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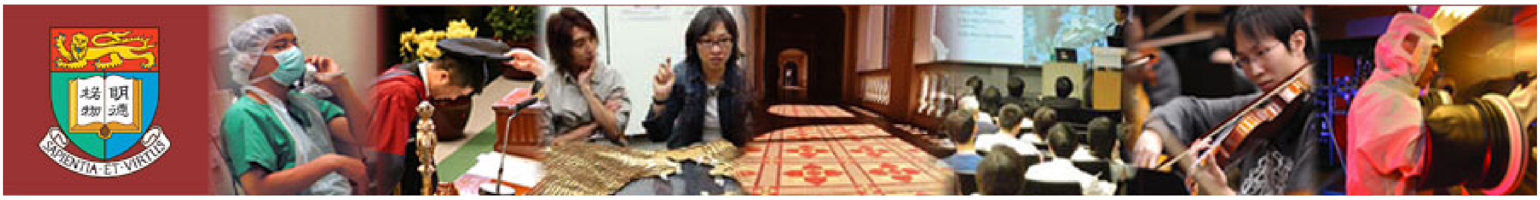

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On Infectious Models for Dependent Default Risk

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***Abstract*—Modeling dependent defaults is a key issue in risk measurement and management. In this paper, we introduce a Markovian infectious model to describe the dependent relationship of default processes of credit entities. The key idea of the proposed model is based on the concept of common shocks adopted in the insurance industry. We compare the proposed model to both one-sector and two-sector models considered in the credit literature using real default data. A log-likelihood ratio test is applied to compare the goodness-of-fit of the proposed model. Our empirical results reveal that the proposed model outperforms both the one-sector and two-sector models.**

***Keywords*-Markov chains; one-sector model; two-sector model; chain reaction of infectious defaults; default risk, common shock.**

I. Introduction

There has been considerable concerns and interest in modeling dependent defaults of credit entities. To model dependent default processes, Copula is one of the major tools. They have been used to specify dependent structures of multivariate distributions in statistics, in particular in survival analysis, (see, for example, [1]). A Copula function maps marginal distributions of random variables to their joint multivariate distribution so that the dependence of these random variables and their marginal behaviors can be modeled separately. This provides the flexibility in modeling the dependent relationships of the random variables. Copulas have been used in modeling dependent relationships of defaults and credit qualities of entities in the literature, see for instance [13]. A comprehensive discussion on Copulas can be found in [2]. Besides Copulas, other methods have been proposed in the literature for modeling dependent defaults. Some examples include the Poisson mixture model in [3], the correlated default models in [10], Moody's Binomial Expansion Techniques (BET) [15], etc.

Davis and Lo [7], [8] introduced an infectious default model, where a contagion model for concentration risk in a large portfolio of bonds was adopted. Their model can be regarded as an extension of Moody's BET, where defaults are assumed to be independent. Despite its capability in modeling dependent defaults, the computational cost of the infectious default model could be expensive if the number of bonds in a portfolio is large. In some recent literature, it has been shown empirically that the default of a bond causes the widening of credit spreads of other firms, see for example Das et al. [10]. This provides empirical evidence and motivation for studying and modeling the impact of default of a bond on the likelihood of defaults of other bonds. Indeed, this situation may not be unlike the spreading of certain infectious diseases, like influenza.

In [5], a Greenwood's model in [9] was introduced to model the impact of default of a bond on the likelihood of defaults of other bonds. The original form of the Greenwood's model was a one-factor model. It was then extended to a two-sector model in [5]. They also derived the joint probability distribution function for the duration of a default crisis, (i.e. the default cycle), and the severity of defaults during the crisis period. Two concepts, namely, Crisis Value-at-Risk (CRVaR) and Crisis Expected Shortfall (CRES), were also used to assess the impact of a default crisis. The Greenwood's model was also extended to a network of sectors in [4], [6]. In [14], a common shock model for correlated insurance losses was discussed. Explicit formulas for the correlations between pairs of insurance business lines were derived in terms of the magnitude of the common shock. Furthermore, the probability distributions for claim counts and severities were obtained.

