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Perfect function transfer in two and three dimensions without initialization

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We find analytic models that can perfectly transfer, without state initialization or remote collaboration, arbitrary functions in two- and three-dimensional interacting bosonic and fermionic networks. This provides for the possible experimental implementation of state transfer through bosonic or fermionic atoms trapped in optical lattices. A significant finding is that the state of a spin qubit can be perfectly transferred through a fermionic system. Families of Hamiltonians are described that are related to the linear models and that enable the perfect transfer of arbitrary functions. Furthermore, we propose methods for eliminating certain types of errors.

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I. INTRODUCTION

The study of the transfer of a quantum state using *naturally available* interactions in a spin chain dates back to Bose's work [1]. The motivation is to enable transfer over short distances within a quantum information-processing system. However, in most realistic cases it fails to perfectly perform such a transfer. In 2004, Chirstandl *et al.* [2] found that perfect state transfer (PST) is possible in spin-1/2 networks if parameters in the system's Hamiltonian are carefully engineered. Since that time, several general results have been obtained for state transfer. For example, the fidelity can be made arbitrarily large if there is no limit to the number of sequential gates that can be applied [3]. The communication can also be improved if the sender encodes the message state over a set of multiple spins [4–7], and ideal state transmissions have also been proposed using quantum dots [8,9]. (See also the review [10] and references therein). The possibility of perfect transfer triggered some general results ([10], and references therein, [11]) and interest in finding preengineered models that analytically demonstrate perfect state transfer. For example, Ref. [12] pointed out that PST can be realized without initialization of the chain by making use of the Heisenberg representation.

The strategy for PST in Refs. [2,13] is to map indices of the number of sites of a spin chain onto the magnetic quantum numbers of an angular momentum operator. The nearest-neighbor interaction thus becomes the x component of a quasiangular momentum. It was recently pointed out that the nearest-neighbor interaction can correspond to either the bosonic or spinless fermionic representation of quasiangular momenta operators in one dimension [14]. As with any analytical solvable model in physics, the simplicity and beauty of the result clarifies the physical picture and this one provides *the example* of "perfect state transfer." On the other hand, it is obvious that state transfer should not be limited by a restriction to one-dimensional chains and thus prompts the following question: Can the strategy in Ref. [2] be applied to the two- and three-dimensional systems? Here, we propose an analytical solution to the problem of perfectly transferring an unknown function (termed perfect function transfer or PFT) with one or more variables from a processor at one site to another processor at another site in systems that are two or three dimensional. The interaction required for PST in our model can be implemented using the Bose-Hubbard model (see, e.g., [15] and references therein) or fermions in an optical lattice [16]. In these systems there is a family of Hamiltonians that can perform PFT, obtained via *dressing* transformations.

This paper is organized as follows. In Sec. II we provide the physical Hamiltonians that can be used to transfer states. In Sec. III we provide a generalization of the previous work that enables the transfer of a function through a lattice. This enables us to show how to transfer a function perfectly through the lattice, which we do in Sec. IV. In Sec. V, we show the perfect state transfer of spin qubits between sites. In Sec. VI we generalize these results to include errors in the engineered couplings, dressed states for error avoidance, and the description of the transfer method for a three-dimensional lattice. Finally, we conclude in Sec. VII.

II. LINEAR BOSE MODEL

Consider the dynamics of cold bosonic atoms loaded in a two-dimensional square optical lattice with N sites along the horizontal direction and M sites along the vertical direction governed by the general linear nearest-neighbor Bose model Hamiltonian [15,17–19]:

$$H = -\sum_{j=1}^{M-1}\sum_{k=1}^{N-1}\left[J_{jk}^{(1)}b_{j,k}^{\dagger}b_{j+1,k} + J_{jk}^{(2)}b_{j,k}^{\dagger}b_{j,k+1} + \text{H.c.}\right], \quad (1)$$

where $b_{j,k}^{\dagger}$ ($b_{j,k}$) is the bosonic creation (annihilation) operator at the two-dimensional site (j,k). Equation (1) describes hopping bosons in the absence of on-site repulsion. The hopping (or tunneling) matrix element along the horizontal and vertical directions between nearest-neighbor sites may be given by

