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Solar system tests of Hořava-Lifshitz gravity

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In the present paper we consider the possibility of observationally constraining Hořava gravity at the scale of the Solar System, by considering the classical tests of general relativity (perihelion precession of the planet Mercury, deflection of light by the Sun and the radar echo delay) for the spherically symmetric black hole Kehagias-Sfetsos solution of Hořava-Lifshitz gravity. All these gravitational effects can be fully explained in the framework of the vacuum solution of Hořava gravity. Moreover, the study of the classical general relativistic tests also constrain the free parameter of the solution. From the analysis of the perihelion precession of the planet Mercury we obtain for the free parameter ω of the Kehagias-Sfetsos solution the constraint $\omega \geq 3.212 \times 10^{-26} \text{ cm}^{-2}$, the deflection of light by the Sun gives $\omega \geq 4.589 \times 10^{-26} \text{ cm}^{-2}$, while the radar echo delay observations can be explained if the value of ω satisfies the constraint $\omega \geq 9.179 \times 10^{-26} \text{ cm}^{-2}$.

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I. INTRODUCTION

Recently, a renormalizable gravity theory in four dimensions which reduces to Einstein gravity with a non-vanishing cosmological constant in the infrared (IR) energy scale (corresponding to large distances), but with improved ultraviolet (UV) energy scale behaviors (corresponding to very small distances), was proposed by Hořava [1, 2]. Quantum field theory has had considerable success experimentally, but from a theoretical point of view it predicts infinite values for physical quantities. Thus for certain Feynman diagrams containing loops, the calculations leads to an infinite result. This is known as the ultraviolet divergence of quantum field theory, because small loop sizes correspond to high energies. Infrared Divergence is the divergence caused by the low energy behavior of a quantum theory. The Hořava theory admits a Lifshitz scale-invariance in time and space, exhibiting a broken Lorentz symmetry at short scales, while at large distances higher derivative terms do not contribute, and the theory reduces to standard general relativity (GR). The Hořava theory has received a great deal of attention and since its formulation various properties and characteristics have been extensively analyzed, ranging from formal developments [3], cosmology [4], dark energy [5] and dark matter [6], spherically symmetric or rotating solutions [7–11], to the weak field observational tests [12, 13]. At large distances, higher derivative terms do not contribute and the theory reduces to standard gen-

eral relativity if a particular coupling λ , which controls the contribution of the trace of the extrinsic curvature has a specific value. Indeed, λ is running, and if $\lambda = 1$ is an IR fixed point, standard general relativity is recovered. Therefore, although a generic vacuum of the theory is the anti-de Sitter one, particular limits of the theory allow for the Minkowski vacuum, a physical state characterized by the absence of ordinary (baryonic) matter. In this limit post-Newtonian coefficients coincide with those of pure GR. Thus, the deviations from conventional GR can be tested only beyond the post-Newtonian corrections, that is for a system with strong gravity at astrophysical scales.

In this context, IR-modified Hořava gravity seems to be consistent with the current observational data, but in order to test its viability more observational constraints are necessary. In Ref. [14], potentially observable properties of black holes in the Hořava-Lifshitz gravity with Minkowski vacuum were considered, namely, the gravitational lensing and quasinormal modes. Quasinormal modes are the modes of energy dissipation of a perturbed object or field. Black holes have many quasinormal modes that describe the exponential decrease of asymmetry of the black hole in time as it evolves towards the spherical shape. It was shown that the bending angle is seemingly smaller in the considered Hořava-Lifshitz gravity than in GR, and the quasinormal modes of black holes are longer lived, and have larger real oscillation frequency in Hořava-Lifshitz gravity than in GR. In Ref. [15], by adopting the strong field limit approach, the properties of strong field gravitational lensing for the Hořava-Lifshitz black hole were considered, and the angular position and magnification of the relativistic images were obtained. Compared with the Reissner-Norström black hole, a significant difference in the parameters was found. Thus, it was argued that this may offer a way to distinguish

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a deformed Hořava-Lifshitz black hole from a Reissner-Nordström black hole. In general relativity the Reissner-Nordström black hole solution describes the gravitational field of a charged black hole. In Ref. [16], the behavior of the effective potential was analyzed, and the time-like geodesic motion in the Hořava-Lifshitz spacetime was also explored. In Ref. [17], the basic physical properties of matter forming a thin accretion disk in the vacuum spacetime metric of the Hořava-Lifshitz gravity models were considered. It was shown that significant differences as compared to the general relativistic case exist, and that the determination of these observational quantities could discriminate, at least in principle, between standard GR and Hořava-Lifshitz theory, and constrain the parameter of the model.