In this paper, we adopt the concept of common shocks and consider the situation that the joint default probability depends on the current number of defaulted bonds in the sector. We wish to model explicitly the impact of the current number of defaulted bonds on the likelihood of defaults of other surviving bonds. Here we first focus on a one-sector model where the probability of defaults of other surviving bonds depends on the current number of defaulted bonds. Then, instead of considering the two-sector model [4], [5], we introduce a novel causality relationship to describe the defaults of two sectors. A set of random variables called infectious factors are introduced to describe the effect of the defaults in a sector on the other sector. In this paper, we give a certain expression of the infectious factor, then we

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apply this to develop our two-sector model. This provides an elegant way to compare the likelihoods of defaults of two sectors. We then conduct empirical studies on the proposed model using real default data.

The rest of the paper is structured as follows. In Section II, we present a new one-sector model. Section III discusses the new two-sector model. In section IV, we provide empirical studies on the proposed models based on real default data. The final section concludes the paper with some future research issues.

II. A New One-sector Model

In this section we present an extended version of the Greenwood's model to describe the chain reaction of infectious defaults. The key idea of this new one-sector model is to model the impact of the current number of defaulted bonds on the joint default probability in a sector.

Let $\mathcal{T}$ be the time index set $\{0, 1, 2, \ldots, \}$ of our model. To model the uncertainty, we consider a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where $\mathcal{P}$ is a real-world probability. Suppose that

$$X := \{X_t\}_{t \in \mathcal{T}} \quad \text{and} \quad Y := \{Y_t\}_{t \in \mathcal{T}}$$

denote two stochastic processes on the $(\Omega, \mathcal{F}, \mathcal{P})$, where X_t and Y_t represent the numbers of surviving bonds and the defaulted bonds at $t \in \mathcal{T}$, respectively. We assume that the initial conditions are given as follow:

$$X_0 = x_0, \quad Y_0 = y_0 \quad \text{where} \quad x_0 + y_0 = N \ .$$

Note that for each $t \in \mathcal{T}$, the sum of the numbers of the defaulted bonds and the surviving bonds at time instant $t+1$ must equal the number of surviving bonds at time t.

$$X_{t+1} + Y_{t+1} = X_t \ .$$

In this new one-sector model, for each $t \in \mathcal{T}$, let α_t be the probability that the default of a surviving bond is infected by the defaulted bonds at time t. Then, under the one-sector model, the joint probability distribution of $\{X_{t+1}, Y_{t+1}\}$ given $\{X_t, Y_t\}$ is:

$$\begin{aligned} & p_{x_t,y_t}(x_{t+1}, y_{t+1}) \\ & = P\{(X_{t+1}, Y_{t+1}) = (x_{t+1}, y_{t+1}) \mid (X_t, Y_t) = (x_t, y_t)\} \\ & = \binom{x_t}{y_{t+1}} {\alpha_t}^{y_{t+1}} (1-\alpha_t)^{x_{t+1}} \\ & = \binom{x_t}{x_t - x_{t+1}} {\alpha_t}^{x_t - x_{t+1}} (1-\alpha_t)^{x_{t+1}}. \end{aligned} \tag{1}$$

Here we consider the situation that the joint default probability depends on the current number of defaulted bonds. We assume that

$$\alpha_t = \begin{cases} a_1 & \text{if } y_t > 0 \\ a_0 & \text{if } y_t = 0. \end{cases}$$

Indeed, the model we considered here is "self-exciting" in the sense that the joint future default probability law switches over time depending on which region the current number of defaults lies, (i.e., $y_t = 0$ or $y_t > 0$). This idea is not unlike the idea of self-exciting threshold autoregressive models pioneered by Tong [16], [17], [18].

We now define h_t as an indicator function for the presence of default bonds at time t. Namely,

$$h_t = \begin{cases} 1 & \text{if } y_t > 0 \\ 0 & \text{if } y_t = 0. \end{cases}$$

Under this one-sector model, the default probability at each time t depends on the number of defaulted bonds at time $t-1$. We have a two-dimensional Markov chain process in the state space $S = \{(x, y) : 0 \le x, y \le X_0\}$ and the transition probability matrix is of size $(X_0 + 1)^2 \times (X_0 + 1)^2$. The recursive formula to evaluate the joint probability distribution (x_t, y_t) can therefore be deduced as follows:

$$\begin{aligned} & p_t(m, n) \\ & = P\{(X_t, Y_t) = (m, n)\} \\ & = p_{t-1}(m+n, 0) \binom{m+n}{n} a_0^n (1-a_0)^m \\ & + \sum_{i=1}^{N-(m+n)} p_{t-1}(m+n, i) \binom{m+n}{n} a_1^n (1-a_1)^m. \end{aligned} \tag{2}$$

