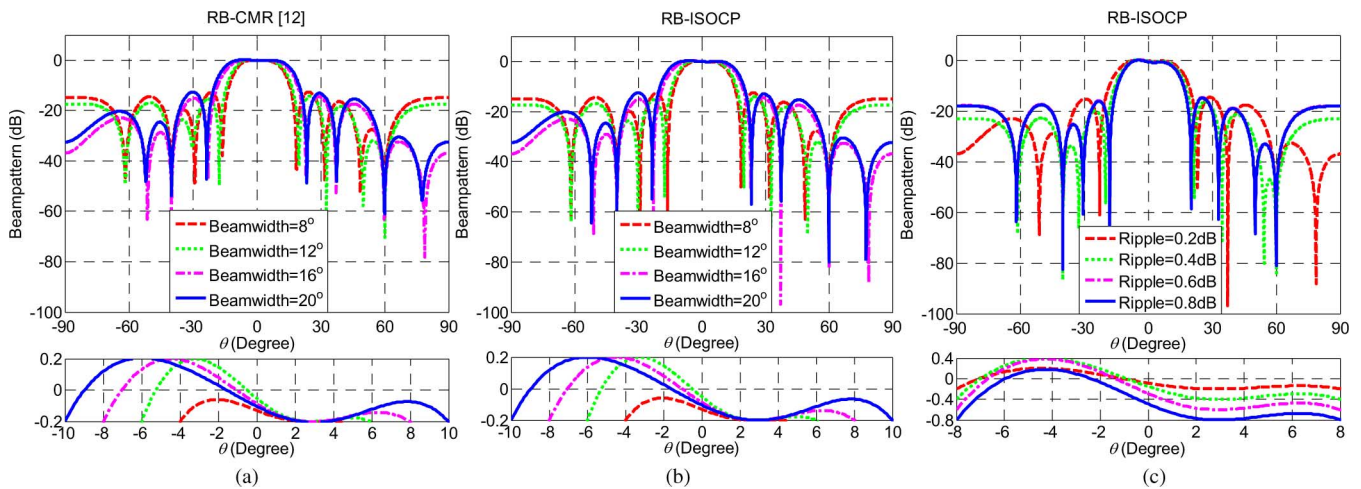


Fig. 2. (a) Beam patterns of the RB-CMR with different beamwidths and a fixed ripple $r_{db} = 0.2$ dB. (b) Beam patterns of the RB-ISOCP with different beamwidths and a fixed ripple $r_{db} = 0.2$ dB. (c) Beam patterns of the RB-ISOCP with different ripples and a fixed beamwidth of 16° .

ratio (INR) of 20 dB are assumed to impinge on the array from far-field at angles $\theta_1 = -40^\circ$ and $\theta_2 = 60^\circ$. The DOA of the SOI is assumed to be $\theta_0 = 3^\circ$, whereas the nominal direction is 0° .

1) *Infinite Sample Case*: Firstly, we test the performance of the proposed method with an ideal array covariance matrix \mathbf{R}_x by assuming that the number of snapshots N is infinite. The signal-to-noise ratio (SNR) of the SOI is 10 dB. Let the ripple be $r_{db} = 0.4$ dB and the ROI be $\Omega = [-4^\circ, 4^\circ]$, i.e., the designed beamwidth of the ROI is 8° . The maximum norm of the linear update vector δ is chosen to be $\delta_{\max} = 0.3$. Fig. 1(c) shows beam patterns obtained using the RB-ISOCP with different number of iterations. It can be seen that the proposed method converges in five iterations, so that the beam patterns so obtained after five and ten iterations are nearly identical.

As a comparison, we consider the robust beamformer with constraints on magnitude response (RB-CMR) studied in [12] (see also Matlab code therein). The optimal pattern based on the autocorrelation of \mathbf{w} is shown in Fig. 1(a), and it will be used as a gold standard to assess the performance of the proposed approach in this particular problem. It can be seen from Fig. 1(b) that the pattern obtained after spectral factorization (RB-CMR [12]) is quite different from the optimal one and its interference rejection level are significantly degraded. This is mainly attributed to the numerical error caused by spectral factorization. It should be noted there may be other spectral factorization methods with better numerical behaviors. However, we did not intend to modify the code used in [12] for a fair comparison. On

the other hand, Figs. 1(c) show the proposed beam patterns obtained within ten iterations. It can be seen that the proposed RB-ISOCP exhibits deeper nulls than RB-CMR, and its beam pattern is very close to the optimal one.

