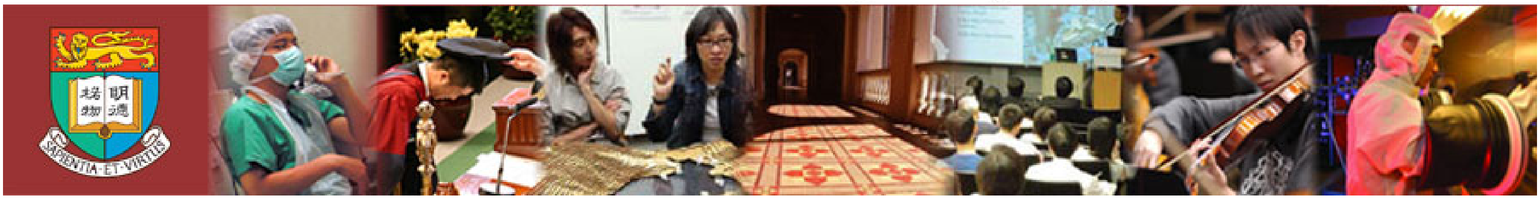

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Reflection and transmission of line-source excited pulsed EM fields at a thin, high-contrast layer with dielectric and conductive properties

Adrianus T. de Hoop
Delft University of Technology,
Faculty of Electrical Engineering,
Mathematics and Computer Science,
Laboratory of Electromagnetic Research,
Mekelweg 4, 2628 CD Delft, the Netherlands
Email: a.t.dehoop@tudelft.nl

Lijun Jiang
The University of Hong Kong,
Department of Electrical and Electronic Engineering,
Pokfulam Road, Hong Kong
Email: jianglj@hku.hk

***Abstract*—A methodology is presented for analytically modeling the reflection and transmission of line-source excited pulsed electromagnetic fields fields at a thin, planar, high-contrast layer with dielectric and conductive properties. Closed-form time-domain expressions are derived for the field components in a two-dimensional setting via an extension of the 'Cagniard-DeHoop' method.**

I. Formulation of the problem

Position in the configuration is specified by the coordinates $\{x, y, z\} \in \mathbb{R}^3$ with respect to a Cartesian reference frame with the origin $\mathcal{O}$ and the three mutually perpendicular base vectors $\{\boldsymbol{i}_x, \boldsymbol{i}_y, \boldsymbol{i}_z\}$ of unit length each. In the indicated order, the base vectors form a right-handed system. The time coordinate is $t \in \mathbb{R}$.

A line source of electric current with volume source density

$$J_y(x,z,t) = I(t)\delta(x, z-h), \tag{1}$$

with electric current $I(t)$ and located at $\{x = 0, -\infty < y < \infty, z = h\}$, with $h > 0$, in a homogeneous, isotropic, lossless medium with electric permittivity $\epsilon > 0$ and magnetic permeability $\mu > 0$, generates a two-dimensional, y-independent electromagnetic field with non-vanishing components $\{H_x, E_y, H_z\}(x, z, t)$. The pertaining EM field equations are

$$\partial_x H_z - \partial_z H_x + \epsilon\,\partial_t E_y = -I(t)\delta(x, z-h), \tag{2}$$

$$\partial_x E_y + \mu\,\partial_t H_z = 0, \tag{3}$$

$$\partial_z E_y - \mu\,\partial_t H_x = 0. \tag{4}$$

In the plane $\{z = 0\}$, a thin layer of vanishingly small thickness d and with high contrasts in dielectric and conductive properties is present. Its properties are modeled by the thin-sheet boundary conditions [2]

$$\lim_{z\downarrow 0} E_y(x,z,t) = \lim_{z\uparrow 0} E_y(x,z,t) = E_y(x,0,t) \text{ for all } x \in \mathbb{R}, t \in \mathbb{R}, \tag{5}$$