$$J_{jk}^{(1)} = \int d^3\vec{r}\, w^*(\vec{r}-\vec{r}_{j,k})[T + V_{\text{lat}}(\vec{r})]w(\vec{r}-\vec{r}_{j+1,k}), \quad (2)$$

$$J_{jk}^{(2)} = \int d^3\vec{r}\, w^*(\vec{r}-\vec{r}_{j,k})[T + V_{\text{lat}}(\vec{r})]w(\vec{r}-\vec{r}_{j,k+1}), \quad (3)$$

where $w(\vec{r}-\vec{r}_{j,k})$ is a single-atom Wannier function at lattice site (j,k), $V_{\rm lat}(\vec{r})$ denotes the optical lattice potential, and T is the kinetic energy of a single atom.

We will consider the limit when the on-site repulsion is negligibly small or the case in which the total boson number is 1. The Hamiltonian in Eq. (1) is also equivalent to that of the on-chip coupled cavities (e.g., in Refs. [20,21]) or coupled superconducting quantum interference devices (SQUIDs) where the coupling, J_{jk}, between each pair of resonators can be tuned.

III. ANGULAR MOMENTUM AND ENGINEERED LINEAR BOSE MODEL

By generalizing Ref. [2] the indices of the number of sites of a two-dimensional lattice can be mapped into the magnetic quantum numbers of the two total angular momenta l_1,l_2 such that $l_1=\frac{M-1}{2}$,$l_2=\frac{N-1}{2}$, and $m_1=-\frac{M+1}{2}+j$,$m_2=-\frac{N+1}{2}+k$. For instance, the magnetic numbers for the first site are $(-\frac{M-1}{2},-\frac{N-1}{2})$.

With this mapping, the bosonic operator (or spinless fermionic operator) $b^\dagger_{j,k}$ at site $j=m_1+\frac{M+1}{2}$ and $k=m_2+\frac{N+1}{2}$ corresponds to the $\mathrm{SU}_1(2)\times\mathrm{SU}_2(2)$ irreducible spherical tensor bosonic operator $A^\dagger_{l_1m_1,l_2m_2}$, or in a shorthand notation $A^\dagger_{m_1,m_2}$. The three components of the horizontal (vertical) quasiangular momentum vector $\vec{L}^{(1)}$ ($\vec{L}^{(2)}$) can be expressed in terms of the atomic creation and annihilation operators as

$$\begin{aligned}
L^{(1)}_x &= \sum_{jk} C^{(1)}_j (b^\dagger_{j,k} b_{j+1,k} + b^\dagger_{j+1,k} b_{j,k}),\\
L^{(2)}_x &= \sum_{ik} C^{(2)}_k (b^\dagger_{j,k} b_{j,k+1} + b^\dagger_{j,k+1} b_{j,k}),
\end{aligned} \tag{4}$$

$$\begin{aligned}
L^{(1)}_y &= i\sum_{jk} C^{(1)}_j (b^\dagger_{j,k} b_{j+1,k} - b^\dagger_{j+1,k} b_{j,k}),\\
L^{(2)}_y &= i\sum_{ik} C^{(2)}_k (b^\dagger_{j,k} b_{j,k+1} - b^\dagger_{j,k+1} b_{j,k}),
\end{aligned} \tag{5}$$

$$L^{(1)}_z = \sum n_{j,k}\left[j-\frac{1}{2}(M+1)\right], \tag{6}$$

$$L^{(2)}_z = \sum n_{j,k}\left[k-\frac{1}{2}(N+1)\right], \tag{7}$$

where $C^{(1)}_j=\frac{1}{2}\sqrt{j(M-j)}$ and $C^{(2)}_k=\frac{1}{2}\sqrt{k(N-k)}$. The particle number operator at site (j,k) is $n_{j,k}=b^\dagger_{j,k}b_{j,k}$ and the total particle number operator is $\hat{n}=\sum n_{j,k}$.