It is the purpose of the present paper to consider the classical tests (perihelion precession, light bending, and the radar echo delay, respectively) of general relativity for static gravitational fields in the framework of Hořava-Lifshitz gravity. To do this we shall adopt for the geometry outside a compact, stellar type object (the Sun), the static and spherically symmetric metric obtained by Kehagias and Sfetsos [10]. For the Kehagias and Sfetsos (KS) metric, we first consider the motion of a particle (planet), and analyze the perihelion precession. In addition to this, by considering the motion of a photon in the static KS field, we study the bending of light by massive astrophysical objects and the radar echo delay, respectively. All these gravitational effects can be explained in the framework of the KS geometry. Existing data on light-bending around the Sun, using long-baseline radio interferometry, ranging to Mars using the Viking lander, the perihelion precession of Mercury, and recent Lunar Laser Ranging results can all give significant and detectable Solar System constraints, associated with Hořava-Lifshitz gravity. More exactly, the study of the classical general relativistic tests, constrain the parameter of the solution. We will also compare our results with the phenomenological constraints on the Kehagias-Sfetsos solution from solar system orbital motions obtained in [12]. In order to obtain reliable constraints on the parameter of the model, the analysis must not be limited to the perihelion precession of the planet Mercury, but it should be extended to take into account the other recent observational data on the perihelion precession of the other planets of the Solar System.

In the context of the classical tests of GR the Dadhich, Maartens, Papadopoulos and Rezanian (DMPR) solution of the spherically symmetric static vacuum field equations in brane world models was also extensively analyzed [18]. It was found that the existing observational solar system data constrain the numerical values of the bulk tidal parameter and of the brane tension.

This paper is organized as follows. In Sec. II, we present the action and specific solutions of static and spherically symmetric spacetimes in Hořava-Lifshitz gravity. In Sec. III, we analyze the classical Solar System tests for the case of the KS asymptotically flat solution

[10] of Hořava-Lifshitz gravity. We conclude our results in Sec. IV.

II. BLACK HOLES IN HOŘAVA-LIFSHITZ GRAVITY

In the Hořava-Lifshitz gravity theory, Lorentz symmetry is broken in the ultraviolet. The breaking manifests in the strong anisotropic scalings between space and time, $\vec{x} \rightarrow l\vec{x}$, $t \rightarrow l^z t$. In $(3+1)$ -dimensional spacetimes, the theory is powercounting renormalizable, provided that $z \geq 3$. At low energies, the theory is expected to flow to $z = 1$, whereby the Lorentz invariance is accidentally restored. Such an anisotropy between time and space can be easily realized, when writing the metric in the Arnowitt-Deser-Misner (ADM) form [19]. The formalism supposes that the spacetime is foliated into a family of spacelike surfaces Σ_t , labeled by their time coordinate t , and with coordinates on each slice given by x^i . Using the ADM formalism, the four-dimensional metric is parameterized in the following form

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt). \quad (1)$$

The Einstein-Hilbert action is given by

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{g} N \left(K_{ij} K^{ij} - K^2 + R^{(3)} - 2\Lambda \right), \quad (2)$$

where G is Newton's constant, $R^{(3)}$ is the three-dimensional curvature scalar for g_{ij} , and K_{ij} is the extrinsic curvature, defined as

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (3)$$

where the dot denotes a derivative with respect to t .

The IR-modified Hořava action is given by [8]

$$\begin{aligned} S = \int dt d^3x \sqrt{g} N & \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda_g K^2) - \frac{\kappa^2}{2\nu_g^4} C_{ij} C^{ij} \right. \\ & + \frac{\kappa^2 \mu}{2\nu_g^2} \epsilon^{ijk} R_{il}^{(3)} \nabla_j R^{(3)l}_k - \frac{\kappa^2 \mu^2}{8} R_{ij}^{(3)} R^{(3)ij} \\ & + \frac{\kappa^2 \mu^2}{8(3\lambda_g - 1)} \left(\frac{4\lambda_g - 1}{4} (R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2 \right) \\ & \left. + \frac{\kappa^2 \mu^2 \omega}{8(3\lambda_g - 1)} R^{(3)} \right], \quad (4) \end{aligned}$$

where κ , λ_g , ν_g , μ , ω and Λ_W are constant parameters. $K = K^i_i$ is the contraction of the intrinsic curvature, while ∇_j is the covariant derivative with respect to the three-dimensional metric. C^{ij} is the Cotton tensor, defined as

$$C^{ij} = \epsilon^{ikl} \nabla_k \left(R^{(3)j}_l - \frac{1}{4} R^{(3)} \delta^j_l \right). \quad (5)$$

In the UV, this theory is power counting renormalizable, at least around the flat space (vacuum) solution. In the IR, the terms of lowest dimension should dominate. Note that the last term in Eq. (4) represents a ‘soft’ violation of the ‘detailed balance’ condition, which modifies the IR behavior. More specifically, if one maintains Hořava’s original detailed balance, and try to recover Einstein-Hilbert in the low energy regime, then one obtains a negative cosmological constant. The introduction of the last term in Eq. (4) corrects this, and provides a positive cosmological constant. Since $\omega \propto \mu^2$, this IR modification term, $\mu^4 R^{(3)}$, with an arbitrary cosmological constant, represents the analogs of the standard Schwarzschild-(A)dS solutions, which were absent in the original Hořava model.

