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Adaptive rendezvous of multiple mobile agents with nonlinear dynamics and preserved network connectivity

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Abstract—This paper investigates rendezvous of multiple nonlinear dynamical mobile agents with a virtual leader in a dynamic proximity network. It is assumed that only a fraction of agents in the group have access to the information on the position and velocity of the virtual leader. To avoid fragmentation, a bounded connectivity-preserving rendezvous algorithm is proposed for the multi-agent systems. Under the assumption that the initial network is connected, local adaptation strategies for the rendezvous algorithm are introduced that enable all agents to synchronize with the virtual leader even when only one agent is informed, without requiring any knowledge of the agent dynamics. Simulation results on an example are given to numerically verify the theoretical results.

Keywords: Distributed control, rendezvous, multi-agent system, network connectivity.

I. INTRODUCTION

In the past three decades, coordinated control for multi-agent systems has been a very important topic in diverse fields such as biology, physics, computer science and control engineering [1], [2], [3], [4], [5], [6], [7], [8], [9]. Many different coordinated control protocols such as consensus [10], [11], [12], [13], flocking [14], [15], [16], [17], [18], [19], [20], swarm [21], and rendezvous [22] are proposed from the field of control engineering, and have the properties of distributed control, local interactions and self-organization [23].

Most of the existing works for coordinated control of multi-agent systems are concerned with agents described by single-integrator or double-integrator dynamics [23]. However, the agents may be governed by nonlinear intrinsic dynamics. In fact, nonlinear dynamics are commonly considered in synchronization of complex dynamical networks [24], [25], [26]. Second-order consensus-type problems with nonlinear agent dynamics are investigated in the fixed networks [27], [28], [29] and switching networks [30].

In this paper, we investigate the rendezvous of multiple mobile agents with nonlinear dynamics, which aims to guide all agents to move with the same velocity and converge to the same position. In order to avoid fragmentation, we construct a bounded potential function to guarantee the connectivity of underlying networks. This paper extends the algorithm

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in [22] to the case of nonlinear agent dynamics while extending that in [30] to bounded potential function. Under the assumption that the initial network is connected, we introduce local adaptation strategies for both the weights on the velocity navigational feedback and the coupling strengths that enable all agents in the group to synchronize with the virtual leader.

The remainder of the paper is organized as follows. Section II describes the rendezvous problem to be solved in this paper. Section III establishes new results on the rendezvous problem. Section IV presents one simulation example. Section V draws conclusions to the paper.

II. PROBLEM STATEMENT

Consider N agents, labeled as $1, 2, \dots, N$, moving in an n -dimensional Euclidean space. The motion of each agent is governed by

$$\begin{aligned} \dot{q}_i &= p_i, \\ \dot{p}_i &= f(p_i) + u_i, \end{aligned} \quad (1)$$

where $q_i \in \mathbf{R}^n$ is the position vector of agent i , $p_i \in \mathbf{R}^n$ is its velocity vector, $f(p_i) \in \mathbf{R}^n$ is its intrinsic dynamics, and $u_i \in \mathbf{R}^n$ is its control input.

The problem of rendezvous with a virtual leader is to design the control input u_i , $i = 1, 2, \dots, N$, such that

$$\begin{aligned} \lim_{t \rightarrow \infty} \|q_i(t) - q_\gamma(t)\| &= 0, \\ \lim_{t \rightarrow \infty} \|p_i(t) - p_\gamma(t)\| &= 0, \end{aligned}$$

for all $i = 1, 2, \dots, N$, where q_γ and p_γ are respectively the position and velocity of the virtual leader, which is specified by

$$\begin{aligned} \dot{q}_\gamma &= p_\gamma, \\ \dot{p}_\gamma &= f(p_\gamma). \end{aligned} \quad (2)$$

Assumption 1: The vector field $f : \mathbf{R}^n \rightarrow \mathbf{R}^n$ satisfies

$$(x - y)^T [f(x) - f(y)] \leq (x - y)^T \Delta (x - y), \quad \forall x, y \in \mathbf{R}^n,$$

for some positive definite diagonal constant matrix Δ .

Assumption 1 is a Lipschitz-like condition, which is satisfied by many well known systems such as the chaotic Chua circuit [26], [31].

In this paper, we assume that only a fraction of agents are informed about the virtual leader. Since some agents do not have any information on the virtual leader, certain connectivity conditions are required to guarantee the convergence of the coordinated motion. It is commonly assumed that the underlying topology of the network can maintain its connectivity frequently enough throughout the motion process. However, for a given set of initial conditions,

this assumption is very difficult to satisfy and verify. In particular, the connectivity of the initial network generally cannot guarantee the connectivity of the network throughout a long-term dynamic evolution of a group of agents.