For the tensor product structure of the two SU(2) groups to hold, it is essential that $\vec{L}^{(1)}$ and $\vec{L}^{(2)}$, generators of the groups $\mathrm{SU}_1(2)$ and $\mathrm{SU}_2(2)$, commute. It is not obvious, but $[\vec{L}^{(1)},\vec{L}^{(2)}]=0$ *does* hold for bosons *and* fermions. (Fermions will be discussed later.) This can be seen by directly calculating the commutators of different components. However, unlike in one dimension, the Jordon-Wigner transformation cannot be used to directly replace a fermionic operator by corresponding Pauli matrices $\sigma^+_{j,k}$. If the $b^\dagger_{j,k}\Leftrightarrow\sigma^+_{j,k}$ in (1), then the corresponding $\vec{L}^{(1)}$ and $\vec{L}^{(2)}$ will not commute.

Supposing that the $J_{j,k}$ are preengineered in Eq. (1) as in Ref. [2], such that $J^{(1)}_{j,k}=J^{(1)}C^{(1)}_j$ and $J^{(2)}_{j,k}=J^{(2)}C^{(2)}_k$, then the time evolution becomes

$$U(t)=\exp\left[i\left(J^{(1)}L^{(1)}_x+J^{(2)}L^{(2)}_x\right)t\right]. \tag{8}$$

Note that the parameters $J^{(1)}_{j,k}=J^{(1)}C^{(1)}_j$ are constructed such that they are independent of k. Although the required "fine tuning" has not yet been accomplished in present-day experiments, a certain pattern of inhomogeneous $J_{j,k}$ on different sites has been implemented experimentally [22]. Theoretically, there is no fundamental obstacle to achieving the required $J_{j,k}$.

The irreducible tensor operator $A^\dagger_{m_1,m_2}\Leftrightarrow b^\dagger_{j,k}$ in the Heisenberg representation evolves as

$$\begin{aligned}
U^\dagger(t)A^\dagger_{m_1,m_2}U(t) = \sum e^{i\frac{\pi}{2}(m_1'-m_1)}d^{l_1}_{m_1'm_1}[J^{(1)}(t)]\\
\times e^{i\frac{\pi}{2}(m_2'-m_2)}d^{l_2}_{m_2'm_2}[J^{(2)}(t)]A^\dagger_{m_1',m_2'},
\end{aligned} \tag{9}$$

where $d^l_{m'm}$ is the small Wigner D function. When $J^{(1)}=J^{(2)}=J$ and $t_0=\pi/J$, this expression reduces to a simple form:

$$U^\dagger(t_0)\,b^\dagger_{j,k}\,U(t_0)=r_1r_2b^\dagger_{M-j+1,N-k+1}\,, \tag{10}$$

where the factors $r_1=\exp(-i\pi\frac{M-1}{2})$ and $r_2=\exp(-i\pi\frac{N-1}{2})$ are analogous to the *signature* in nuclear structure theory [23]. These factors determine the interference of quantum states traversing the chain and depend on the number of lattice sites [14]. Choosing the number of sites appropriately can produce interference such that $r_1r_2=1$. Along the two directions after t_0, we have

$$\begin{aligned}
e^{iL^{(1)}_x\pi}b^\dagger_{j,k}e^{-iL^{(1)}_x\pi} &= r_1b^\dagger_{M-j+1,k},\\
e^{iL^{(2)}_x\pi}b^\dagger_{j,k}e^{-iL^{(2)}_x\pi} &= r_2b^\dagger_{j,N-k+1},
\end{aligned}$$

which reduces the problem to one-dimensional chains.

IV. PERFECT STATE TRANSFER

Let us first consider the case of a bosonic system that can be applied to various linear optical or atomic systems. Suppose that a known or unknown function f is encoded in the bottom-left site (1,1), such that $f(x)$ is mapped to the state $f(b^\dagger_{1,1})|\mathbf{0}\rangle$, where $|\mathbf{0}\rangle=|0\rangle^{\otimes N}$. Thus, a general function can be perfectly transferred to the top-right (M,N) by

$$U(t_0)f(b^\dagger_{1,1})|\mathbf{0}\rangle=f(b^\dagger_{M,N})|\mathbf{0}\rangle. \tag{11}$$

Note that the sites, other than (1,1) and (M,N), of the two-dimensional network are not necessarily in the ground state and can be in an arbitrary state $g(b^\dagger_{j,k})|\mathbf{0}\rangle$ so that initialization of the chain is not required [12]. However, for simplicity and without loss of generality, we will assume that the system is in the state $|\mathbf{0}\rangle$.