The fundamental constants of the speed of light c , Newton’s constant G , and the cosmological constant Λ are defined as

$$c^2 = \frac{\kappa^2 \mu^2 |\Lambda_W|}{8(3\lambda_g - 1)^2} \quad G = \frac{\kappa^2 c^2}{16\pi(3\lambda_g - 1)} \quad \Lambda = \frac{3}{2} \Lambda_W c^2. \quad (6)$$

The Hořava-Lifshitz theory is associated with the breaking of the diffeomorphism invariance, required for the anisotropic scaling in the UV [2].

Throughout this work, we consider the static and spherically symmetric metric given by

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (7)$$

where $e^{\nu(r)}$ and $e^{\lambda(r)}$ are arbitrary functions of the radial coordinate r .

Imposing the specific case of $\lambda_g = 1$, which reduces to the Einstein-Hilbert action in the IR limit, one obtains the following solution of the vacuum field equations in Hořava gravity,

$$e^{\nu(r)} = e^{-\lambda(r)} = 1 + (\omega - \Lambda_W) r^2 - \sqrt{r[\omega(\omega - 2\Lambda_W) r^3 + \beta]}, \quad (8)$$

where β is an integration constant [8].

By considering $\beta = -\alpha^2/\Lambda_W$ and $\omega = 0$ the solution given by Eq. (8) reduces to the Lu, Mei and Pope (LMP) solution [9], given by

$$e^{\nu(r)} = 1 - \Lambda_W r^2 - \frac{\alpha}{\sqrt{-\Lambda_W}} \sqrt{r}. \quad (9)$$

Alternatively, considering now $\beta = 4\omega M$ and $\Lambda_W = 0$, one obtains the Kehagias and Sfetsos’s (KS) asymptotically flat solution [10], given by

$$e^{\nu(r)} = 1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}. \quad (10)$$

If the limit $4M/\omega r^3 \ll 1$, from Eq. (10) we obtain the standard Schwarzschild metric of general relativity, $e^{\nu(r)} = 1 - 2M/r$, which represents a ‘Post-Newtonian’ approximation of the KS solution of the second order in the speed of light. We shall use the KS solution for analyzing the Solar System constraints of the theory. Note

that there are two event horizons at

$$r_{\pm} = M \left[1 \pm \sqrt{1 - 1/(2\omega M^2)} \right]. \quad (11)$$

To avoid a naked singularity at the origin, one also needs to impose the condition

$$\omega M^2 \geq \frac{1}{2}. \quad (12)$$

Note that in the GR regime, i.e., $\omega M^2 \gg 1$, the outer horizon approaches the Schwarzschild horizon, $r_+ \simeq 2M$, and the inner horizon approaches the central singularity, $r_- \simeq 0$. One should also note that the KS solution is obtained without requiring the projectability condition, which was assumed in the original Hořava theory. However, it is important to emphasize that static and spherically symmetric exhibiting the projectability condition have been obtained in the literature [20].

III. SOLAR SYSTEM TESTS FOR HOŘAVA-LIFSHITZ GRAVITY BLACK HOLES

At the level of the Solar System there are three fundamental tests, which can provide important observational evidence for GR and its generalizations, and for alternative theories of gravitation in flat space. These tests are the precession of the perihelion of Mercury, the deflection of light by the Sun, and the radar echo delay observations, respectively, and have been used to successfully test the Schwarzschild solution of general relativity and some of its generalizations. In this Section we consider these standard Solar System tests of general relativity in the case of the KS asymptotically flat solution [10] of Hořava-Lifshitz gravity. Throughout the next Sections we use the natural system of units with $G = c = 1$.

A. The perihelion precession of the planet Mercury

The motion of a test particle in the gravitational field of the metric given by Eq. (7) can be derived from the variational principle

$$\delta \int \sqrt{-e^{\nu} \dot{t}^2 + e^{\lambda} \dot{r}^2 + r^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2)} ds = 0, \quad (13)$$

where the dot denotes d/ds . It may be verified that the orbit is planar, and hence without any loss of generality we can set $\theta = \pi/2$. Therefore we will use ϕ as the angular coordinate. Since neither t nor ϕ appear explicitly in Eq. (13), their conjugate momenta are constant,

$$e^{\nu} \dot{t} = E = \text{constant}, \quad r^2 \dot{\phi} = L = \text{constant}. \quad (14)$$

The line element, given by Eq. (7), and taking into account Eqs. (14), provides the following equation of motion for r

$$\dot{r}^2 + e^{-\lambda} \frac{L^2}{r^2} = e^{-\lambda} (E^2 e^{-\nu} - 1). \quad (15)$$

