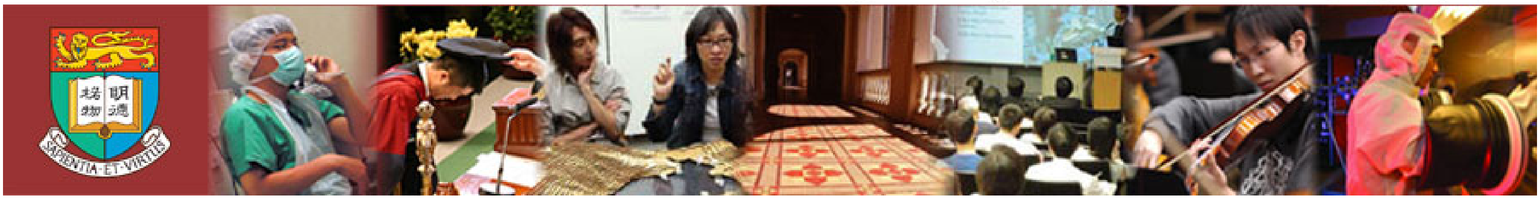

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Author(s)	Chen, MS; Sha, WEI; Sun, YX; Huang, ZX; Wu, XL
Citation	The 2nd International Conference on Education Technology and Computer (ICETC 2010), Shanghai, China, 22-24 June 2010. In Proceedings of 2nd ICETC, 2010, v. 5, p. 126-128
Issued Date	2010
URL	http://hdl.handle.net/10722/137735
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Adaptive Frequency sweep analysis for electromagnetic problems using the Thiele interpolating continued fractions

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***Abstract*—A direct rational approximation method based on Thiele interpolating continued fractions theory is proposed for fast frequency sweep analysis of electromagnetic problems. And an adaptive algorithm is also formed. Compared with the conventional rational approximation method, the proposed method can get a rational approximation directly without a great number of matrix inverse computations and doesn't need to allocate much memory for high derivatives of the dense impedance matrix. Meanwhile, the computation of surface currents by continued fractions can be sped up as compared with the traditional rational approximation. Numerical simulations for broad band scattering analysis of different shaped objects are discussed to shown the effectiveness of the present method.**

***Index Terms* — Computational Electromagnetics, Thiele interpolating continued fractions, method of moments, fast frequency sweep analysis.**

I. INTRODUCTION

The solution of electromagnetic scattering problems by different shaped objects is one of the most challenging problems in modern computational techniques [1][2], which have attracted a great amount of attention over the past few decades. As an efficient tool, method of moments (MOM) is not only a suitable but also an accurate and standard method which has been widely used for solution the EM integral equations. For many practical applications [3], there is need for solution of EM problems over a broad frequency band [4~6]. However, as a frequency-domain method, MOM need to compute the problems at different frequency points one by one, especially for highly frequency dependent problems, which is very time consuming.

To overcome this problem, a lot of techniques have been developed. Take the model-based parameter estimation (MBPE) [7] and its special case asymptotic waveform evaluations (AWE) [8][9] for example, which are the most popular frequency sweep analysis tools for frequency domain methods. In the AWE technique, a Taylor series expansion is generated at a given frequency points to approximate the equivalent surface current, and the rational function approach is applied to improve the accuracy. As compared with using MOM at each of the frequency points, the AWE method is found to be superior in terms of the CPU time to obtain frequency response. However, the memory needed is greatly increased on account of the high derivatives of the dense impedance matrix. In MBPE method, data are extracted from more than one expansion point, and finally a rational function is also obtained.

In this paper, a new approach based on Thiele interpolating continued fractions is proposed. As compared with the MBPE and AWE method, the presented technique has the following strong points: firstly, it doesn't need to allocate much memory for high derivatives of the dense impedance matrix and can get a rational approximation directly without a great number of matrix inverse computations. Secondly, the computation of surface currents by continued fractions can be sped up as compared with the traditional rational approximation. Finally, an adaptive method is also formed for scattering analysis of a given frequency band and numerical simulation for different shaped objects is considered.