The Hamiltonian (1) does not include the on-site repulsion $H_U=\sum n_{j,k}(n_{j,k}-1)$. When the repulsion is very strong and the total boson number $\hat{n}$ is much smaller than the number of lattice sites, the system tends to have at most one atom at each site because of the energy gap from H_U. (Entanglement could still possibly lead to errors, however.)

In the case that there is at most one particle in the whole system (the total boson number $\hat{n}$ is zero or one), a function can be transferred perfectly. In this case, an arbitrary state $|\phi\rangle$ of the whole system having one particle can be annihilated by the on-site repulsion Hamiltonian $H_U|\phi\rangle = 0$. Although the linear Hamiltonian H in Eq. (1) does not commute with H_U, the equality $e^{-iHt-iH_Ut}|\phi\rangle = e^{-iHt}|\phi\rangle$ remains valid if and only if $|\phi\rangle$ is a state with the total particle number being zero and one. Therefore, this case is equivalent to the one in Eq. (11) and the transfer will be perfect if we are able to prepare an initial state $|\phi\rangle_{1,1} = \alpha|\mathbf{0}\rangle + \beta b^\dagger_{1,1}|\mathbf{0}\rangle$, where only the first site is occupied. Thus, the state can be perfectly transferred to $|\phi\rangle_{M,N} = \alpha|\mathbf{0}\rangle + \beta b^\dagger_{M,N}|\mathbf{0}\rangle$. In the cases that the total boson number is larger, ref. [24] discussed interesting state transfers by using spinor bosons.

V. PERFECT STATE TRANSFER OF SPIN QUBITS BETWEEN SITES

Although the results in Sec. IV are directly applicable to so-called spinless fermions $c^\dagger_{j,k}$, by replacement $b^\dagger_{j,k} \Leftrightarrow c^\dagger_{j,k}$, they can be generalized to realistic cold fermionic atoms $c^\dagger_{j,k,\sigma}$ (or a spherical tensor $A^\dagger_{l_1m_1,l_2m_2,s\sigma}$) at site (j,k) with a total spin s and components σ. Here, we consider the example of $s = 1/2$. One can engineer a spin-independent Hamiltonian similar to Eq. (1) with quasiangular momenta in two directions,

$$L^{(1)}_x = \sum_{jk\sigma} C^{(1)}_j (c^\dagger_{j,k,\sigma} c_{j+1,k,\sigma} + c^\dagger_{j+1,k,\sigma} c_{j,k,\sigma}),$$

$$L^{(2)}_x = \sum_{ik\sigma} C^{(2)}_k (c^\dagger_{j,k,\sigma} c_{j,k+1,\sigma} + c^\dagger_{j,k+1,\sigma} c_{j,k,\sigma}),$$

where the sum is over up and down spins $\sigma = \downarrow, \uparrow$. The same construction applies to other components of the angular momenta. The most general state that can be prepared at site $(1,1)$ is $|\phi\rangle_{1,1} = (\alpha + \beta c^\dagger_{1,1,\uparrow} + \gamma c^\dagger_{1,1,\downarrow} + \delta c^\dagger_{1,1,\uparrow} c^\dagger_{1,1,\downarrow})|\mathbf{0}\rangle$. This state can be transferred perfectly using $U(t_0)|\phi\rangle_{1,1} = |\phi\rangle_{M,N}$. We can also define the spin operators at a given site in terms of the Pauli matrices $\vec{\sigma} = (X,Y,Z)$, $\vec{S}_{jk} = \sum_{\sigma\sigma'} \langle\sigma'|\frac{\vec{\sigma}}{2}|\sigma\rangle c^\dagger_{j,k,\sigma'} c_{j,k,\sigma}$. The total spin operator is then

$$\vec{S} = \sum_{jk} \vec{S}_{jk}, \tag{12}$$

which commutes with the quasiangular momenta $\vec{L}^{(1)}$ and $\vec{L}^{(2)}$ and can generate an arbitrary gate on $|\phi\rangle_{1,1}$.