The change of variable $r = 1/u$ and the substitution $d/ds = Lu^2 d/d\phi$ transforms Eq. (15) into the form

$$\left(\frac{du}{d\phi}\right)^2 + e^{-\lambda}u^2 = \frac{1}{L^2}e^{-\lambda}(E^2e^{-\nu} - 1). \quad (16)$$

By formally representing $e^{-\lambda} = 1 - f(u)$, we obtain

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = f(u)u^2 + \frac{E^2}{L^2}e^{-\nu-\lambda} - \frac{1}{L^2}e^{-\lambda}. \quad (17)$$

By taking the derivative of the previous equation with respect to ϕ we find

$$\frac{d^2u}{d\phi^2} + u = F(u), \quad (18)$$

where

$$F(u) = \frac{1}{2} \frac{dG(u)}{du}, \quad (19)$$

and we have denoted

$$G(u) \equiv f(u)u^2 + \frac{E^2}{L^2}e^{-\nu-\lambda} - \frac{1}{L^2}e^{-\lambda}.$$

A circular orbit $u = u_0$ is given by the root of the equation $u_0 = F(u_0)$. Any deviation $\delta = u - u_0$ from a circular orbit must satisfy the equation

$$\frac{d^2\delta}{d\phi^2} + \left[1 - \left(\frac{dF}{du}\right)_{u=u_0}\right] \delta = O(\delta^2), \quad (20)$$

which is obtained by substituting $u = u_0 + \delta$ into Eq. (18). Therefore, in the first order in δ , the trajectory is given by

$$\delta = \delta_0 \cos\left(\sqrt{1 - \left(\frac{dF}{du}\right)_{u=u_0}} \phi + \beta\right), \quad (21)$$

where δ_0 and β are constants of integration. The angles of the perihelia of the orbit are the angles for which r is minimum, and hence u or δ is maximum. Therefore, the variation of the orbital angle from one perihelion to the next is

$$\phi = \frac{2\pi}{\sqrt{1 - \left(\frac{dF}{du}\right)_{u=u_0}}} = \frac{2\pi}{1 - \sigma}. \quad (22)$$

The quantity σ defined by the above equation is called the perihelion advance, which represents the rate of advance of the perihelion. As the planet advances through ϕ radians in its orbit, its perihelion advances through $\sigma\phi$ radians. From Eq. (22) σ is given by

$$\sigma = 1 - \sqrt{1 - \left(\frac{dF}{du}\right)_{u=u_0}}, \quad (23)$$

or, for small $(dF/du)_{u=u_0}$, by

$$\sigma = \frac{1}{2} \left(\frac{dF}{du}\right)_{u=u_0}. \quad (24)$$

For a complete rotation we have $\phi \approx 2\pi(1 + \sigma)$, and the advance of the perihelion is

$$\delta\phi = \phi - 2\pi \approx 2\pi\sigma. \quad (25)$$

The observed value of the perihelion precession of the planet Mercury is $\delta\phi_{Obs} = 43.11 \pm 0.21$ arcsec per century [21]. As a first step in the study of the perihelion precession in Hořava-Lifshitz gravity, the relevant functions are given by

$$f(u) = -\frac{\omega}{u^2} + \frac{\omega}{u^2} \sqrt{1 + \frac{4Mu^3}{\omega}},$$

$$G(u) = -\omega \left(1 + \frac{1}{L^2u^2}\right) \left(1 - \sqrt{1 + \frac{4M}{\omega}u^3}\right) + \frac{1}{L^2}(E^2 - 1),$$

and

$$F(u) = \frac{\omega}{L^2u^3} \left(1 - \sqrt{1 + \frac{4M}{\omega}u^3}\right) + 3M \left(1 + \frac{1}{L^2u^2}\right) \frac{u^2}{\sqrt{1 + \frac{4M}{\omega}u^3}}, \quad (26)$$

respectively.

The circular orbits are given by the roots of the equation $F(u_0) = u_0$, which is given by

$$3Mu_0^2 - \frac{M}{L^2} = \frac{\omega}{L^2u_0^3} + \sqrt{1 + \frac{4M}{\omega}u_0^3} \left(u_0 - \frac{\omega}{L^2u_0^3}\right). \quad (27)$$

In order to solve Eq. (27) we represent the parameter ω as $\omega = \omega_0/M^2$, and u_0 as $u_0 = x_0/M$, where ω_0 and x_0 are dimensionless parameters, respectively. Then Eq. (27) can be written as

$$3x_0^2 - b^2 = \frac{\omega_0 b^2}{x_0^3} + \sqrt{1 + \frac{4}{\omega_0}x_0^3} \left(x_0 - \frac{\omega_0 b^2}{x_0^3}\right), \quad (28)$$

where $b^2 = M^2/L^2$.