Then we employ the maximum likelihood method to estimate the parameters, based on the observed data $x_0, x_1, x_2, \ldots, x_N$ and $h_0, h_1, h_2, \ldots, h_N$. The estimation of the parameters can be obtained as follow:

$$\hat{a}_0 = \frac{\sum_{t=0}^{N-1} (1-h_t) y_{t+1}}{\sum_{t=0}^{N-1} (1-h_t) x_t} \quad \text{and} \quad \hat{a}_1 = \frac{\sum_{t=0}^{N-1} h_t y_{t+1}}{\sum_{t=0}^{N-1} h_t x_t}.$$

III. A New Two-sector Model

In this section, we consider defaults of bonds in two sectors, say sector A and sector B. In [5], it describes a causality relationship goes from the defaulted bonds in one sector to surviving bonds in another sector under its two-sector model. Here we consider the causality relationship in another way. We develop the model for the chain reaction of the infectious defaults in sector A, in which the default bonds are influenced by the defaults in sector B.

Let $\mathcal{T}$ denote the time index set $\{0, 1, 2, \ldots\}$ of our model. Again we fix a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where $\mathcal{P}$ is a real-world probability. Suppose that $X := \{X_t\}_{t \in \mathcal{T}}$, $X^{(2)} := \{X_t^{(2)}\}_{t \in \mathcal{T}}$, $Y := \{Y_t\}_{t \in \mathcal{T}}$ and $Y^{(2)} := \{Y_t^{(2)}\}_{t \in \mathcal{T}}$ denote two stochastic processes on the $(\Omega, \mathcal{F}, \mathcal{P})$. Here X_t and Y_t represent respectively the numbers of surviving bonds and the defaulted bonds in sector A at time $t \in \mathcal{T}$, while $X_t^{(2)}$ and $Y_t^{(2)}$ represent respectively the numbers of surviving bonds and the defaulted bonds in sector B at time t. We assume the initial conditions are given as follow:

$$X_0 = x_0, \quad Y_0 = y_0 \quad \text{where} \quad x_0 + y_0 = N$$

and

$$X_0^{(2)} = x_0^{(2)}, \quad Y_0^{(2)} = y_0^{(2)} \quad \text{where} \quad x_0^{(2)} + y_0^{(2)} = N^{(2)}$$

To illustrate, we assume that the defaults in sector B at time t can be described by the infectious factor β_t. The infectious factor β_t satisfies three important properties:
(i) $\beta_t > 0$, at each $t \in \mathcal{T}$;
(ii) Given $\{X_{t-1}^{(2)}, Y_{t-1}^{(2)}\}$, $E(\beta_t) = 1$ at each $t \in \mathcal{T}$;
(iii) When the defaults get severe, its value raises.
And we let α_t denote the probability that the default of a surviving bond is infected by the defaulted bonds at time t in sector A, where we assume $\alpha_t = a_1$, if $y_t > 0$, and $\alpha_t = a_0$, if $y_t = 0$.

Here we present the key idea in this model. For each $t \in \mathcal{T}$, let $\alpha_t\beta_t$ denote the probability that the default of a surviving bond in Sector A is infected by the defaulted bonds at time t from both sector A and sector B. Both the chain reaction of infectious defaults in sector A and the impact of defaults in sector B are considered in this new two-sector model. Then, the joint probability distribution of $\{X_{t+1}, Y_{t+1}\}$ given $\{X_t, Y_t\}$ is:

$$\begin{aligned} & p_{x_t,y_t}(x_{t+1}, y_{t+1}) \\ &= P\{(X_{t+1}, Y_{t+1}) = (x_{t+1}, y_{t+1}) \mid (X_t, Y_t) = (x_t, y_t)\} \\ &= \begin{pmatrix} x_t \\ y_{t+1} \end{pmatrix} (\beta_t\alpha_t)^{y_{t+1}} (1 - \beta_t\alpha_t)^{x_{t+1}} \qquad (3) \\ &= \begin{pmatrix} x_t \\ x_t - x_{t+1} \end{pmatrix} (\beta_t\alpha_t)^{x_t - x_{t+1}} (1 - \beta_t\alpha_t)^{x_{t+1}} \end{aligned}$$

Consequently, property (i) ensures the default probabilities $\alpha_t\beta_t > 0$. Property (ii) indicates that the default probabilities $\beta_t\alpha_t$ fluctuates in accordance with α_t. Property (iii) indicates how defaults in sector B effect the default of bonds in sector A.

