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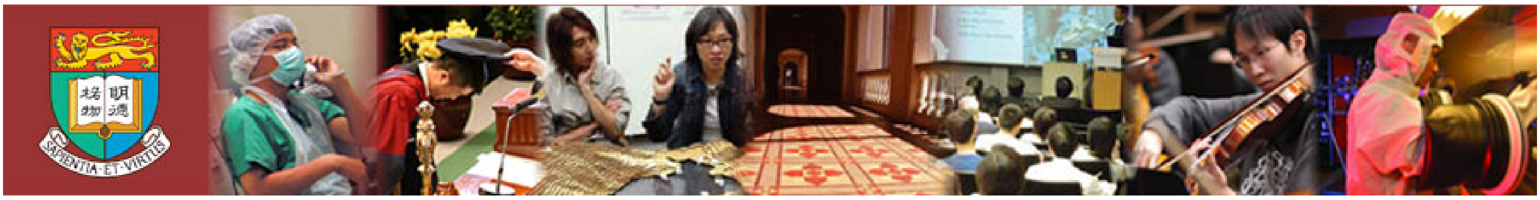

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Finite-Width Gap Excitation and Impedance Models

Yat Hei Lo*, Shiquan He§, Lijun Jiang† and Weng Cho Chew‡
Department of Electrical and
Electronic Engineering
The University of Hong Kong
Room 204, Chow Yei Ching Building
Pokfulam Road, Hong Kong
Email: *yhlo@eee.hku.hk, §sqhe@hku.hk, †ljiang@eee.hku.hk, ‡wcchew@hku.hk
‡On leave from University of Illinois, Urbana-Champaign, USA.

***Abstract*—In this paper, we present a new method for the feed model for the method of moments (MoM). It is derived from a more accurate model with the realistic size of the excitation, in order to replace the commonly-used delta-gap excitation model. This new model is formulated around the electric field integration equations (EFIE) where the terms for magnetic current and magnetic field can be removed. Hence it is much simpler to implement and reduces the numerical complexity. In addition, a variational formulation is derived to provide second order accuracy of the input admittance calculation. Moreover, this new formulation can be easily extended such that one can insert passive load elements of finite size onto the distributive network, without complicated modification of the MoM analysis. This allows simulation of many realistic networks which include load elements such as resistors, capacitors and inductors.**

I. Introduction

The modeling of wire antennas, or antennas of arbitrary shapes, involves the use of a source or a feed model. There has been a need to construct a model which is realistic, and at the same time, applicable to the formulation applied. One common modeling algorithm applied for antennas utilizes the electric field integration equation (EFIE) and method of moments (MoM) [1]. In most cases, a delta-gap source model is used. The excitation is considered as an unphysically infinitesimally small port, with a width of almost zero. Then, using MoM with RWG basis [2], the voltage applied can be modeled as a voltage jump across a set of basis functions which assemble the delta-gap. This method allows very simple implementation of the source model [3].

The delta-gap model is by far, not ideal for the reason that no physical feed can be infinitesimally small. There have been efforts to consider a more realistic excitation model with a finite gap size. In fact, an accurate model has been formulated [4], and the topic of realistic source models continues [5]. When considering an accurate source model, based on physical principles such as Huygens and the reciprocity theorem [4], it is inevitable that the computation becomes more expensive. This is due to the fact that the applied voltage is equivalent to an incident electric field $\mathbf{E}_{inc}$, where $\mathbf{E}_{inc}$ has to be generated by a magnetic frill current $\mathbf{M}_s$. In the formulation of EFIE with PEC, it is not preferable to include $\mathbf{M}$, and hence $\mathbf{H}$ into the calculations. Otherwise one must also consider the curl of the Green's function in the integral equation, which is sometimes referred to as the $\mathcal{K}$-operator [6].

Here, we derive an alternative model which is based on [4] but removing the need for the frill current, hence the $\mathcal{K}$-operator in the calculations. The model allows the use of variational formulation for accurate calculation of the input admittance or impedance. The formulation presented here also allows the insertion of passive load elements, with a finite size. Some preliminary results for insertion of lumped capacitors are presented. We demonstrate for the first time that the variational form could accelerate the convergence

II. Formulation

Consider an arbitrary structure being driven by a voltage source at a gap as shown in Fig. 1(a). By equivalence principle, it can be accurately modeled by surrounding the gap a magnetic frill current $\mathbf{M}_s$ as in Fig. 1(b). This yields an electric field of $\mathbf{E}_a$ within the gap. The relation between $\mathbf{M}_s$ and $\mathbf{E}_a$ is given by [4]

$$\mathbf{M}_s = -\hat{n} \times \mathbf{E}_a. \tag{1}$$

The value of $\mathbf{E}_a$ provides the voltage jump between the gap as if a voltage is applied across the gap. For gaps much smaller than the wavelength, $\mathbf{M}_s$ can be assumed to be uniform. This is dual to the case of a solenoid, where a circulating magnetic current generates an electric field inside the gap, which becomes the incident field $\mathbf{E}_a$ for EFIE formulation.