In the case of the planet Mercury we have $a = 57.91 \times 10^{11}$ cm and $e = 0.205615$, while for the values of the mass of the Sun and of the physical constants we take $M = M_\odot = 1.989 \times 10^{33}$ g, $c = 2.998 \times 10^{10}$ cm/s, and $G = 6.67 \times 10^{-8}$ cm³g⁻¹s⁻², respectively [24]. Mercury also completes 415.2 revolutions each century. With the use of these numerical values we first obtain $b^2 = M/a(1 - e^2) = 2.66136 \times 10^{-8}$. By performing a first order series expansion of the square root in Eq. (28), we obtain the standard general relativistic

equation $3x_0^2 - x_0 + b^2 = 0$, with the physical solution $x_0^{(GR)} \approx b^2$. In the general case, the value of x_0 also depends on the numerical value of ω_0 , and, for a given ω_0 , x_0 must be obtained by numerically solving the nonlinear algebraic equation Eq. (28). The variation of x_0 as a function of ω_0 is represented in Fig. 1.

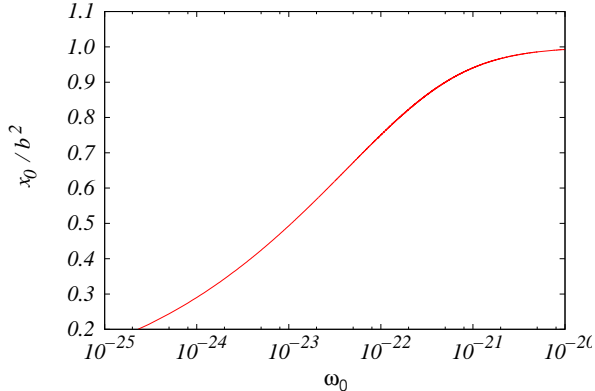


FIG. 1: Variation of x_0 as a function of ω_0 .

The perihelion precession is given by $\delta\phi = \pi (dF(u)/du)|_{u=u_0}$, and in the variables x_0 and ω_0 can be written as

$$\delta\phi = \pi \frac{3\sqrt{\omega_0}\Phi(\omega_0, x_0)}{x_0^4 (4x_0^3 + \omega_0)^{3/2}}, \quad (29)$$

where

$$\begin{aligned} \Phi(\omega_0, x_0) = & 2(x_0^3 + \omega_0)x_0^5 + \\ & b^2 \left[2x_0^6 + \left(6\omega_0 - 4\sqrt{\omega_0(4x_0^3 + \omega_0)} \right) x_0^3 \right. \\ & \left. + \omega_0^2 - \sqrt{\omega_0^3(4x_0^3 + \omega_0)} \right]. \end{aligned} \quad (30)$$

In the ‘‘Post-Newtonian’’ limit $4x_0^3/\omega_0 \ll 1$, we obtain the classical general relativistic result $\delta\phi_{GR} = 6\pi b^2$. The variation of the perihelion precession angle as a function of ω_0 is represented in Fig. 2.

The gravity analysis of radio Doppler and range data generated by the Deep Space Network with Mariner 10 during two of its encounters with Mercury in March 1974 and March 1975 determined the observed value of the perihelion precession of the planet Mercury as $\delta\phi_{Obs} = 43.11 \pm 0.20$ arcsec per century [24]. Therefore the range of variation of the perihelion precession is $\delta\phi_{Obs} \in (42.91, 43.31)$. This range of observational values fixes the range of ω_0 as $\omega_0 \in (6.95508 \times 10^{-16}, 6.98748 \times 10^{-16})$. The general relativistic formula for the precession, gives $\delta\phi_{GR} = 42.94$ arcsec per century. The perihelion precession can be also obtained by using elliptic integrals, or, in the case of the arbitrary central potentials, by using the method developed in [25].

Recently, new observational results on the extra-precession of the perihelion of the planets of the Solar

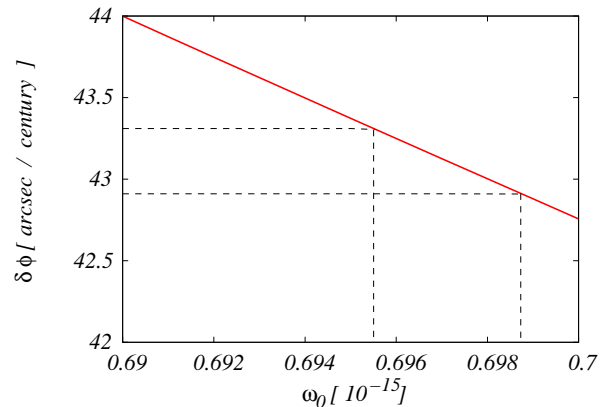


FIG. 2: Variation of the planetary precession angle $\delta\phi$ (in arcseconds per century) as a function of ω_0 .

System have been obtained. These results can be used as a test for gravitational models in the Solar System, like, for example, the determinations of the PPN parameter β and of the coefficient J_2 of the oblateness of the Sun [22]. One can also use these results to estimate a possible advance in the planets perihelia, and an anomalous precession to the usual Newtonian/Einsteinian secular precession of the longitude of the perihelion of Saturn was found [23]. Once the precision of the observations would improve, these data could also be used to constrain through perihelion precession the value of the parameter ω for the Kehagias-Sfetsos black hole solution.

