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Combined Semi-definite Relaxation and Sphere Decoding Method for Multiple Antennas Systems

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Abstract—In this paper, a new detection method which combines the semi-definite programming relaxation (SDR) with the sphere decoding (SD) is proposed for 256-QAM multiple-input multiple-output (MIMO) system. In this method, the SDR algorithms are engaged to obtain a primary result. Then, a hyper-sphere is constructed which is centered at the received signal and has its radius equals to the Euclidean distance between the primary result and the received signal. Finally, the SD searching strategy is employed to determine the final result which satisfies the principle of maximum likelihood. Simulation results show that the proposed method can offer optimum BLER performance as well as lower computational complexity than the conventional SD detectors.

Keywords- *Semi-definite Relaxation; MIMO System; Sphere decoding*

I. INTRODUCTION

Multiple antennas system which is also called multiple-input multiple-output (MIMO) system has been considered as a promising solution to provide high data rate for future wireless communications. However, due to the limitations of either unsatisfactory performance or high complexity of the detection methods, MIMO system is still far from reaching full practical implementation. The maximum likelihood (ML) detection can provide the best block-error-rate (BLER) performance, but its computational complexity is NP-hard. In order to overcome this challenge, some other detectors had been proposed. Equalization-based detectors such as zero-forcing (ZF) detector and minimum mean squared error (MMSE) detector have very low complexity, but they suffer from unacceptable degradations in BLER performance. Sphere decoding (SD) [1, 2] is able to provide optimum BLER performance with less complexity than ML decoding by searching only a subset of the entire lattice space within a hyper-sphere. Nevertheless, it is still impractical when the constellation size is large and the signal-to-noise ratio (SNR) is low. This is mainly caused by the large initial radius of the hyper-sphere which results in a very time-consuming searching process. In the traditional SD decoders, either the result of ZF or the result of MMSE is employed to determine the initial hyper-sphere. Due to the poor BLER performance of ZF or MMSE for cases with large constellation size, such as 64-QAM and 256-QAM, the complicated searching process becomes unacceptable.

Recently, the decoding algorithms based on semi-definite programming (SDP) approach have been suggested for MIMO detector. It has become more and more attractive simply because of the fact that semi-definite relaxation (SDR) problems can be conveniently solved in polynomial time [3],

[4]. The SDR was firstly applied for BPSK, 4-QAM [5], [6], and 16-QAM signals, such as bound-constrained SDR (BC-SDR) [7] and virtually-antipodal SDR (VA-SDR) [8]. SDR detection of these signals all exhibit acceptable BLER performance and relatively low complexity. Extension of SDR methods for 256-QAM signals has been proposed in [9], [10]. Although the SDR detectors can offer significantly low computational complexity, as well as better BLER performance than ZF and MMSE, their BLER performance is still worse than SD detector because of the relaxation process.

The purpose of this paper is to propose a new detection method for 256-QAM signals which combines the SDR with the SD. In this method, the SDR algorithms are engaged to obtain a primary result. Then, a hyper-sphere is constructed which is centered at the received signal and has its radius equals to the Euclidean distance between the primary result and the received signal. Finally, the SD searching strategy is employed to determine the final result which satisfies the principle of maximum likelihood. This method can offer optimum BLER performance as well as lower computational complexity than the traditional SD detectors.

The paper is organized as follows. In Section II, the concept of SDR and its application to MIMO detection will be introduced. Then the principle of the combined SDR-SD detection algorithm will be presented in Section III. Finally, results of computer simulation conducted to verify the validity of the proposed method will be reported in Section IV. Section V is the conclusion.

II. PROBLEM FORMULATION

The MIMO system with M_t transmit antennas and M_r receive antennas is modeled as:

$$\mathbf{r}_c = \mathbf{H}_c \mathbf{x}_c + \mathbf{n}_c \quad (1)$$

where $\mathbf{r}_c \in \mathbb{C}^{M_r}$ is the received signal vector, and $\mathbf{x}_c \in \mathbb{C}^{M_t}$ is the transmitted signal vector; $\mathbf{H}_c \in \mathbb{C}^{M_r \times M_t}$ is the channel matrix, with elements h_{ij} representing the transfer function from the j -th transmit antenna to i -th receive antenna; $\mathbf{n}_c \in \mathbb{C}^{M_r}$ is an independently and identically distributed zero-mean Gaussian noise vector with elements having a fixed variance. The complex transmission given by (1) can be equivalently represented in real matrix form as:

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (2)$$

$$\text{where } \mathbf{r} = \begin{bmatrix} \text{Re}(\mathbf{r}_c) \\ \text{Im}(\mathbf{r}_c) \end{bmatrix}, \mathbf{H} = \begin{bmatrix} \text{Re}(\mathbf{H}_c) & -\text{Im}(\mathbf{H}_c) \\ \text{Im}(\mathbf{H}_c) & \text{Re}(\mathbf{H}_c) \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \text{Re}(\mathbf{x}_c) \\ \text{Im}(\mathbf{x}_c) \end{bmatrix}$$

with $\text{Re}(\bullet)$ and $\text{Im}(\bullet)$ being the real and imaginary parts of (\bullet) , respectively.

The 256-QAM ML detection aims at finding the solution of the following optimization problem:

$$\begin{cases} \min \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 & (3.1) \\ \text{st. } \mathbf{x} \in \{\pm 1, \pm 3, \pm 5, \pm 7, \pm 9, \pm 11, \pm 13, \pm 15\}^{N_t} & (3.2) \end{cases}$$

It is well known that ML detection can provide the best BLER performance. However, since this optimization problem is a non-convex problem, its computational complexity is unacceptably high.

In order to reduce the complexity, the original ML problem is transformed to a semidefinite problem by converting the objective function and relaxing the feasible set as follows:

Define a rank-1 semidefinite matrix $\mathbf{\Omega}$, which is given by:

$$\mathbf{\Omega} = [\mathbf{x}^T \quad 1]^T [\mathbf{x}^T \quad 1] \quad (4)$$

It is easy to find that the ML detection problem given in (3) can be rewritten as:

$$\begin{cases} \min Tr \left\{ \mathbf{\Omega} \begin{bmatrix} \mathbf{H}^T \mathbf{H} & -\mathbf{H}^T \mathbf{r} \\ -\mathbf{r}^T \mathbf{H} & \mathbf{r}^T \mathbf{r} \end{bmatrix} \right\} & (5.1) \\ \text{st. } \mathbf{\Omega} = \mathbf{\Omega}^T \in \mathbb{R}^{(N_t+1) \times (N_t+1)} & (5.2) \\ \mathbf{\Omega}_{1,1} = \mathbf{\Omega}_{1,2} \mathbf{\Omega}_{2,1} \in \mathbb{R}^{N_t \times N_t} & (5.3) \\ \mathbf{\Omega}_{1,2} \in \{\pm 1, \pm 3, \pm 5, \pm 7, \pm 9, \pm 11, \pm 13, \pm 15\}^{N_t} & (5.4) \\ \mathbf{\Omega}_{2,2} = 1 & (5.5) \end{cases}$$

It can be observed that the extremely high complexity of the ML detection is due to the presence of the two non-convex constraints (5.3) and (5.4). Thus, relaxation of these constraints will be engaged to transform the original problem into a semidefinite problem, which can then be efficiently solved in polynomial time. In what follows, three kinds of SDR decoders will be presented, which are derived from three different relaxation processes.

A. SDR1 (BC-SDR)

The constraint (5.4) implies $1 \leq x_i^2 \leq 225$, where x_i denotes the i -th component of \mathbf{x} . Then, (5.2) along with (5.3) can be relaxed into $\mathbf{Y} \succeq 0$. A new symbol \mathbf{Y} is introduced here to distinguish from the aforementioned $\mathbf{\Omega}$ since they are actually different matrixes after relaxation. Consequently, the BC-SDR problem is obtained as:

$$\begin{cases} \min Tr \left\{ \mathbf{Y} \begin{bmatrix} \mathbf{H}^T \mathbf{H} & -\mathbf{H}^T \mathbf{r} \\ -\mathbf{r}^T \mathbf{H} & \mathbf{r}^T \mathbf{r} \end{bmatrix} \right\} & (6.1) \\ \text{st. } \mathbf{Y} \in \mathbb{R}^{(N_t+1) \times (N_t+1)} \succeq 0 & (6.2) \\ \mathbf{1}\mathbf{1} \preceq \text{Diag}\{\mathbf{Y}_{1,1}\} \preceq 225\mathbf{I}, & (6.3) \\ \mathbf{Y}_{2,2} = 1 & (6.4) \end{cases}$$

The BC-SDR problem (6) can then be solved by any of the SDP solvers, such as Sedumi [4], based on interior point methods. Once the solution is found, the solution of the relaxed problem should be converted to a feasible and approximation solution.