To model the infectious factor β_t, we assume that β_t is a function of $x_{t-1}^{(2)}, y_t^{(2)}$:

$$\beta_t = \frac{y_t^{(2)} + \xi}{\alpha_{t-1}^{(2)} x_{t-1}^{(2)} + \xi}$$

where $\alpha_t^{(2)}, t \in \mathcal{T}$ is the default probability defined in the new one-sector model for sector B and ξ is a positive constant.

The assumption of β_t is reasonable, since it satisfies the three properties mentioned above: (i) Obviously, $\beta_t > 0$ and (ii) Given $\{X_{t-1}^{(2)}, Y_{t-1}^{(2)}\}$,

$$\begin{aligned} & p_{x_{t-1}^{(2)}, y_{t-1}^{(2)}}(x_t^{(2)}, y_t^{(2)}) \\ &= P\{(X_t^{(2)}, Y_t^{(2)}) = (x_t^{(2)}, y_t^{(2)}) \mid (X_{t-1}^{(2)}, Y_{t-1}^{(2)}) = (x_{t-1}^{(2)}, y_{t-1}^{(2)})\} \\ &= \begin{pmatrix} x_{t-1}^{(2)} \\ x_{t-1}^{(2)} - x_t^{(2)} \end{pmatrix} (\alpha_{t-1}^{(2)})^{x_{t-1}^{(2)} - x_t^{(2)}} (1 - \alpha_{t-1}^{(2)})^{x_t^{(2)}}. \qquad (4) \end{aligned}$$

Thus $E(y_t^{(2)}) = \alpha_{t-1}^{(2)} x_{t-1}^{(2)}$ and so

$$E(\beta_t) = E\left(\frac{y_t^{(2)} + \xi}{\alpha_{t-1}^{(2)} x_{t-1}^{(2)} + \xi}\right) = \frac{E(y_t^{(2)}) + \xi}{\alpha_{t-1}^{(2)} x_{t-1}^{(2)} + \xi} = 1.$$

(iii) From $\beta_t = \frac{y_t^{(2)} + \xi}{\alpha_{t-1}^{(2)} x_{t-1}^{(2)} + \xi}$ one can see that when the defaults get severer at time t, $y_t^{(2)}$ increases which leads to the raise of β_t. And if the infectious factor β_t is a constant and equals one, this two-sector model reduces to the one-sector model we discussed above.

In the two-sector model, let h_t be a function of y_t given by:

$$h_t = \begin{cases} 1 & \text{if } y_t > 0 \\ 0 & \text{if } y_t = 0. \end{cases}$$

Then we use the maximum likelihood method to estimate the parameters, based on the observed data $x_0, x_1, x_2, \ldots, x_N$ and $h_0, h_1, h_2, \ldots, h_N$. The likelihood function is given as follows:

$$\begin{aligned} & L(a|x_0, x_1, \ldots, x_N, h_0, h_1, \ldots, h_N) \\ &= \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} (1 - a_{h_0}\beta_0)^{x_1} (a_{h_0}\beta_t)^{x_0 - x_1} \times \\ & \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} (1 - a_{h_1}\beta_1)^{x_2} (a_{h_1}\beta_1)^{x_1 - x_2} \times \cdots \times \\ & \begin{pmatrix} x_{N-1} \\ x_N \end{pmatrix} (1 - a_{h_{N-1}}\beta_{N-1})^{x_N} (a_{h_{N-1}}\beta_{N-1})^{x_{N-1} - x_N}. \end{aligned}$$