Next, we show the beam patterns of the conventional RB-CMR and the proposed RB-ISOCP with different beamwidths and a fixed ripple of 0.2 dB. From Figs. 2(a) and (b), it can be seen that all beam patterns obtained from these two methods are comparable and they are very close to the optimal ones in the sense that the main beam specifications are precisely satisfied. However, the proposed RB-ISOCP can generally form deeper nulls at the directions of the interferences. Similar arguments hold for other simulation results as shown in Fig. 2(c), where the beamwidth is fixed and the ripple varies. The above results suggest that the proposed approach is effective in finding nearly optimal solutions.

2) *Finite Sample Case*: It is known that any kinds of array imperfections would result in uncertainties of the array covariance matrix. In this example, the uncertainties caused by insufficient snapshots will be considered to evaluate the robustness of the proposed method. The simulation settings are summarized as follows: The ripple is 0.2 dB and the beamwidth of the ROI is 16° . Fig. 3(a) shows the resultant beam patterns of the proposed RB-ISOCP with different numbers of snapshots N . It can be seen that the sidelobe level of the proposed RB-ISOCP is severely affected by insufficient snapshots, although the beam gain in the ROI can be well controlled. Also, its performance is improved as the number of snapshots increases.

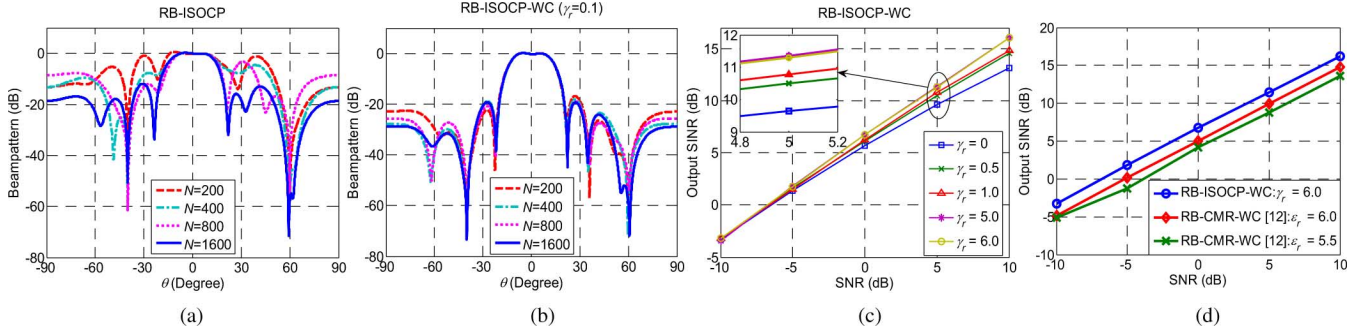


Fig. 3. (a) Beam patterns of RB-ISOCP with different numbers of snapshots. (b) Beam patterns of RB-ISOCP-WC with different numbers of snapshots and a fixed relative regularization factor $\gamma_r = 0.1$. (c) The output SINR of RB-ISOCP versus SNR with different relative regularization factors. (d) Comparison of the output SINR of RB-ISOCP-WC and RB-CMR-WC. Common settings: The ripple is $r_{db} = 0.2$ dB and the beamwidth is 16° .