B. SDR2

Considering the constraint (3.2), the signal \mathbf{x} could also be expressed as:

$$\mathbf{x} = \mathbf{V}\mathbf{q}^T \quad (7)$$

where $\mathbf{V} = [\mathbf{I} \quad 4\mathbf{I}]$, $\mathbf{q} = [\mathbf{q}_1 \quad \mathbf{q}_2]$, $\mathbf{I} \in \mathbb{R}^{N_t \times N_t}$ and $\mathbf{q}_1, \mathbf{q}_2 \in \{\pm 1, \pm 3\}^{N_t}$.

Table 1 The Value of x_j

$\mathbf{q}_2 \backslash \mathbf{q}_1$	-3	-1	1	3
-3	-15	-7	1	9
-1	-13	-5	3	11
1	-11	-3	5	13
3	-9	-1	7	15

Table I gives the value of x_j for the possible combinations of q_{1j} and q_{2j} , where x_j , q_{1j} and q_{2j} denote the j -th element of \mathbf{x} , \mathbf{q}_1 and \mathbf{q}_2 , respectively.

By substituting (7) into (4), and defining a matrix \mathbf{W} , which is given by:

$$\mathbf{W} = [\mathbf{q} \quad 1]^T [\mathbf{q} \quad 1] \quad (8)$$

Thus, we obtain the SDR problem given by:

$$\begin{cases} \min Tr \left\{ \mathbf{W} \begin{bmatrix} \mathbf{V}^T \mathbf{H}^T \mathbf{H} \mathbf{V} & -\mathbf{V}^T \mathbf{H}^T \mathbf{r} \\ -\mathbf{r}^T \mathbf{H} \mathbf{V} & \mathbf{r}^T \mathbf{r} \end{bmatrix} \right\} & (9.1) \\ \text{st. } \mathbf{1}\mathbf{1} \preceq \text{Diag}\{\mathbf{W}_{1,1}\} \preceq 9\mathbf{I}, \mathbf{I} \in \mathbb{R}^{2N_t \times 2N_t} & (9.2) \\ \text{Diag}\{\mathbf{W}_{1,1}\} \pm 4\text{Diag}\{\mathbf{W}_{1,2}\} + 3\mathbf{I} \succeq 0, \mathbf{I} \in \mathbb{R}^{2N_t \times 2N_t} & (9.3) \\ \mathbf{W} \in \mathbb{R}^{(2N_t+1) \times (2N_t+1)} \succeq 0 & (9.4) \\ \mathbf{W}_{2,2} = 1 & (9.5) \end{cases}$$

Once the optimum solution of (9) is obtained, the first $2N_t$ elements of the last row in the solution $\widehat{\mathbf{W}}$ can be considered as $\widehat{\mathbf{q}} = [\widehat{\mathbf{q}}_1 \ \widehat{\mathbf{q}}_2]$, and thus the optimum solution $\widehat{\mathbf{x}}$ is reconstructed by using (7). Finally, the feasible solution is found.

C. SDR3(VA-SDR)

It is worth noting that when the constraint (3.2) is expected to be satisfied, the signal \mathbf{x} could be expressed as:

$$\mathbf{x} = \mathbf{U}\mathbf{p}^T \quad (10)$$

where $\mathbf{U} = [\mathbf{I} \ 2\mathbf{I} \ 4\mathbf{I} \ 8\mathbf{I}]$, $\mathbf{p} = [\mathbf{p}_1 \ \mathbf{p}_2 \ \mathbf{p}_3 \ \mathbf{p}_4]$, $\mathbf{I} \in \mathbb{R}^{N_t \times N_t}$ and $\mathbf{p}_i \in \{\pm 1\}^{N_t}$, $i=1, 2, 3, 4$. Substituting (10) into (4), and defining a matrix \mathbf{Z} , which is given by:

$$\mathbf{Z} = [\mathbf{p} \ 1]^T [\mathbf{p} \ 1] \quad (11)$$

Thus, we obtain the VA-SDR problem given by:

$$\begin{cases} \min Tr \left\{ \mathbf{Z} \begin{bmatrix} \mathbf{U}^T \mathbf{H}^T \mathbf{H} \mathbf{U} & -\mathbf{U}^T \mathbf{H}^T \mathbf{r} \\ -\mathbf{r}^T \mathbf{H} \mathbf{U} & \mathbf{r}^T \mathbf{r} \end{bmatrix} \right\} & (12.1) \\ \text{st. } \mathbf{Z} \in \mathbb{R}^{(4N_t+1) \times (4N_t+1)} \succeq 0 & (12.2) \\ \text{Diag}(\mathbf{Z}) = \mathbf{I} & (12.3) \end{cases}$$