The two possible directions of the spin, $\pm 1/2$, can also represent a spin qubit at site (j,k) when the spin is an electron spin in two-dimensional quantum dot [25] or when it is associated with the two states of fermionic atoms in an optical lattice [26,27]. The states of the qubit are defined by $|0\rangle_{j,k} = c^\dagger_{j,k,\uparrow}|\mathbf{0}\rangle$ and $|1\rangle_{j,k} = c^\dagger_{j,k,\downarrow}|\mathbf{0}\rangle$. An arbitrary state $|\psi\rangle_{j,k} = \beta|0\rangle_{j,k} + \gamma|1\rangle_{j,k}$ of this qubit can be perfectly transferred to $|\psi\rangle_{M-j+1,N-k+1}$ by $U(t_0)$. Any generator $\vec{S}$ can be obtained this way so that any rotation to the state can be constructed. [See the analog in Eq. (9).] Also, with the same method an entangled state in several sites can be transferred perfectly. For example, an entangled state $\beta|0\rangle_{1,1}|0\rangle_{1,2} + \gamma|1\rangle_{1,1}|1\rangle_{1,2}$ is transferred to $\beta|0\rangle_{M,N}|0\rangle_{M,N-1} + \gamma|1\rangle_{M,N}|1\rangle_{M,N-1}$ after time t_0. It should be emphasized that since the possibility of perfect state transfer in a spin network of higher dimension remains unclear, the perfect transfer via fermionic networks (or media) is a promising solution when transferring spin qubits in higher dimensions.

VI. GENERALIZATIONS

Here, we discuss several generalizations. First, we show that imperfections in the engineered couplings lead to a systematic error. These are the least difficult errors to treat, thus showing promise for practical utility. We then provide a method for using a dressed-state encoding to avoid particular types of errors. One example of the utility of this is to avoid a type of collective error. Taken together, these techniques could lead to methods of noise prevention applicable to a wide range of transfer conditions. Finally, we discuss the generalization to higher-dimensional lattice systems.

A. Imperfect transfer

Let us return to the linear boson model and consider the experimental implementation of the $J^{(1,2)}_{j,k}$. Imperfections of the engineered couplings, $J^{(1,2)}_{j,k}$, will imply deviations from the ideal values and thus perfect transfers. We provide here the first-order perturbation effect of such an imperfection. The first-order evolution operator is written as

$$U_\Delta(t) = U(t)[1 - iW(t)],$$

where

$$W(t) = \int_0^t ds \sum_{\substack{m_1m_2\\ m'_1m'_2}} \Delta_{m_1m_2m'_1m'_2} A^\dagger_{m_1,m_2}(s) A_{m'_1,m'_2}(s),$$

$A^\dagger_{m_1,m_2}(s) = U^\dagger(s) A^\dagger_{m_1,m_2} U(s)$, and $\Delta_{jk,j'k'}$ is the deviation from the ideal $J^{(1,2)}_{j,k}$, such that

$$\begin{aligned} U^\dagger_\Delta(t_0)\, b^\dagger_{1,1}\, U_\Delta(t_0) &= b^\dagger_{M,N} + i[W(t_0), b^\dagger_{M,N}] \\ &= b^\dagger_{M,N} + i \sum_{m,n} \Delta_{mn\frac{M-1}{2}\frac{N-1}{2}} A^\dagger_{m,n}, \end{aligned}$$

where

$$\begin{aligned} \Delta_{mn\frac{M-1}{2}\frac{N-1}{2}} = \sum \frac{\Delta_{m_1m_2m'_1m'_2}}{J} I_{m_1m_2m'_1m'_2} \\ \times e^{i\frac{\pi}{2}(n+m-m_1-m_2+m'_1+m'_2-\frac{N+M}{2}+1)}. \end{aligned}$$