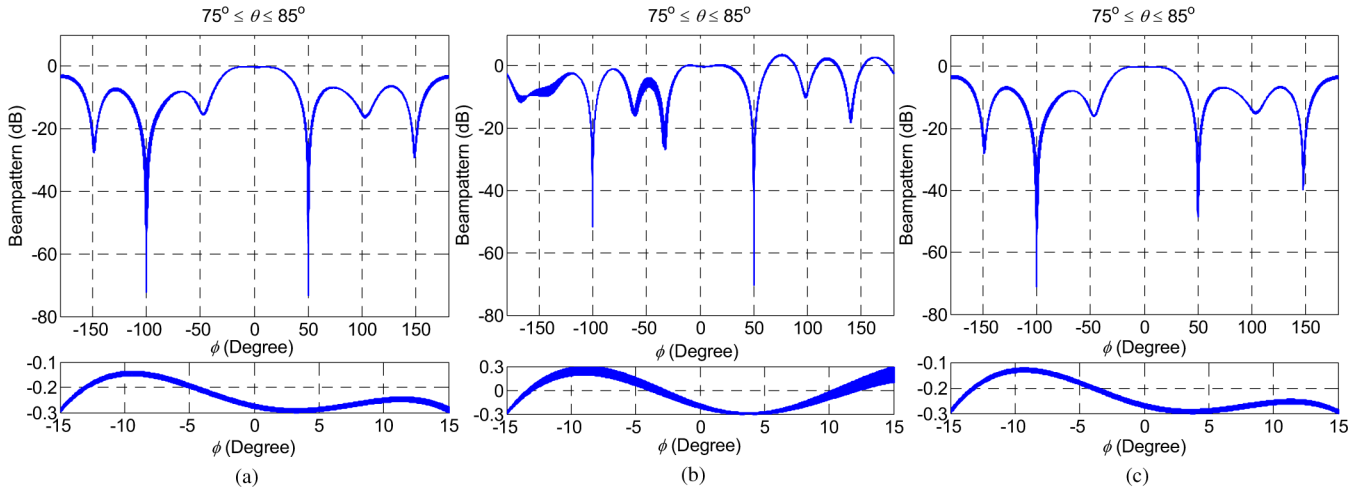


Fig. 4. Beam patterns of RB-ISOCP and RB-ISOCP-WC for a ten-element UCA. The ROI is $\Omega = [\theta_L, \theta_U] \times [\phi_L, \phi_U] = [75^\circ, 85^\circ] \times [-15^\circ, 15^\circ]$, and the ripple is $r_{db} = 0.3$ dB. (a) RB-ISOCP with infinite samples. (b) RB-ISOCP with $N = 100$. (c) RB-ISOCP-WC with $N = 100$ and $\gamma_r = 0.1$.

To further improve the limited performance due to insufficient snapshots, the proposed RB-ISOCP-WC is employed. Following the approach in [12], γ is chosen as $\gamma = \gamma_r \mathbf{R}_x(1, 1)$, where γ_r denotes the relative regularization factor, and it is chosen as $\gamma_r = 0.1$ for illustrative purpose. It can be seen in Fig. 3(b) that the performance is greatly improved by considering the uncertainty of the array covariance matrix. Next, with a fixed number of snapshots $N = 200$, a hundred of Monte-Carlo simulations are run to estimate the output signal-to-interference-plus-noise ratio (SINR) of the proposed RB-ISOCP-WC versus the SNR ranging from -10 dB to 10 dB. It can be seen from Fig. 3(c) that the RB-ISOCP-WC gives better performance than the RB-ISOCP (i.e., $\gamma_r = 0$). Also, a small regularization factor is sufficient to achieve a satisfactory improvement. For larger regularization factors, the proposed method gives nearly the same performance, as we can see that the curves of $\gamma_r = 5$ and $\gamma_r = 6$ almost overlap.

For a comparison, the conventional robust beamforming with constraints on magnitude response using worst-case optimization (RB-CMR-WC) [12] is also considered. As suggested in [12], the regularization factor for the RB-CMR-WC should be selected as $\varepsilon = \varepsilon_r \mathbf{R}_x(1, 1)$, where ε_r denotes the relative regularization factor. Comparing our definition of γ with ε in (16) of [12], $\gamma(\gamma_r)$ should generally be larger than $\varepsilon(\varepsilon_r)$ for the same uncertainty matrix Δ . For a fair comparison, we choose $\gamma_r = 6$ for the proposed algorithm and try to find the best performance of the RB-CMR-WC from the set of different relative regularization factors given by $\varepsilon_r = [0, 0.5, \dots, 6]$. Fig. 3(d) shows the performance comparison of the proposed RB-ISOCP-WC and the RB-CMR-WC. For clarity, only the two best results of the

latter are shown. It can be seen that the RB-CMR-WC achieves the best performance when $\varepsilon_r = 6$, which is outperformed by the proposed RB-ISOCP-WC. As discussed previously, the inferior performance of the RB-CMR-WC is probably due to the invalid assumption of Hermitian Toeplitz array covariance matrix and the numerical error introduced in spectral factorization. Therefore, a larger regularization factor may be required for the RB-CMR-WC to combat such unexpected errors. Nevertheless, the major advantage of the proposed approach over the conventional method is its usefulness in designing robust beamformers with arbitrary geometries, as we shall demonstrate in the next example.