B. Light deflection by the Sun

The deflection angle of light rays passing nearby the Sun in the KS geometry is given, with the use of general relation derived in [26], by

$$\phi(r) = \phi(\infty) + \int_r^\infty \frac{[f(r)]^{-1/2}}{\sqrt{f(r_0)(r/r_0)^2/f(r) - 1}} \frac{dr}{r}, \quad (31)$$

where r_0 is the distance of the closest approach, and we have denoted

$$f(r) = 1 + \omega r^2 - \sqrt{r(\omega^2 r^3 + 4\omega M)}. \quad (32)$$

By introducing a new variable x by means of the transformation $r = r_0 x$, Eq. (31) can be written as

$$\phi(r_0) = \phi(\infty) + \int_1^\infty \frac{[f(r_0 x)]^{-1/2}}{\sqrt{f(r_0) x^2/f(r_0 x) - 1}} \frac{dx}{x} \quad (33)$$

By representing ω as $\omega = \omega_0/M^2$, and r_0 as $r_0 = x_0 M$, we obtain

$$\phi(x_0) = \phi(\infty) + \int_1^\infty \frac{[g(\omega_0, x_0, x)]^{-1/2}}{\sqrt{g_0(\omega_0, x_0) x^2/g(\omega_0, x_0, x) - 1}} \frac{dx}{x}, \quad (34)$$

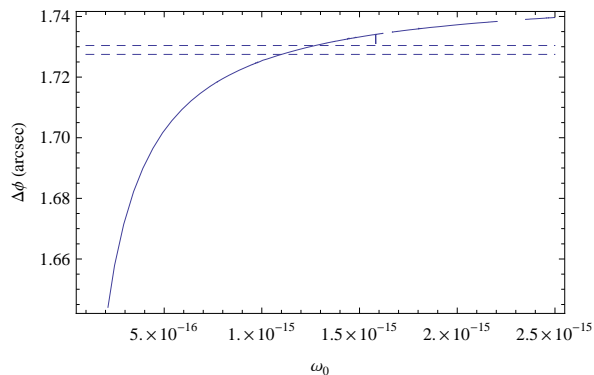


FIG. 3: The light deflection angle $\Delta\phi$ (in arcseconds) as a function of the parameter ω_0 .

where we have denoted

$$g(\omega_0, x_0, x) = 1 + \omega_0 x_0^2 x^2 - \sqrt{x_0 x (\omega_0^2 x_0^3 x^3 + 4\omega_0)}, \quad (35)$$

and

$$g_0(\omega_0, x_0) = 1 + \omega_0 x_0^2 - \sqrt{x_0 (\omega_0^2 x_0^3 + 4\omega_0)}, \quad (36)$$

respectively.

For the Sun, by taking $r_0 = R_\odot = 6.955 \times 10^{10}$ cm, where R_\odot is the radius of the Sun, we find for x_0 the value $x_0 = 4.71194 \times 10^5$. The variation of the deflection angle $\Delta\phi = 2|\phi(x_0) - \phi(\infty)| - \pi$ is represented, as a function of ω_0 , in Fig. 3. In the "Post-Newtonian" limit $4x_0^3/\omega_0 \ll 1$, we obtain the classical general relativistic result $\Delta\phi = 4M_\odot/R_\odot = 1.73''$.

We consider now the constraints on the Hořava-Lifshitz gravity arising from the solar system observations of the light deflection by the Sun. The best available data come from long baseline radio interferometry [27], which gives $\delta\phi_{LD} = \delta\phi_{LD}^{(GR)}(1 + \Delta_{LD})$, with $\Delta_{LD} \leq 0.0017$, where $\delta\phi_{LD}^{(GR)} = 1.7275$ arcsec. The observational constraints of light deflection restricts the value of ω_0 to $\omega_0 \in (1.1 \times 10^{-15}, 1.3 \times 10^{-15})$.

C. Radar echo delay

A third Solar System test of general relativity is the radar echo delay [21]. The idea of this test is to measure the time required for radar signals to travel to an inner planet or satellite in two circumstances: a) when the signal passes very near the Sun and b) when the ray

does not go near the Sun. The time of travel of light t_0 between two planets, situated far away from the Sun, is given by

$$t_0 = \int_{-l_1}^{l_2} dy, \quad (37)$$

where dy is the differential distance in the radial direction, and l_1 and l_2 are the distances of the planets to the Sun, respectively. If the light travels close to the Sun, in the metric given by Eq. (7) the time travel is

$$t = \int_{-l_1}^{l_2} \frac{dy}{v} = \int_{-l_1}^{l_2} e^{[\lambda(r) - \nu(r)]/2} dy, \quad (38)$$