Since the first $4N_t$ elements of the last row in the solution $\widehat{\mathbf{Z}}$ can be considered as $\widehat{\mathbf{p}} = [\widehat{\mathbf{p}}_1 \ \widehat{\mathbf{p}}_2 \ \widehat{\mathbf{p}}_3 \ \widehat{\mathbf{p}}_4]$, thus the optimum solution $\widehat{\mathbf{x}}$ is reconstructed by using (10). Finally, the solution is converted to a feasible and approximation solution.

All of the SDR problems above can be solved in polynomial time efficiently. Then we need to convert the optimum solution of the relaxed problem into a feasible solution. If the optimum solution has rank of one, it is the exact feasible solution. If not, there are some methods such as dominant eigenvector approximation, randomization or rank-1 approximation to convert it to a feasible solution.

III. SDR-SD DETECTION ALGORITHM

A. Review of Sphere decoding

The Sphere decoding searches within a hyper-sphere the lattice points \mathbf{x} which satisfy:

$$\|\mathbf{r} - \mathbf{H}\mathbf{x}\| < d^2 \quad (13)$$

where d is the initial radius of the sphere. The searching of the lattice points is a back-substitution algorithm. \mathbf{H} is firstly reduced into an upper triangular matrix by using the QR decomposition:

$$\mathbf{H} = \mathbf{Q} \begin{bmatrix} \mathbf{R} \\ 0 \end{bmatrix} \quad (14)$$

where $\mathbf{Q} \in \mathbb{R}^{N_r \times N_r}$ is orthogonal and $\mathbf{R} \in \mathbb{R}^{N_r \times N_t}$ is upper triangular matrix. Let $\mathbf{Q} = [\mathbf{Q}_1 \ \mathbf{Q}_2]$, where $\mathbf{Q}_1 \in \mathbb{R}^{N_r \times N_t}$ and $\mathbf{Q}_2 \in \mathbb{R}^{N_r \times (N_r - N_t)}$. (13) can be written as

$$\begin{aligned} \|\mathbf{r} - \mathbf{H}\mathbf{x}\| &< d^2 \\ \left\| \mathbf{Q} \begin{bmatrix} \mathbf{R} \\ 0 \end{bmatrix} \mathbf{x} - \mathbf{r} \right\| &< d^2 \\ \left\| \begin{bmatrix} \mathbf{R} \\ 0 \end{bmatrix} \mathbf{x} - \begin{bmatrix} \mathbf{Q}_1^T \\ \mathbf{Q}_2^T \end{bmatrix} \mathbf{r} \right\| &< d^2 \\ \|\mathbf{R}\mathbf{x} - \mathbf{Q}_1^T \mathbf{r}\| &< d^2 - \|\mathbf{Q}_2^T \mathbf{r}\| \end{aligned} \quad (15)$$

Let $\mathbf{r}' = \mathbf{Q}_1^T \mathbf{r}$, $d' = d^2 - \|\mathbf{Q}_2^T \mathbf{r}\|$, (15) becomes

$$\|\mathbf{R}\mathbf{x} - \mathbf{r}'\| < d' \quad (16)$$

Thus, the boundary of x_{N_t} is

$$\left[\frac{r'_{N_t} - d'}{R_{N_t, N_t}} \right] \leq x_{N_t} \leq \left[\frac{r'_{N_t} + d'}{R_{N_t, N_t}} \right] \quad (17)$$

The searching process is done from N_t th dimension to 1st dimension, the signals from previously detected dimensions are subtracted from the received signal. Then we can obtain the intervals of \mathbf{x} from N_t th dimension to 1st dimension and determine all the lattice points in the sphere.