The gap is now filled with PEC, while surrounded by $\mathbf{M}_s$, as shown in Fig. 1(b). Only the tangential component of $\mathbf{E}_a$ (we denote as $\mathbf{E}_{z,a}$) just outside the PEC is considered. Whereas $\mathbf{E}_{z,a}$ induces a current on the PEC which in turns generates a scattered field $\mathbf{E}_p$. The tangential components of $\mathbf{E}_a$ and $\mathbf{E}_p$ shall be equal and opposite, i.e. $\mathbf{E}_{z,p} = -\mathbf{E}_{z,a}$.

The importance of $\mathbf{M}_s$ is that it induces a voltage jump across the gap. This voltage jump can be modeled as the equivalent incident field. Therefore, instead of calculating the excitation in terms of $\mathbf{M}_s$, one can use the equivalent $\mathbf{E}_{z,a}$. It induces the voltage across the gap given by

$$\int_{-\infty}^{\infty} E_{z,a} dz = V, \tag{2}$$

where V is the voltage applied across the gap.

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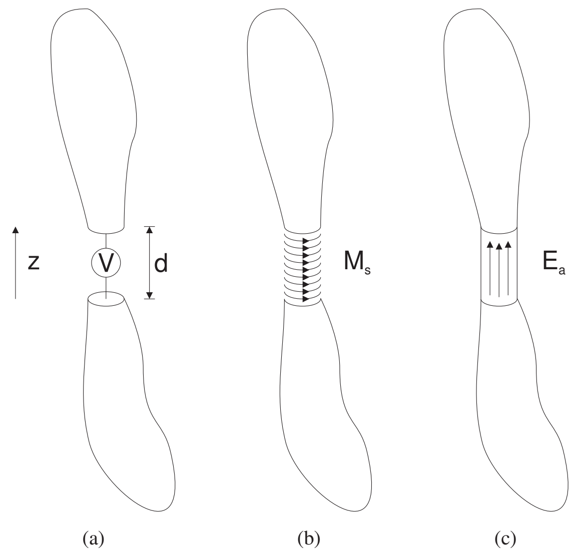


Fig. 1. Equivalent models of an arbitrary structure driven by a voltage source

A. Integral Equation

The integral equation of a PEC cylinder driven by a ribbon current is given by

$$\begin{aligned} -E_{z,a} &= i\omega\mu\hat{z}\cdot\int_{-\infty}^{\infty}\overline{\mathbf{G}}(z,z')\cdot\mathbf{J}_p(z')dz'2\pi a \\ &= E_{z,p}, \end{aligned} \tag{3}$$

where $\mathbf{E}_{z,a}$ is the incident field generated by the magnetic frill current as described above; $\mathbf{J}_p$ is the induced current on the surface of the PEC. In order to satisfy the voltage jump as expressed in (2), $\mathbf{E}_{z,a}$ may be expressed as a pulse function with a uniform electric field of V/d where d is the gap distance [7].

B. Variational Formula for Input Admittance

The input impedance of an antenna can be simply determined from the ratio between the applied voltage and resulting current that flows across the source gap. Usually, one considers the input admittance and the impedance is simply the reciprocal. After $\mathbf{J}_p$ is solved for, the input admittance is defined as

$$Y_{in} = \frac{I}{V} = \frac{\langle \mathbf{E}_a, \mathbf{J}_p\rangle}{V^2}. \tag{4}$$

The above definition is so called the direct form, which is not variational. To derive the variational form of the admittance, we introduce an additional term, which becomes zero in the limit of the exact solution of $\mathbf{J}_p$, then (4) becomes

$$Y_{in} = \frac{\langle \mathbf{E}_a, \mathbf{J}_p\rangle}{V^2} + \frac{\langle \mathbf{E}_t, \mathbf{J}_p\rangle}{V^2}, \tag{5}$$

where $\mathbf{E}_t$ is the total electric field given by

$$\mathbf{E}_t = \mathbf{E}_a + \mathbf{E}_p. \tag{6}$$

To prove that (5) is variational, we take the first-order variation about the exact solution, which becomes

$$\delta Y_{in} \cong \frac{1}{V^2}\left[\langle \mathbf{E}_a, \delta\mathbf{J}\rangle + \langle \mathbf{E}_{te}, \delta\mathbf{J}\rangle + \langle \delta\mathbf{E}, \mathbf{J}_{pe}\rangle\right], \tag{7}$$

where the additional subscript e denotes exact solutions. By reciprocity, we have

$$\langle \delta\mathbf{E}, \mathbf{J}_{pe}\rangle = \langle \mathbf{E}_{pe}, \delta\mathbf{J}\rangle. \tag{8}$$