$v = e^{(\nu - \lambda)/2}$ is the speed of light in the presence of the gravitational field. The time difference is

$$\Delta t = t - t_0 = \int_{-l_1}^{l_2} \left\{ e^{[\lambda(r) - \nu(r)]/2} - 1 \right\} dy. \quad (39)$$

Since $r = \sqrt{y^2 + R_\odot^2}$, where R_\odot is the radius of the Sun, we have

$$\Delta t = \int_{-l_1}^{l_2} \left\{ e^{[\lambda(\sqrt{y^2 + R_\odot^2}) - \nu(\sqrt{y^2 + R_\odot^2})]/2} - 1 \right\} dy. \quad (40)$$

The first experimental Solar System constraints on time delay have come from the Viking lander on Mars [21]. In the Viking mission two transponders landed on Mars and two others continued to orbit round it. The latter two transmitted two distinct bands of frequencies, and thus the Solar coronal effect could be corrected for. However, recently the measurements of the frequency shift of radio photons to and from the Cassini spacecraft as they passed near the Sun have greatly improved the observational constraints on the radio echo delay. For the time delay of the signals emitted on Earth, and which graze the Sun, one obtains $\Delta t_{RD} = \Delta t_{RD}^{(GR)}(1 + \Delta_{RD})$, with $\Delta_{RD} \leq (2.1 \pm 2.3) \times 10^{-5}$ [28].

For the case of the Earth-Mars-Sun system we have $R_E = l_1 = 1.525 \times 10^{13}$ cm (the distance Earth-Sun) and $R_P = l_2 = 2.491 \times 10^{13}$ cm (the distance Mars-Sun). With these values the standard general relativistic radar echo delay has the value $\Delta t_{RD}^{(GR)} \approx 4M_\odot \ln(4l_1 l_2 / R_\odot^2) \approx 2.4927 \times 10^{-4}$ s. With the use of Eq. (40), it follows that the time delay for the KS black hole solution of Hořava-Lifshitz gravity can be represented as

$$\Delta t_{RD} = 2 \int_{-l_1}^{l_2} \frac{\omega (y^2 + R_\odot^2) \left[\sqrt{1 + (4M_\odot/\omega) (y^2 + R_\odot^2)^{-3/2}} - 1 \right]}{1 - \omega (y^2 + R_\odot^2) \left[\sqrt{1 + (4M_\odot/\omega) (y^2 + R_\odot^2)^{-3/2}} - 1 \right]} dy. \quad (41)$$

By introducing a new variable ξ defined as $y = 2\xi M_\odot$, and by representing again ω as $\omega = \omega_0/M_\odot^2$, we obtain for the time delay, the following expression

$$\Delta t_{RD} = 16\omega_0 M_\odot \int_{-\xi_1}^{\xi_2} \frac{(\xi^2 + a^2) \left[\sqrt{1 + (1/2\omega_0)(\xi^2 + a^2)^{-3/2}} - 1 \right]}{1 - 4\omega_0(\xi^2 + a^2) \left[\sqrt{1 + (1/2\omega_0)(\xi^2 + a^2)^{-3/2}} - 1 \right]} d\xi, \quad (42)$$

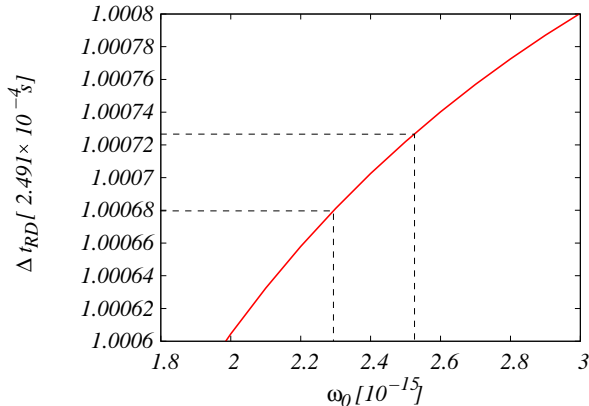


FIG. 4: Variation of the time delay Δt_{RD} as a function of ω_0 .

where $a^2 = R_\odot^2/4M_\odot^2$, $\xi_1 = l_1/2M_\odot$, and $\xi_2 = l_2/2M_\odot$, respectively. The variation of the time delay as a function of ω_0 is represented in Fig. 4. In the limiting case $4M/\omega r^3 \ll 1$ we reobtain again the standard general relativistic result [21].

The observational values of the radar echo delay are consistent with the KS black hole solution in Hořava-Lifshitz gravity if $\omega_0 \in (2.0199 \times 10^{-15}, 2.2000 \times 10^{-15})$. The general relativistic value $\Delta t_{RD} = 2.4927 \times 10^{-4}$ is obtained for $\omega_0 \approx 4 \times 10^{-15}$.