B. Combined SDR and SD Method

One of the key issues of SD is the selection of the initial radius d of the hyper-sphere. If the radius is too large, the sphere contains very large number of lattice points, and hence results in very high search complexity. If the radius is too small, the sphere may contain no lattice points and the searching has to be restarted with a new initial radius [11, 12]. A traditional way to attain the initial radius is the ZF equalization. The radius is the distance between the received point and the ZF equalized point. For the MIMO systems with small constellation size, such as 8-QAM and 16-QAM, the ZF-initialed SD has acceptable complexity. However, for MIMO system with large constellation size, such as 256-QAM, the complexity of the ZF-initialed SD becomes unacceptably high. For this reason, the feasible solution of the SDR problem is proposed to be the initial point of SD for 256-QAM MIMO systems. The radius becomes the distance between the received point and the feasible point. Since the SDR detections have much better BLER performance than ZF for 256-QAM MIMO systems, the radius given by SDR tends to be smaller. Thus, the number of lattice points to be visited inside the sphere is smaller, which means it can reduce the complexity of SD.

IV. SIMULATION RESULTS

Matlab simulation has been used to assess the performances of the proposed SDR-SD used in the 4×4 MIMO systems transmitting 256-QAM in a block-fading channel. The three SDR detectors in Section II are separately combined with SD. Their BLER performances are compared with the stand-alone SD detector and SDR detectors and the results are shown in Figure 1. It can be seen that the three SDR-SD detectors can offer the same BLER performance as the SD detector, and have much better BLER performance than the stand-alone SDR detectors. Figure 2 shows the comparison of the complexity of the different detection algorithms. In this paper, the complexity of a detection algorithm is measured by the computational time required. It can be seen that all three combined SDR-SD detectors are faster than the stand-alone SD detector. Furthermore, the SDR1-SD has the lowest complexity.

V. CONCLUSION

In this paper, the concept of SDR and its application to MIMO detection is presented. Then the principle of the combined SDR-SD detection algorithm is proposed. The simulation results show that the SDR-SD detectors have much lower complexity compared with the stand-alone SD detector while maintaining the optimum BLER performance.

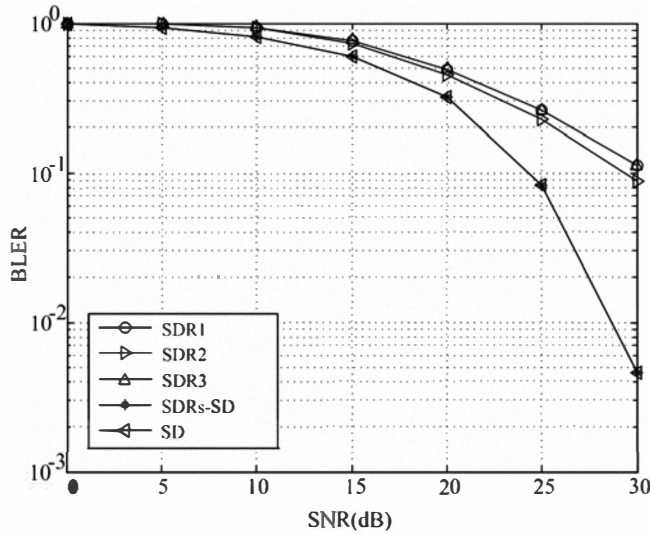


Figure 1. Comparison of BLER performance.

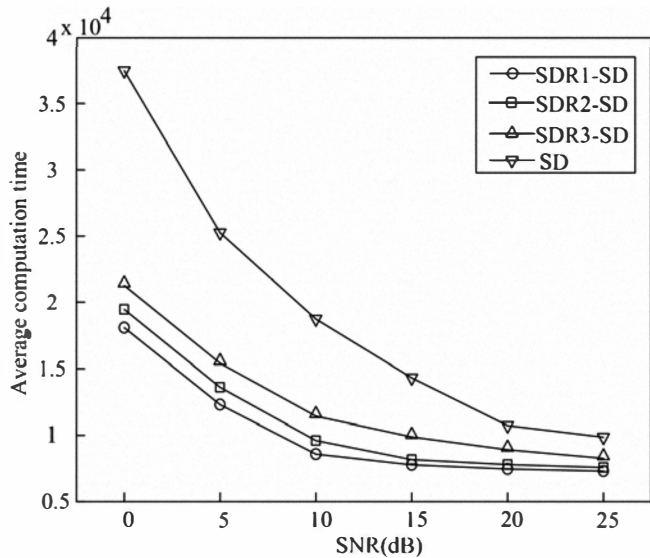


Figure 2. Comparison of computational time.

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