Using the above, and by (6), (7) can be written as

$$\begin{aligned} \delta Y_{in} &\cong \frac{1}{V^2}\left[\langle \mathbf{E}_a, \delta\mathbf{J}\rangle + \langle \mathbf{E}_{te}, \delta\mathbf{J}\rangle + \langle \mathbf{E}_{pe}, \delta\mathbf{J}\rangle\right] \\ &= \frac{2}{V^2}\langle \mathbf{E}_{te}, \delta\mathbf{J}\rangle \\ &= 0. \end{aligned} \tag{9}$$

Equation (9) is true because the tangential component of $\mathbf{E}_{te}$ is zero for an exact solution. This shows that when a first-order error of $\mathbf{J}_p$ exists, error of Y_{in} remains zero to the first order. This variational formulation yields a more accurate calculation of the input admittance, hence the input impedance. Using (6), (5) can also be rewritten as

$$Y_{in} = 2\frac{\langle \mathbf{E}_a, \mathbf{J}_p\rangle}{V^2} + \frac{\langle \mathbf{E}_p, \mathbf{J}_p\rangle}{V^2}. \tag{10}$$

C. Lumped Gap Impedance Elements

Very often, one needs to introduce additional passive elements such as capacitors, inductors or resistors in a distributive network. For example, in order to avoid undesirable reflections from the end of a terminating port, one can introduce small losses by inserting resistors or capacitors. Collectively, it is a problem of inserting lumped impedance elements into MoM analysis.

A common practice of achieving lumped elements is similar to that of a delta-gap excitation model, where an infinitesimally small element is inserted [8]. The impedance is then given by the corresponding voltage drop across the edge where the element is inserted.

Here, our modeling of the gap impedance is similar as the gap excitation model described above. Consider a structure containing an impedance element Z_g of finite size. The voltage drop across the element is given by V_g where

$$V_g = Z_g I_g. \tag{11}$$

We consider the case of method of moments using RWG basis and Galerkin testing [2]. Then $\mathbf{J}_p$ can be expressed as

$$\mathbf{J}_p \cong \sum_{n=1}^{N} I_n\mathbf{f}_n(\mathbf{r}), \tag{12}$$

where $\mathbf{f}_n(\mathbf{r})$ is the RWG basis function. With the insertion of a lumped element, there exists a set of basis functions which are in the region of the lumped element. We call those set of basis function $\mathbf{f}_z$. At the boundary between the lumped element and normal PEC there exists another set of basis functions. We call those $\mathbf{f}_i$. Figure 2(a) shows an illustration of a simple example of a strip and its equivalent model is shown in Fig. 2(b).

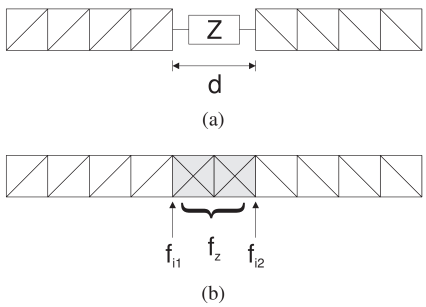


Fig. 2. Equivalent lumped element model with a finite size.

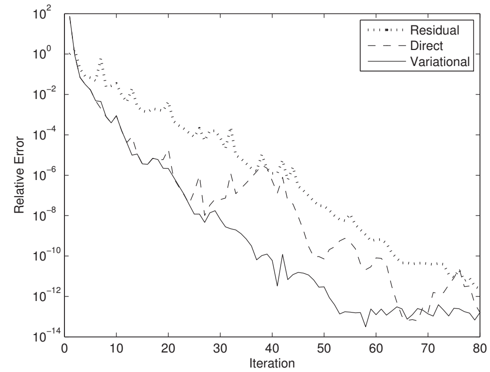


Fig. 3. Convergence of the strip model using BiConjugate Gradient iteration and variational formulation.

The shaded area indicates the position of the lumped element inserted. Then $\mathbf{f}_z$ includes the edges inside the shaded area, and $\mathbf{f}_i$ are those edges at the boundary between the shaded and unshaded areas.