B. Example-II: UCA

In Example-I, we have shown that the proposed method works well in the case of ULAs. Though the conventional RB-CMR in [12] is able to achieve comparable performance as ours, its application to other array geometries, say two-dimensional arrays, may not be straightforward as described previously. On the other hand, the proposed method does not have such limitation and it is applicable to arbitrary arrays. The general experimental settings are as follows: The number of sensor elements of the UCA is $M = 10$, the radius is $5\lambda/2\pi$. Two equal power interferences with an INR of 20 dB are assumed to impinge on the array from far-field at angles $(\theta_1, \phi_1) = (82^\circ, 50^\circ)$ and $(\theta_2, \phi_2) = (78^\circ, -100^\circ)$. The DOA of the SOI is assumed to be $(\theta_0, \phi_0) = (80^\circ, 3^\circ)$, whereas the nominal direction of the SOI is $(80^\circ, 0^\circ)$. The ROI is $\Omega = [\theta_L, \theta_U] \times [\phi_L, \phi_U] = [75^\circ, 85^\circ] \times [-15^\circ, 15^\circ]$, and the target ripple is $r_{db} = 0.3$ dB.

1) *Infinite Sample*: Similar to the first example, we firstly consider the performance of the RB-ISOCP with ideal array covariance matrix \mathbf{R}_r . Fig. 4(a) shows the beampatterns of the RB-ISOCP in discretized θ -planes within the region $\theta \in [75^\circ, 85^\circ]$. It can be seen that the ROI is very flat and the array gain in this region can be well controlled according to the prescribed ripple size. Moreover, deep nulls are imposed at the directions of interferences.

2) *Finite Sample*: In this experiment, the performance of the proposed method is evaluated when there are uncertainties in the array covariance matrix due to insufficient samples. For simplicity, the experimental settings are identical to the previous infinite sample case, except that the array covariance matrix is approximated from one hundred snapshots. Fig. 4(b) shows the beampattern obtained using the RB-ISOCP within the region $\theta \in [75^\circ, 85^\circ]$. Compared with the results for the case of ideal array covariance matrix in Fig. 4(a), it can be seen that the sidelobe level degrades significantly, because the RB-ISOCP fails to take the uncertainty of the array covariance matrix into account. Fig. 4(c) shows the beampattern obtained using the RB-ISOCP-WC with a relative regularization factor of $\gamma_r = 0.1$. As expected, the performance can be greatly improved, and the result is close to that in the case of ideal array covariance matrix.

IV. CONCLUSIONS

An iterative SOCP method for designing robust beamformers with magnitude response constraints has been presented. A locally optimal solution to the original non-convex problem is efficiently obtained by solving a sequence of convex SOCP subproblems, which are obtained via a local linearization of the magnitude squared response of the array. The proposed method is further extended to handle uncertainties of the array covariance matrix due to array imperfections. By incorporating uncertainties in form of bounded variation in the design procedure, the robustness of the beamformers can be significantly improved. Design results show that the proposed method is an attractive alternative to traditional design methods in tackling the robust beamforming problem, especially for arrays with arbitrary geometries.

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Performance Improvement of a U-Slot Patch Antenna Using a Dual-Band Frequency Selective Surface With Modified Jerusalem Cross Elements

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Abstract—A dual-band FSS consisting of regular Jerusalem cross elements was first used to study its impact on the bandwidths and resonant frequencies of a U-slot patch antenna. Based on the simulation experience of the first partial study, another FSS with modified Jerusalem cross elements was proposed to improve the bandwidths, antenna gains, and return losses of a smaller U-slot patch antenna at 2.45 and 5.8 GHz for Bluetooth and WLAN applications, respectively. Measured data of the return loss, radiation pattern, and antenna gain of this smaller U-slot patch antenna were also presented. It is proven that the smaller U-slot patch antenna implanted with a FSS consisting of modified Jerusalem cross elements has a good performance with sufficient bandwidth and higher gain and is capable of dual-band operation.

Index Terms—Bandwidths, dual-band FSS, Jerusalem cross elements, return loss, U-slot patch antenna.

I. INTRODUCTION

For wireless communications, multi-band and wide-band patch antennas will become the requirements for accurately transmitting the voice, data, video, and multimedia information in wireless communication systems, such as ultra wide-band measurement applications, intelligent transportation systems (ITS), wireless local area networks

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