II. THEORY AND FORMULATIONS

Consider an arbitrarily shaped three-dimensional (3D) conducting object illuminated by an incident field ($\mathbf{E}^{inc}, \mathbf{H}^{inc}$). The electric field integral equation (EFIE) and magnetic field integral equation (MFIE) are given by

$$\hat{\mathbf{n}} \times \mathbf{L}(\mathbf{J}) = \hat{\mathbf{n}} \times \mathbf{E}^{inc}(\mathbf{r}) \qquad \mathbf{r} \in S \qquad (1)$$

$$\frac{1}{2}\mathbf{J}(\mathbf{r}) + \hat{\mathbf{n}} \times \mathbf{K}(\mathbf{J}) = \hat{\mathbf{n}} \times \mathbf{H}^{inc}(\mathbf{r}) \qquad \mathbf{r} \in S \qquad (2)$$

where $\mathbf{J}(\mathbf{r})$ denotes the unknown surface current density and the integral operators $\mathbf{L}$ and $\mathbf{K}$ are defined by

$$\mathbf{L}(\mathbf{J}) = jk_0\eta_0 \iint_S \left(\mathbf{J}(\mathbf{r}')g(\mathbf{r},\mathbf{r}') + \frac{1}{k_0^2}\nabla'\bullet\mathbf{J}(\mathbf{r}')\nabla g(\mathbf{r},\mathbf{r}') \right) dS' \quad (3)$$

$$\mathbf{K}(\mathbf{J}) = \iint_S \mathbf{J}(\mathbf{r}') \times \nabla g(\mathbf{r},\mathbf{r}') dS' \qquad (4)$$

in which S denotes the surface of the object, k_0 is the free-space wavenumber, η_0 is the free-space wave impedance, $\hat{\mathbf{n}}$ is an outwardly directed normal, and $g(\mathbf{r},\mathbf{r}')$ is well-known free-space Green's function given by

$$g(\mathbf{r},\mathbf{r}') = \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \quad (5)$$

Either EFIE or MFIE can be used to solve for $\mathbf{J}(\mathbf{r})$. However, for a given closed S, $\mathbf{L}$ can be singular at certain frequencies when the exterior medium is lossless. Consequently, (1) may give an erroneous solution at these frequencies. A similar problem occurs in MFIE as well. To eliminate this problem, one can combine EFIE and MFIE to find

$$\alpha\hat{\mathbf{n}}\times\hat{\mathbf{n}}\times\mathbf{L}(\mathbf{J}) + (1-\alpha)\eta_0\left[\frac{1}{2}\mathbf{J}(\mathbf{r}) + \hat{\mathbf{n}}\times\mathbf{K}(\mathbf{J})\right] = \alpha\hat{\mathbf{n}}\times\hat{\mathbf{n}}\times\mathbf{E}^{inc}(\mathbf{r}) + (1-\alpha)\eta_0\hat{\mathbf{n}}\times\mathbf{H}^{inc}(\mathbf{r}) \qquad \mathbf{r}\in S \quad (6)$$

which is known as the combined field integral equation (CFIE). The combination parameter α is usually chosen between 0.2 and 0.8.

By MOM solution with Rao-Wilton- Glisson (RWG) basis functions [10], the integral equation will result in a matrix equation of the form

$$\mathbf{Z}(k)\mathbf{I}(k) = \mathbf{V}(k) \quad (7)$$

To get the solution of above equation over broad frequency band, the Thiele interpolating continued fractions of $\mathbf{I}(k)$ is presented of the following form

$$\mathbf{I}(k) \approx \mathbf{R}_n(k) = a_0 + \frac{k-k_0|}{|a_1} + \frac{k-k_1|}{|a_2} + \cdots + \frac{k-k_{n-1}|}{|a_n} \quad (8)$$