Since $|d^{l_1}_{nm_1}(\theta)| \leqslant 1$, the integrals $I_{m_1m_2m'_1m'_2} = \int_0^\pi d\theta d^{l_1}_{mm_1}(\theta) d^{l_2}_{nm_2}(\theta) d^{l_1}_{\frac{M-1}{2}m'_1}(\theta) d^{l_2}_{\frac{N-1}{2}m'_2}(\theta)$ are less than π. If we consider the nearest-neighbor $\Delta_{m_1m_2m'_1m'_2}$ to be a constant Δ, it is easy to provide an upper bound $\Delta_{mn\frac{M-1}{2}\frac{N-1}{2}}$, which is $2\pi L\Delta/J$ for one-dimensional and $\pi(2L+1)^2\Delta/J$ for two-dimensional lattices. This implies that while one should engineer the system so that Δ/J is as small as possible, the reliability of the function transfer depends on the size of the system and is better for smaller systems. The transfer fidelity for one particle at (1,1) is $1 - i\Delta_{\frac{M-1}{2}\frac{N-1}{2}\frac{M-1}{2}\frac{N-1}{2}}$. Although the imperfection has an effect, it causes only a systematic error that can be corrected by using fairly standard methods. For example, the

above state $|\phi\rangle_{1,1}$ can be transferred imperfectly into $|\phi\rangle_{M,N} = \alpha|\mathbf{0}\rangle + \beta' b^\dagger_{M,N}|\mathbf{0}\rangle$, where $\beta' = \beta - i\Delta_{\frac{M-1}{2}\frac{N-1}{2}\frac{M-1}{2}\frac{N-1}{2}}$. If we measure the particle number at site (M,N) for this state, after many measurements, we find α and β'. Since the error $\Delta_{\frac{M-1}{2}\frac{N-1}{2}\frac{M-1}{2}\frac{N-1}{2}}$ is an intrinsic property of this network, it does not change for each measurement and could be determined. Therefore, we can recover β from $\beta = \beta' + i\Delta_{\frac{M-1}{2}\frac{N-1}{2}\frac{M-1}{2}\frac{N-1}{2}}$.

B. Dressed states for error protection

An arbitrary time-independent unitary transformation W does not change the commutation relations among angular momentum components if it corresponds to a similarity transformation in the Heisenberg picture. Indeed, the whole family of Hamiltonians generated by an arbitrary W can transfer functions perfectly. However, this map can introduce new effects. For a spin system, this transformation W corresponds to the so-called dressed qubit [28]. The bosonic Hilbert space is infinite dimensional and thus allows more flexibility for transformations, including continuous-variable transformations (which are not possible in spin chains).

As a simple example, let us first consider transformations $W = \exp[-i\theta(L_z^{(1)} + L_z^{(2)})]$. Under the dressing transformation, the Hamiltonian $H_l = J(L_x^{(1)} + L_x^{(2)})$ becomes

$$H' = WHW^\dagger = \cos\theta H + \sin\theta J\left(L_y^{(1)} + L_y^{(2)}\right).$$

The *dressed state* $|\theta\rangle = W|\mathbf{0}\rangle$ allows perfect transfer of any function f using $U(t_0)$. The function is transferred via the coherent state $|\theta\rangle$ as

$$f(e^{-i(M+N+1)\theta} b^\dagger_{1,1})|\theta\rangle \rightarrow f(e^{-i(M+N+1)\theta} b^\dagger_{M,N})|\theta\rangle. \quad (13)$$

Another example is a dressing transformation that entangles an individual site (j,k) and a collective bath, $W_{jk} = \exp[\lambda(b_{jk} - b^\dagger_{jk})B]$ and $W = \prod W_{jk}$, where B is an operator of the collective bath and λ is a small constant. Consider the effect of this transformation to first order in λ. The dressed Hamiltonian is

$$H' = W(H + H_B)W^\dagger = H + H_B + \lambda V,$$

where $V = \sum_{jk}(b^\dagger_{jk}B_{jk} + b_{jk}B^\dagger_{jk})$, H_B is the bath Hamiltonian, and the operators $B_{jk} = (J_j + J_{j-1} + J_k + J_{k-1})B + [H_B, B]$. This introduces a weak system-bath interaction with a specific V via the dressing transformation. Clearly, it is possible to introduce different types of interactions using different types of dressing transformations. The perfect state transfer from site $(1,1)$ to (N,M) is described by the initial density matrix at $(1,1)$,