IV. DISCUSSIONS AND FINAL REMARKS

In the present paper, we have considered observational possibilities for testing the Kehagias and Sfetsos solution of the vacuum field equations in Hořava-Lifshitz gravity at the level of the Solar System. We have found that this solution can give a satisfactory description of the perihelion precession of the planet Mercury, and of the other gravitational phenomena in the Solar System. The classical tests of general relativity (perihelion precession, light deflection and radar echo delay) give strong constraints on the numerical value of the parameter ω of the model. The parameter ω , having the physical dimensions of length^{-2} , is constrained by the perihelion precession of the planet Mercury to a value of $\omega \geq 7 \times 10^{-16}/M_\odot^2 = 3.212 \times 10^{-26} \text{ cm}^{-2}$. The deflection angle of the light rays by the Sun can be fully explained in Hořava-Lifshitz gravity with the parameter ω having the value $\omega \geq 10^{-15}/M_\odot^2 = 4.5899 \times 10^{-26} \text{ cm}^{-2}$,

while the radar echo delay experiment suggests a value of $\omega \geq 2 \times 10^{-15}/M_\odot^2 = 9.1798 \times 10^{-26} \text{ cm}^{-2}$. Tentatively, and in order to provide a numerical value that could be used in practical calculations, from these values we can give an estimate of ω as

$$\omega = (5.660 \pm 3.1) \times 10^{-26} \text{ cm}^{-2}. \quad (43)$$

The standard deviation in our determination of ω is $3.2 \times 10^{-26} \text{ cm}^{-2}$, the variance is $9.7 \times 10^{-26} \text{ cm}^2$, and the median of the determined values is $4.5 \times 10^{-26} \text{ cm}^2$. It is interesting to note that the values of ω obtained from the study of the light deflection by the Sun and of the radar echo delay experiment are extremely close, while the perihelion precession of the planet Mercury provides a smaller value. Unfortunately, the observational data on the perihelion precession are strongly affected by the solar oblateness, whose value is poorly known [29, 30]. We have also neglected the solar Lense-Thirring effect [31], as well as the effects of the asteroids. Even so, by taking into account the smallness of the parameter ω , it follows that there is a very good agreement between the numerical values for ω obtained from these three Solar System Tests. On the other hand, it is important to note that the light deflection and the radar echo delay observations, which are very similar from a physical point of view, do not give intersecting intervals for ω_0 , since the value of ω_0 obtained from light deflection is in the range $\omega_0 \in (1.1 \times 10^{-15}, 1.3 \times 10^{-15})$, while from the radar echo delay we obtain $\omega_0 \in (2.0199 \times 10^{-15}, 2.2000 \times 10^{-15})$. The values obtained from the light deflection by the Sun are systematically smaller, by a factor of around two, as compared to the values obtained from radar echo delay observations. This non-intersecting range of values could be explained by the differences in the observational errors for the two effects. While the observational error in light deflection is around 0.0017, the corresponding error in the radar echo delay observations is of the order of $10^{-5} - 10^{-6}$. The very small error of the radar echo delay allows a very precise determination of the value of ω_0 . Another possibility for this discrepancy may be related to some intrinsic properties of the model, like, for example, the fact that in Hořava-Lifshitz gravity there is no full diffeomorphism invariance of the Hamiltonian formalism.

In the weak-field and slow-motion approximation, the corrections to the third Kepler law of a test particle in the Kehagias and Sfetsos black hole geometry were obtained in [12]. The corrections were compared to the phe-

nomenologically determined orbital period of the transiting extrasolar planet HD209458b Osiris. The order-of-magnitude lower bound on the parameter ω_0 obtained from this study is $\omega_0 \geq 1.4 \times 10^{-18}$, as compared to the value $\omega_0 \geq 7 \times 10^{-16}$ obtained in the present paper. Tighter constraints are established by the inner planets for which $\omega_0 \geq 10^{-15} - 10^{-12}$ [12]. However, in order to obtain a better precision from these data, a full general relativistic study is needed, as well as a significant improvement in the determination of the values of the orbital periods of the exoplanets.

Thus, the gravitational dynamics of the KS solution is determined by the free parameter ω . In order to explain the observational effects in the Solar System, ω must have an extremely small value, of the order of a few 10^{-26} cm^{-2} . Therefore the explanation of the classical tests of GR must require a very precise fine tuning of this constant at the level of the Solar System. It is also very important for future observations to determine if ω is a local quantity or a universal constant. By assuming that ω is a universal constant, its smallness suggests the possibility that it may have a microscopic origin.

In conclusion, the study of the classical tests of general relativity provide a very powerful method for constraining the allowed parameter space of the Hořava-Lifshitz gravity solutions, and to provide a deeper insight into

the physical nature and properties of the corresponding spacetime metrics. Therefore, this opens the possibility of testing Hořava-Lifshitz gravity by using astronomical and astrophysical observations at the Solar System scale [12]. Of course, this analysis requires developing general methods for the high precision study of the classical tests in arbitrary spherically symmetric spacetimes. In the present paper we have provided some basic theoretical tools necessary for the in depth comparison of the predictions of the Hořava-Lifshitz gravity model with the observational/experimental results.

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