$$\lim_{z\downarrow 0} H_x(x,z,t) - \lim_{z\uparrow 0} H_x(x,z,t) = (G_L + C_L\partial_t)E_y(x,0,t) \text{ for all } x \in \mathbb{R}, t \in \mathbb{R}, \tag{6}$$

in which

$$G_L = \lim_{d\downarrow 0} \int_{z=-d/2}^{d/2} \sigma_L(z)\mathrm{d}z, \tag{7}$$

where $\sigma_L(z)$ is the electric conductivity of the layer and

$$C_L = \lim_{d\downarrow 0} \int_{z=-d/2}^{d/2} \epsilon_L(z)\mathrm{d}z, \tag{8}$$

where $\epsilon_L(z)$ is the electric permittivity of the layer.

It is assumed that the electric current excitation starts to act at the instant $t = 0$ and that prior to this instant the field quantities vanish throughout the configuration.

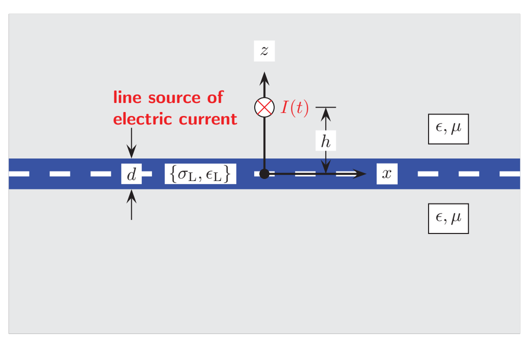

Fig. 1. Configuration with highly contrasting thin sheet.

II. The field problem in the wave slowness domain

The time invariance of the configuration enables the use of the one-sided time Laplace transformation

$$\{\hat{H}_x, \hat{E}_y, \hat{H}_z\}(x,z,s) = \int_{t=0}^{\infty} \exp(-st)\{H_x, E_y, H_z\}(x,z,t)\mathrm{d}t. \tag{9}$$

For our further proceedings, the time Laplace transform parameter s is taken to be real and positive, which in view of Lerch's theorem [7, p. 63–65] is sufficient for the uniqueness of the inversion. Under the transformation (9), the operator ∂_t is replaced with s. Next, the spatial shift invariance of the configuration enables the use of the wave slowness representation

$$\{\hat{H}_x, \hat{E}_y, \hat{H}_z\}(x, z, s) = \frac{s}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \exp(-spx)\{\tilde{H}_x, \tilde{E}_y, \tilde{H}_z\}(p, z, s)\mathrm{d}p, \quad (10)$$

where p is the (complex-valued) wave slowness parameter. With this representation, the operator ∂_x is replaced with $-sp$. Furthermore, using the properties of the Dirac delta distribution,

$$\tilde{J}_y(p, z, s) = \hat{I}(s)\delta(z - h). \quad (11)$$

With this, the field equations (2) – (4) transform into

$$-sp\tilde{H}_z - \partial_z \tilde{H}_x + s\,\epsilon\,\tilde{E}_y = -\hat{I}(s)\delta(z - h), \quad (12)$$
$$-sp\tilde{E}_y + s\,\mu\,\tilde{H}_z = 0, \quad (13)$$
$$\partial_z E_y - s\mu\,\tilde{H}_x = 0, \quad (14)$$

and the boundary conditions (5) – (6) into

$$\lim_{z\downarrow 0} \tilde{E}_y(p, z, s) = \lim_{z\uparrow 0} \tilde{E}_y(p, z, s) = \tilde{E}_y(p, 0, s), \quad (15)$$

$$\lim_{z\downarrow 0} \tilde{H}_x(p, z, s) - \lim_{z\uparrow 0} \tilde{H}_x(p, z, s) = (G_\mathrm{L} + s\,C_\mathrm{L})\tilde{E}_y(p, 0, s). \quad (16)$$