in which $a_i = \varphi[k_0,k_1,\cdots,k_i]$ for $i = 0,1,\cdots,n$. $\varphi[k_0,k_1,\cdots,k_i]$ are the inverse differences of $\mathbf{I}(k)$ at $k_0,k_1,\cdots k_n$, which can be computed recursively as below:

$$\varphi[k_i] = \mathbf{I}[k_i],\ i = 0,1,\cdots,n \quad (9)$$

$$\varphi[k_i,k_j] = \frac{k_j - k_i}{\mathbf{I}[k_j] - \mathbf{I}[k_i]} \quad (10)$$

$$\varphi[k_i,k_j,k_l] = \frac{k_l - k_j}{\varphi[k_i,k_l] - \varphi[k_j,k_l]} \quad (11)$$

$$\varphi[k_i,\cdots,k_j,k_l,k_m] = \frac{k_m - k_l}{\varphi[k_i,\cdots,k_j,k_m] - \varphi[k_i,\cdots,k_j,k_l]} \quad (12)$$

It is easy to verify $R_n(k)$ is a rational function of type $([(n+1)/2]/[n/2])$, which satisfy $\mathbf{R}_n(k_i) = \mathbf{I}(k_i)$, where $[x]$ denotes the greatest integer not exceeding x.

From the description above, one can conclude that the proposed method can form a direct rational approximation and doesn't need to compute and store the high order derivatives of the dense impedance matrix. Meanwhile, since the characteristic of the continued fractions, the computation of surface currents by proposed method can be sped up greatly as compared with the traditional rational approximation.

To get a practical frequency sweep analysis method for electromagnetic engineering, we want to form an adaptive frequency sweep analysis algorithm based on the relative residual

$$err(k) = \frac{\|\mathbf{Z}(k)\mathbf{R}_n(k) - \mathbf{V}(k)\|}{\|\mathbf{V}(k)\|} \quad (13)$$

Assume a frequency sweep method is desired to solve (7) at frequency points $f_u = k_u v/2\pi$ for $u = 1,2,\cdots,N$.

Based on a given order, one will get a rational approximation function by (8). Substituting this function into (11) if $err(k_u)$ is small than some tolerance value ξ for $u = 1,2,\cdots,N$, the frequency sweep is completed. Otherwise the inverse differences of $\mathbf{I}(k)$ will be computed at k_{n+1} to increase the order, while k_{n+1} is chosen to make the (13) get the largest value.

III. NUMERICAL RESULTS

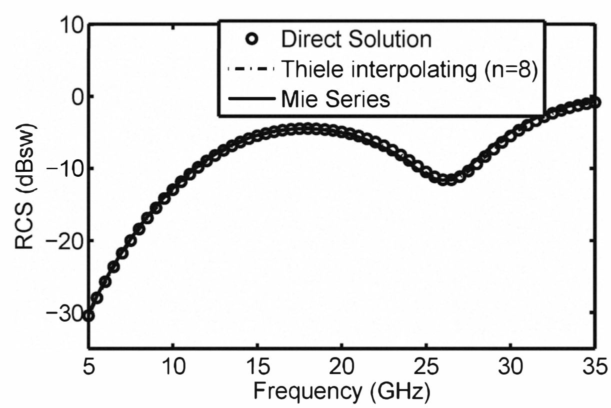

Fig.1 the RCS frequency response of a PEC sphere with radius of 0.318cm

To validate the effectiveness of the proposed method, we firstly consider a PEC sphere with its radius of 0.318cm, which is illuminated by a plane wave propagating in the z direction and E-polarized in the x direction. As shown in figure 1, the frequency band is chosen to be 5GHz~35GHz, the total number of RWG functions is 1470. The order of the Thiele interpolating continued fractions is chosen to be 8, and the interpolating frequency points are chosen based on equidistance node .The simulation result is compared with that of the analytical solution method by Mie series and direct solution of EFIE by method of moments. With the step of 0.5 GHz, it takes the direct method 1739 seconds to obtain the solution over the frequency band, while the Thiele interpolating continued fractions method consumed 214 seconds.