Recall that the voltage V_g depends on I_g, and that I_g is unknown. One cannot utilize (3) directly and let $\mathbf{E}_{z,a} = V_g/d$. Instead the current I_g is given by

$$I_g = \sum \mathbf{f}_i I_i l_i \tag{13}$$

at either end of the lumped element, where I_i are the coefficients of the corresponding basis functions $\mathbf{f}_i$, and l_i are the edge lengths. For elements of size much smaller than the wavelength, one can see I_g at $\mathbf{f}_{i1}$ and $\mathbf{f}_{i2}$ are equal, i.e. the current entering the lumped element is equal to that leaving the element. This follows from the current continuity equation. Given $V_g = Z_g I_g$, and $\mathbf{E}_{z,a} = V_g/d$, for every entries in the impedance matrix $\overline{\mathbf{Z}}$ where $\mathbf{f}_z$ and $\mathbf{f}_i$ intersect, the value of the matrix entry is to be corrected by

$$Z_{zi} \rightarrow Z_{zi} + \left\langle \mathbf{f}_z, \hat{t} \cdot \frac{Z_g l_i}{d} \right\rangle \tag{14}$$

before the matrix system is solved for.

III. Numerical Results

A. Finite Gap Excitation and Variational Formulation

We verify our formulation using a half-wavelength strip of 2m long dipole, with a width of 20mm, and at a frequency of 75MHz. The input impedance of this strip is $(88.68-51.82i)\Omega$ if calculated using a delta-gap excitation model. With our finite gap excitation model, the impedance is calculated as $(87.35-52.56i)\Omega$, using a gap distance of 20mm. Both values are within range of values presented in Table 4.1 of [3]. Figure 3 shows the convergence history of this model when solved using bi-conjugate gradient (BiCG) method. The relative errors of the input impedance, calculated from the direct method using (4) and the variational formulation using (10) against the exact value solved using Gaussian elimination are both shown for comparison.

It should be noted that it may be considered low frequency for such a half-wavelength antenna model, when the mesh density becomes large. In this case, convergence can be slow or even break down when using an iterative solver. In order to ensure proper operation of the matrix solver, loop-tree decomposition, frequency normalization, and basis rearrangement [9] have been carried out before the system is solved. Hence as shown in Fig. 3, the convergence is easily achieved within 100 iterations.

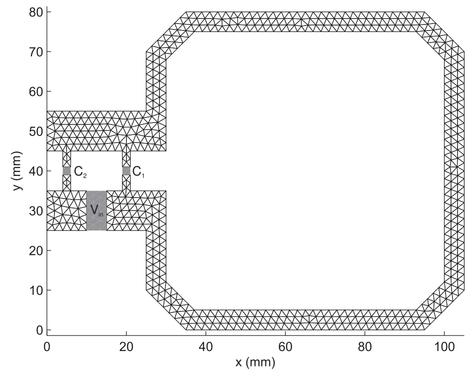
It can be seen that with the variational formulation, the error of Z_{in} reduces reasonably monotonically and rapidly. The fluctuation of the convergence curve is a behavior of BiCG. The convergence of Z_{in} is achieved much faster than that of the residual. It is also obvious that the error using variational formulation could be enhanced over the direct method by two orders of the magnitude. Also much larger fluctuation of the direct method arises from the machine precision error.

B. Finite Gap Lumped Elements

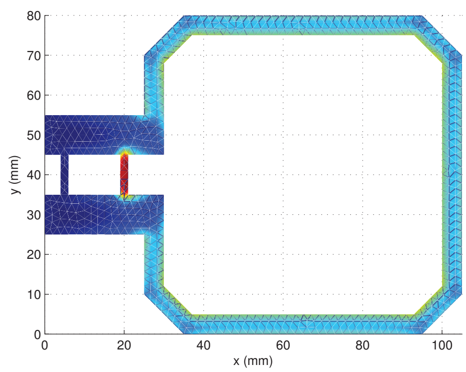
We also considered an example of an inductive coil designed for the generation of uniform magnetic fields for the application of magnetic resonance imaging (MRI). The coil is shown in Fig. 4(a). The operating frequency is 12.5MHz. There are two capacitors connected to the model, whereas C_1 = 825pF and C_2 = 25pF. Using the formulation as described, the resultant current density $\mathbf{J}_p$ is shown as in Fig. 4(b). The result is also compared against that simulated by Ansoft's HFSS software, where they match well albeit the different color scales are used.

IV. Conclusion

We presented a new method for a source feed model. This new model realizes the finite width of a physical feed. By virtue of an equivalent incident electric field which provides the necessary voltage jump across the gap, the formulation removes the requirement of the magnetic frill current for an accurate model. This allows simpler implementation for EFIE algorithms. The input admittance can also be expressed in a variational form for the second order accuracy. The formulation described here can be modified such that, one can insert lumped load elements of finite sizes. The construction of the lumped load elements is similar to that of the source feed model. The incident field arising from the loads are derived from a current dependent voltage drop across the element, and



(a)



(b)

Fig. 4. An example coil for the application of MRI: (a) Geometry and meshing, also shown the finite gap excitation and capacitance ports; (b) Resultant $\mathbf{J}_p$ on the model, solved using method of moments with the gap excitation and gap element formulation.

corresponding entries in the impedance matrix are corrected for.

Acknowledgment

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