III. The slowness-domain field expressions

Defining the *incident field* $\{H^\mathrm{i}_x, E^\mathrm{i}_y, H^\mathrm{i}_z\}$ as the field that would exist in the absence of the contrasting layer, (12) – (14) lead to

$$\partial_z^2 \tilde{E}^\mathrm{i}_y + s^2\gamma^2(p)\tilde{E}^\mathrm{i}_y = -s\,\mu\,\hat{I}(s)\delta(z - h), \quad (17)$$

in which

$$\gamma = (c^{-2} - p^2)^{1/2}, \quad (18)$$

with $\gamma > 0$ for $p \in \mathbb{I}$ and

$$c = (\epsilon\,\mu)^{-1/2}. \quad (19)$$

From (17) it follows that

$$\tilde{E}^\mathrm{i}_y = \mu\,\hat{I}(s)\frac{\exp[-s\gamma(p)Z^\mathrm{i}]}{2\gamma(p)}, \quad (20)$$

in which $Z^\mathrm{i} = |z - h|$. In the half-space $\{(x, y) \in \mathbb{R}^2, z > 0\}$ we now write the total field as the sum of the incident and the *reflected field* $\{H^\mathrm{r}_x, E^\mathrm{r}_y, H^\mathrm{r}_z\}$. In the half-space $\{(x, y) \in \mathbb{R}^2, z < 0\}$ we denote the total field as the *transmitted field* $\{H^\mathrm{t}_x, E^\mathrm{t}_y, H^\mathrm{t}_z\}$. Taking into account that the reflected field and the transmitted field travel away from the scattering thin layer, we write their electric field slowness representation as

$$\tilde{E}^\mathrm{r}_y = \mu\,\tilde{R}(p, s)\hat{I}(s)\frac{\exp[-s\gamma(p)Z^\mathrm{r}]}{2\gamma(p)}, \quad (21)$$

in which $Z^\mathrm{r} = z + h$, with $z > 0$, and

$$\tilde{E}^\mathrm{t}_y = \mu\,\tilde{T}(p, s)\hat{I}(s)\frac{\exp[-s\gamma(p)Z^\mathrm{t}]}{2\gamma(p)}, \quad (22)$$

in which $Z^\mathrm{t} = h - z$, with $z < 0$. In (21), $\tilde{R}(p, s)$ is the *slowness-domain reflection coefficient*; in (22), $\tilde{T}(p, s)$ is the *slowness-domain transmission coefficient*. Using (22) – (24) in (13) – (14) and substituting the result in the boundary conditions (15) – (16), we obtain

$$\tilde{R}(p, s) = -1 + \tilde{T}(p, s), \quad (23)$$
$$\tilde{T}(p, s) = \frac{\beta_\mathrm{L}(p)}{\beta_\mathrm{L}(p) + G_\mathrm{L}/C_\mathrm{L} + s}, \quad (24)$$

with

$$\beta_\mathrm{L}(p) = \frac{2\gamma(p)}{\mu\,C_\mathrm{L}}. \quad (25)$$

IV. The space-time expressions for the electric field constituents

The expressions for the time Laplace transformed electric field constituents are written as

$$\hat{E}^{\mathrm{i,r,t}}_y(x, z, s) = s\,\mu\,\hat{I}(s)\hat{G}^{\mathrm{i,r,s}}(x, z, s), \quad (26)$$

in which

$$\hat{G}^\mathrm{i}(x, z, s) = \frac{1}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\exp\{-s[px + \gamma(p)Z^\mathrm{i}]\}}{2\gamma(p)}\mathrm{d}p, \quad (27)$$

$$\hat{G}^\mathrm{r}(x, z, s) = \frac{1}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \tilde{R}(p, s)\frac{\exp\{-s[px + \gamma(p)Z^\mathrm{r}]\}}{2\gamma(p)}\mathrm{d}p, \quad (28)$$