By solving $\frac{\partial \ln L(a|x_0, x_1, \ldots, x_N)}{\partial a_0} = 0$, we have

$$\sum_{t=0}^{N-1} (1 - h_t) \frac{y_{t+1}}{a_0} + \sum_{t=0}^{N-1} (1 - h_t) \frac{-\beta_t x_{t+1}}{1 - \beta_t a_0} = 0,$$

and

$$a_0 = \frac{\sum_{t=0}^{N-1} (1 - h_t) \frac{y_{t+1}}{\gamma_0 - \beta_t}}{\sum_{t=0}^{N-1} (1 - h_t) \frac{x_t \beta_t}{\gamma_0 - \beta_t}}$$

where $\gamma_0 = \frac{1}{a_0}$. Since $\gamma_0 \gg \beta_t$ we estimate a_0 as

$$\hat{a}_0 \approx \frac{\sum_{t=0}^{N-1} (1 - h_t) y_{t+1}}{\sum_{t=0}^{N-1} (1 - h_t) x_t \beta_t}.$$

Using the same techniques, we can also deduce

$$\hat{a}_1 \approx \frac{\sum_{t=0}^{N-1} h_t y_{t+1}}{\sum_{t=0}^{N-1} h_t x_t \beta_t}.$$

IV. Numerical Examples

In this section, we present the empirical results for the two new models, (i.e., one-sector model and two-sector model), using real default data extracted from the figures in [11]. We compute the estimation results for the two models using the techniques presented in Section II and Section III. Furthermore, we compare the new model with the model in [4], [5] for both the one-sector and two-sector cases.

We use real default data from four different sectors. They include consumer/service sector, energy/natural resources sector, leisure time/media sector and transportation sector. Table I shows the default data taken from [11]. From the table, the proportions of defaults for Consumer, Energy, Media and Transport are 24.11%, 16.9%, 20.46% and 20.00%, respectively. The default probabilities of all four sectors are significantly greater than zero. This means that the default risk of each of the four sectors is quite substantial.

Sectors	Total	Defaults
Consumer	1041	251
Energy	420	71
Media	650	133
Transport	281	59

Table I
DEFAULT DATA TAKEN FROM [11]

We first compute the estimation results of the parameters described in Section II for the one-sector model. From Table II, we report the estimated parameters a_0 and a_1. One can compare the results with the old one-sector model, for which the estimation result α is shown in Table 2. We observe that the default probability α is less than a_1 but greater than a_0 in every sector.

Sectors	Consumer	Energy	Media	Transport
Old One-sector Model				
α	0.0030	0.0021	0.0025	0.0026
New One-sector Model				
a_0	0.0009	0.0012	0.0012	0.0020
a_1	0.0039	0.0033	0.0036	0.0037

Table II
THE DEFAULT PROBABILITY (ONE-SECTOR MODEL).

To compare the new one-sector model with the one-sector model in [4], [5], we adopt the Likelihood Ratio Test (LRT) for the new one-sector model against the one-sector model in [4], [5]. The test statistic of the LRT is the log-likelihood ratio, which follows roughly a χ^2-distribution with one degree of freedom. This is a simple and convenient statistical test for comparing two models. We note that the critical values are 3.843 and 6.637 at significant levels 95% and 99%, respectively. The log-likelihood ratios are presented in the Table III. One can observe that all the log-likelihood ratios, except transport sector, are greater than the critical value 6.637 (with "$^+$"), while the log-likelihood ratio of the transport sector are greater than 3.843 but less than 6.637. We can therefore draw a conclusion that for all the sectors, the new one-sector model is statistically better than the one-sector model in [4], [5] at significant level 95%.

Before we consider the new two-sector infectious model, we need to pair up the sectors. That is to say, for each sector, we have to find its partner (with the highest correlation). We adopt correlation as a measure as in [4], [5], [6] for

Sectors	Consumer	Energy	Media	Transport
Log-likelihood Ratio	65.1^+	17.1^+	34.0^+	5.7

Table III
THE LOG-LIKELIHOOD RATIO: OLD ONE-SECTOR MODEL TO NEW ONE-SECTOR MODEL.