$$(W_{11}|\phi\rangle_{1,1}\langle\phi|W^\dagger_{11}) \otimes \rho_B \rightarrow (W_{NM}|\phi\rangle_{N,M}\langle\phi|W^\dagger_{NM}) \otimes (e^{-iH_Bt_0}\rho_B e^{iH_Bt_0}), \quad (14)$$

where ρ_B is a thermal equilibrium state of the bath. The on-site repulsion is also subject to the dressing transformation in the way that $H' = WH_UW^\dagger$. While it introduces an imperfection for the general cases, Eq. (14) remains the same if the initial state is $W|\phi\rangle_{1,1} = \alpha W|\mathbf{0}\rangle + \beta W b^\dagger_{M,N}|\mathbf{0}\rangle$, where the total particle number in state $|\phi\rangle_{1,1}$ is not larger than 1. The invariance to this collective error is similar to that of decoherence-free subspaces (DFSs) in quantum computation; the state can avoid a family of collective errors. For a discussion of general error correction methods, see Ref. [29].

C. Perfect state transfer in three-dimensional cube

In this case, a particle $b^\dagger_{j,k,n}$ (or fermion $c^\dagger_{j,k,n,\sigma}$) at site (j,k,n) is described by a spherical tensor $A^\dagger_{l_1m_1,l_2m_2,l_3m_3}$. While l_1 and l_2 are the same as in the two-dimensional case, the third direction is characterized by $l_3 = \frac{K-1}{2}$ and $m_3 = -\frac{K+1}{2} + n$. The quasiangular momentum $\vec{L}^{(3)}$ in the third direction is defined analogous to those in (6), for instance, $L_x^{(3)} = \sum_{jk} C_j^{(1)}(b^\dagger_{j,k,n}b_{j,k,n+1} + b^\dagger_{j,k,n+1}b_{j,k,})$. The other components are written by replacing $b^\dagger_{j,k,n} \rightarrow b^\dagger_{j,k}$. The evolution operator is

$$U(t) = \exp\left[-i\left(J^{(1)}L_x^{(1)} + J^{(2)}L_x^{(2)} + J^{(3)}L_x^{(3)}\right)t\right].$$

When $J^{(1)} = J^{(2)} = J^{(3)} = J$ and $t_0 = \pi/J$, this expression reduces to the simple form

$$U^\dagger(t_0)\, b^\dagger_{j,k,n}\, U(t_0) = r_1r_2r_3 b^\dagger_{M-j+1,N-k+1}, \quad (15)$$

where $r_3 = \exp(-i\pi\frac{K-1}{2})$. All results in two dimensions are directly applicable to the three-dimensional, or even higher-dimensional problems.

VII. CONCLUSIONS

We have shown that arbitrary functions can be sent perfectly (without state initialization or remote collaboration) through engineered interacting bosonic and fermionic squares or cubes. As an example, we have analyzed the transfer using ultracold bosonic atoms in optical lattices, described by the Bose-Hubbard model with properly designed site-dependent tunneling amplitudes. In a more general case, we have studied a family of linear and nonlinear Hamiltonians that enable perfect state transfers according to dressing transformations, where noisy factors are considered for various types of transformations. An important consequence of this is that information encoded into spin qubits can be perfectly transferred using fermionic lattices.

The ability to send an encoded state perfectly between points, even in principle, is an important discovery. However, even if imperfections are present, it has been shown that perfect function transfer remains possible. These depend on the encoding used. We have introduced dressing transformations as a useful tool for encoding information in such a way as to cancel errors arising from a bath. This implies, for example, that physically reasonable global transformations could be used to cancel some errors. Therefore, we anticipate future work will uncover more encoded states that transfer well, for example, one may consider DFSs [30–35]. (For reviews, see [36,37].) These might be also good candidates for future investigation because of their ability to enable encoded universality, whether or not they protect against errors. This

is motivated by the results of [7,38], where it was shown that a singlet state on spins 1 and 3, and a generalization of this result, transfer quite well through an unmodulated spin chain without engineered couplings.

Taken together, this work provides methods for robust transfer of an arbitrary function through two- and three-dimensional chains that was, until now, not known to exist, with the exception of Ref. [24]. Furthermore, we have given a Hamiltonian applicable to a variety of physical systems, leading immediately to experimental proposals for testing perfect state transfer.

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