$$\hat{G}^\mathrm{t}(x, z, s) = \frac{1}{2\pi\mathrm{i}} \int_{p=-\mathrm{i}\infty}^{\mathrm{i}\infty} \tilde{T}(p, s)\frac{\exp\{-s[px + \gamma(p)Z^\mathrm{t}]\}}{2\gamma(p)}\mathrm{d}p, \quad (29)$$

are the corresponding Green's function constituents. Note that the integrands, considered as a function of p and with s real and positive, have no poles in the complex p-plane, only branch points. This is a characteristic for the absence of true surface waves like, for example in acoustics, the Rayleigh wave at the stress-free boundary of an elastic solid, the Scholte wave at a fluid/solid boundary and the Stoneley wave at the interface of two elastic solids. This does, however, not imply that no large surface effects can occur.

The time-domain counterparts of (27) – (29) are determined with the aid of an extension of the standard modified Cagniard method [1]. Accordingly, the integrand in the integration with respect to p is continued analytically into the complex p-plane, away from the imaginary axis and, under the application of Cauchy's theorem and Jordan's lemma, the integration along the imaginary p-axis is replaced with one along the hyperbolic path (modified Cagniard path) $px + \gamma(p)Z = \tau$ for $T < \tau < \infty$, where $Z > 0$, $T = D/c$ and $D = (x^2 + Z^2)^{1/2} > 0$, while τ replaces p as the variable of integration. In the relevant Jacobian, the relation $\partial p/\partial\tau = \mathrm{i}\gamma(p)/(\tau^2 - T^2)^{1/2}$ is used. Next, Schwarz's reflection principle of complex function theory is used to combine the integrations in the upper and lower halves of the complex p-plane. Parametrizing the upper part of the modified Cagniard path through

$$\overline{p}(x,Z,\tau) = \frac{x}{D^2}\tau + \mathrm{i}\frac{Z}{D^2}(\tau^2 - T^2)^{1/2} \quad \text{for } T < \tau < \infty, \tag{30}$$

which has the consequence that

$$\begin{aligned}\overline{\gamma}(x,Z,\tau) &= \gamma[\overline{p}(x,Z,\tau)] \\ &= \frac{Z}{D^2}\tau - \mathrm{i}\frac{x}{D^2}(\tau^2 - T^2)^{1/2} \\ &\quad \text{for } T < \tau < \infty,\end{aligned} \tag{31}$$

(20) leads to

$$\hat{G}^{\mathrm{i}}(x,Z^{\mathrm{i}},s) = \frac{1}{2\pi}\int_{\tau=T^{\mathrm{i}}}^{\infty} \exp(-s\tau) \frac{1}{(\tau^2 - T^{\mathrm{i}2})^{1/2}}\mathrm{d}\tau, \tag{32}$$

(21) to

$$\hat{G}^{\mathrm{r}}(x,Z^{\mathrm{r}},s) = \frac{1}{2\pi}\int_{\tau=T^{\mathrm{r}}}^{\infty} \exp(-s\tau) \,\mathrm{Re}\left[-1 + \frac{\beta_{\mathrm{L}}(\overline{p})}{\beta_{\mathrm{L}}(p) + G_{\mathrm{L}}/C_{\mathrm{L}} + s}\right]\frac{1}{(\tau^2 - T^{\mathrm{r}2})^{1/2}}\mathrm{d}\tau, \tag{33}$$

and (22) to

$$\hat{G}^{\mathrm{t}}(x,Z^{\mathrm{t}},s) = \frac{1}{2\pi}\int_{\tau=T^{\mathrm{t}}}^{\infty} \exp(-s\tau) \,\mathrm{Re}\left[\frac{\beta_{\mathrm{L}}(\overline{p})}{\beta_{\mathrm{L}}(p) + G_{\mathrm{L}}/C_{\mathrm{L}} + s}\right]\frac{1}{(\tau^2 - T^{\mathrm{t}2})^{1/2}}\mathrm{d}\tau, \tag{34}$$

where (23) – (25) have been used.