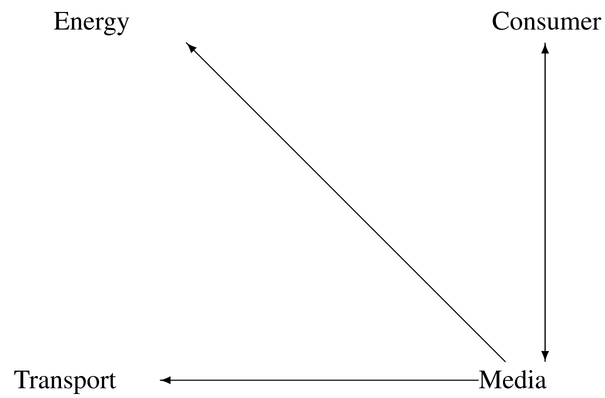

Figure 1. THE PARTNER RELATIONS AMONG TEH SECTORS USING CORRELATIONS.

the correlation between any two sectors. Table IV reports the correlations of the default data from each pair of the sectors. The asterisk "$*$" in the table indicates the pair of sectors which has the largest correlation. Figure 1 gives the partner relations among the four sectors according to the correlations of defaults. For each row (sector A) in Table IV, its partner (sector B) is identified by finding the sector with the largest correlation with sector A in magnitude. We remark that the relation is not necessarily symmetric. This relation is only found symmetric for the sectors media and consumer.

	Consumer	Energy	Media	Transport
Consumer	-	0.0224	0.6013*	0.3487
Energy	0.0224	-	0.1258*	0.1045
Media	0.6013*	0.1258	-	0.3708
Transport	0.3487	0.1045	0.3708*	-

Table IV
CORRELATIONS OF THE SECTORS (TAKEN FROM [4]).

We then construct the two-sector model according to Table IV and Figure 1. We present the estimates of a_0, a_1 with the constant taking values $\xi = 2, 3$. Again we consider the Likelihood Ratio Test (LRT) for the new two-sector model against the existing one-sector model. We compute its log-likelihood ratio, which follows roughly the χ^2-distribution with one degree of freedom. We then compare it with the log-likelihood ratio for the existing one-sector model against existing two-sector model presented in [4], [5]. Those log-

Sector A	Consumer	Energy	Media	Transport
Sector B	**Media**	**Media**	**Consumer**	**Media**
Old Two-sector Model				
α_0	0.0013	0.0018	0.0005	0.0013
α_1	0.0043	0.0023	0.0033	0.0036
Log-likelihood Ratio	69.0^+	1.2	43.4^+	12.8^+
New Two-sector Model				
$\xi = 2$				
a_0	0.0011	0.0012	0.0015	0.0022
a_1	0.0036	0.0034	0.0031	0.0032
Log-likelihood Ratio	88.6^+	11.1^+	59.5^+	7.9^+
$\xi = 3$				
a_0	0.0010	0.0012	0.0014	0.0021
a_1	0.0037	0.0033	0.0032	0.0032
Log-likelihood Ratio	92.7^+	14.7^+	60.1^+	9.1^+

Table V
THE LOG-LIKELIHOOD RATIO.

likelihood ratios greater than the critical value 6.637 are signified with a "+" in Table V. We remark that for all the sectors, except Energy sector, the existing two-sector model is statistically better than the existing one-sector model at both significant levels 99% and 95%. For the Energy sector, however, we find no evidence that the old two-sector model is statistically better. We also remark that the new two-sector model is statistically better than the existing one-sector model for all the four sectors at both significant levels 99% and 95%, when the constant ξ equals 2 or 3. This provides evidence for the use of proposed two-sector model.

To compare the proposed two-sector model with the existing two-sector model, we adopt the usual Bayesian information criteria (BIC). From the experimental results, we remark that for the all sectors, except the transport sector, the log-likelihood ratio for the existing two-sector model against the proposed two-sector model is positive at $\xi = 2, 3$, which means the proposed two-sector model is better. However, in the transport sector, no evidence is found to prove the proposed two-sector model is better.

V. CONCLUDING REMARKS

In this section, we discuss some future research issues. In the two models, (i.e., one-sector model and two-sector model), one can then derive the joint probability distribution function (p.d.f.) for the duration of the default crisis, namely, the default cycle (represented by the random variable T) and the severity of defaults during the crisis period (represented by the random variable W_T). One then apply the proposed models to study default data and calculate the CRVaR and CRES discussed in [5]. Furthermore, the idea of infectious factor can be extended to the case of a network of sectors [4], [6].

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