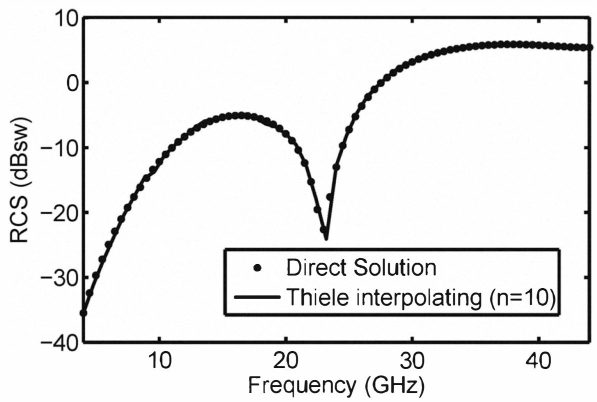

Fig.2 the RCS frequency response of a PEC cube

As a second example, the scattering analysis of a PEC cube with side-length of 0.5cm is considered over the frequency band 4GHz-44GHz.The surface of the object is discretized into 768 triangular elements resulting into 1152 unknowns. With the step of 0.5 GHz, it takes the direct method 2048 seconds to obtain the solution of EFIE over the frequency band, while the Thiele interpolating continued fractions method with its order set to be 10 consumed 275 seconds.

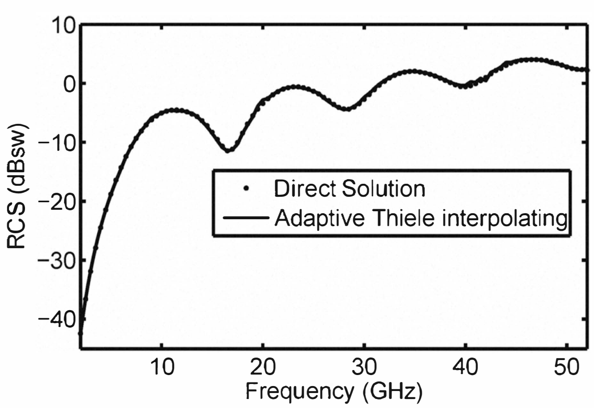

Fig.3 the RCS frequency response of a PEC sphere with radius of0.5cm computed by adaptive Thiele interpolating

Finally, to show the efficiency of the proposed adaptive fast frequency sweep method, the scattering analysis of a PEC sphere with its diameter of 1cm over the frequency band 2GHz~52GHz. The primary order is chosen to be 8 to solve CFIE, and the final order is 15 and the final interpolating frequency points are {2GHz 4.8GHz 8.25 GHz 10.04 GHz 14.5 GHz 16.6 GHz 20.75 GHz 23 GHz 24.3GHz 27 GHz 33.25 GHz 37.1 GHz 39.5 GHz 45.75 GHz 47.8 GHz 52 GHz }. The total CPU time for direct solution method with a step of 0.5GHz is 1442 seconds, while the adaptive Thiele interpolating method consumed only 221 seconds.

IV. CONCLUSION

A new method based on Thiele interpolating continued fractions to accomplish fast frequency sweep analysis is proposed, without computing and storing the high order derivatives, which can get rational approximation of the surface currents directly. Meanwhile, an adaptive frequency sweep algorithm is formed based on the relative residual method.

ACKNOWLEDGMENT

This work is supported by Anhui Provincial Natural Science Foundation under Grant No 090412047 and Science and Technology Planning Project of Anhui Province under Grant No 09020204029, and Science and Technological Fund of Anhui Province for Outstanding Youth, and partially by the Key Program of National Natural Science Foundation of China under Grant No 60931002.

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