In (32), Lerch's uniqueness theorem of the one-sided Laplace transformation [7, pp. 63–65] directly yields

$$G^{\mathrm{i}}(x,Z^{\mathrm{i}},t) = \frac{1}{2\pi(t^2 - T^{\mathrm{i}2})^{1/2}}H(t - T^{\mathrm{i}}). \tag{35}$$

If in (33) and (34) the term in brackets in the integrands had been independent of s, Lerch's theorem would, here as well, directly provide the corresponding function of time. With the aid of the Schouten–Van der Pol theorem of the one-sided Laplace transformation [3], [4, pp. 124–126], [5], [6, pp. 232–236] we proceed further, however, and apply further rules of the inverse Laplace transformation to obtain

$$G^{\mathrm{r}}(x,Z^{\mathrm{r}},t) = \left[-\frac{1}{2\pi(t^2 - T^{\mathrm{r}2})^{1/2}} + \frac{1}{2\pi}\int_{\tau=T^{\mathrm{r}}}^{t} \mathrm{Re}\{\beta_{\mathrm{L}}(\overline{p})\exp\{-[\beta_{\mathrm{L}}(\overline{p}) + G_{\mathrm{L}}/C_{\mathrm{L}}](t-\tau)\}\} \frac{1}{(\tau^2 - T^{\mathrm{r}2})^{1/2}}\mathrm{d}\tau\right]H(t - T^{\mathrm{r}}) \tag{36}$$

and

$$G^{\mathrm{t}}(x,Z^{\mathrm{t}},t) = \left[\frac{1}{2\pi}\int_{\tau=T^{\mathrm{t}}}^{t} \mathrm{Re}\{\beta_{\mathrm{L}}(\overline{p})\exp\{-[\beta_{\mathrm{L}}(\overline{p}) + G_{\mathrm{L}}/C_{\mathrm{L}}](t-\tau)\}\} \frac{1}{(\tau^2 - T^{\mathrm{t}2})^{1/2}}\mathrm{d}\tau\right]H(t - T^{\mathrm{t}}). \tag{37}$$

(Note that, due to the presence of τ in $\beta_{\mathrm{L}}(\overline{p})$, the right-hand sides of (36) and (37) are not time convolutions.) Since $\beta_{\mathrm{L}}(\overline{p})$ is complex-valued, its occurrence in the exponential function is expected to lead to oscillatory phenomena whose magnitude is the more pronounced the larger the imaginary part of $\beta_{\mathrm{L}}(\overline{p})$ is with respect to its real part. Evidently, this ratio is position dependent and the phenomenon is expected to be larger the larger the ratio $|x|/Z^{\mathrm{r,t}}$ is, i.e., close to the boundary, or, which is equivalent, at large horizontal offsets.

Equation (26) finally leads to the time-domain expressions for the electric field constituents

$$E_y^{\mathrm{i,r,t}}(x,z,t) = \mu\,\partial_t\left[I(t) \overset{(t)}{*} G^{\mathrm{i,r,t}}(x,z,t)\right], \tag{38}$$

where $\overset{(t)}{*}$ denotes time convolution. Further investigation into the effect of the different parameters on the wave shapes requires a study via the numerical evaluation of the relevant integrals in (36) – (38).

V. Conclusion

Via an extension of the Cagniard-DeHoop method for analyzing pulsed field behavior in layered configurations, closed-form analytic time-domain expressions have been derived for the electromagnetic field constituents that are generated by a pulsed line source of electric current in the presence of a thin, highly contrasting layer with dielectric and conductive properties in a two-dimensional model setting. The expressions indicate that drastic changes in pulse shape can be expected due to the interaction of the field with the layer. Numerical results based on the field expressions obtained can serve as an indication as to the possibilities of applying the pertaining thin-sheet (approximate) boundary conditions in codes for computational electromagnetics.

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