Suppose that all agents have the same influencing/sensing radius r . Let $\varepsilon \in (0, r]$ be the given constants. Call $G(t) = (V, E(t))$ an undirected dynamic graph consisting of a set of vertices $V = \{1, 2, \dots, N\}$ indexed by the set of agents and a time-varying set of links $E(t) = \{(i, j) | i, j \in V\}$ such that

- i) initial links are generated by $E(0) = \{(i, j) | \|q_i(0) - q_j(0)\| < r - \varepsilon, i, j \in V\}$;
- ii) if $(i, j) \notin E(t^-)$ and $\|q_i(t) - q_j(t)\| < r - \varepsilon$, then (i, j) is a new link being added to $E(t)$;
- iii) if $\|q_i(t) - q_j(t)\| \geq r$, then $(i, j) \notin E(t)$.

We will use a symmetric indicator function $\sigma(i, j) \in \{0, 1\}$ to describe whether there is an edge between agent i and agent j at time t , which is defined as

$$\sigma(i, j)[t] = \begin{cases} 0, & \text{if } ((\sigma(i, j)[t^-] = 0) \cap (\|q_i(t) - q_j(t)\| \geq r - \varepsilon)) \\ & \cup ((\sigma(i, j)[t^-] = 1) \cap (\|q_i(t) - q_j(t)\| \geq r)), \\ 1, & \text{if } ((\sigma(i, j)[t^-] = 1) \cap (\|q_i(t) - q_j(t)\| < r)) \\ & \cup ((\sigma(i, j)[t^-] = 0) \cap (\|q_i(t) - q_j(t)\| < r - \varepsilon)). \end{cases}$$

It is clear that there is a hysteresis in the indicator function for adding new edges to the graph, which means that a new edge will not be added to the graph until the distance between any two unconnected agents decreases to $r - \varepsilon$. This property is crucial in proving the convergence of the algorithm.

In this paper, the control law u_i for agent i takes the form of

$$u_i = \alpha_i + \beta_i + \gamma_i, \quad i = 1, 2, \dots, N, \quad (3)$$

where α_i is the gradient-based term that enforces the position of each agent to converge to a common value, β_i is the consensus term that regulates the velocity of each agent to a common value, and γ_i is the navigational feedback term that drives agent i to track the virtual leader.

III. MAIN RESULTS

A. Description of the Algorithm

The control input (3) is specified as

$$\begin{aligned} u_i &= - \underbrace{\sum_{j \in \mathcal{N}_i(t)} \nabla_{q_i} \psi(\|q_{ij}\|)}_{\alpha_i} - \underbrace{\sum_{j \in \mathcal{N}_i(t)} m_{ij}(p_i - p_j)}_{\beta_i} \\ &\quad - \underbrace{h_i c_1 (q_i - q_\gamma) - h_i c_{2i} (p_i - p_\gamma)}_{\gamma_i}, \\ \dot{m}_{ij} &= k_{ij} (p_i - p_j)^\top (p_i - p_j), \\ \dot{c}_{2i} &= k_i (p_i - p_\gamma)^\top (p_i - p_\gamma), \end{aligned} \quad (4)$$

where $q_{ij} = q_i - q_j$ and $\mathcal{N}_i(t)$ is the neighborhood of agent i at time t , defined as

$$\mathcal{N}_i(t) = \{j : \sigma(i, j)[t] = 1, j \neq i, j = 1, 2, \dots, N\}.$$

For notational convenience, denote

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N \end{bmatrix}, \quad p = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix}, \quad c_2 = \begin{bmatrix} c_{21} \\ c_{22} \\ \vdots \\ c_{2N} \end{bmatrix}$$

and $m = (m_{ij})_{N \times N}$.

Define an energy function for the multi-agent system as

$$\begin{aligned} Q(q, p, q_\gamma, p_\gamma, m, c_2) &= \frac{1}{2} \sum_{i=1}^N U_i(q, q_\gamma) \\ &+ \frac{1}{2} \sum_{i=1}^N (p_i - p_\gamma)^\top (p_i - p_\gamma) \\ &+ \frac{1}{2} \sum_{i=1}^N \left[\sum_{j \in \mathcal{N}_i(t)} \frac{(m_{ij} - \theta)^2}{2k_{ij}} + h_i \frac{(c_{2i} - \theta)^2}{k_i} \right], \end{aligned} \quad (5)$$

where

$$U_i(q, q_\gamma) = \sum_{j \in \mathcal{N}_i(t)} \psi(\|q_i - q_j\|) + h_i c_1 (q_i - q_\gamma)^\top (q_i - q_\gamma), \quad (6)$$

and θ is a positive constant chosen such that

$$\theta \geq \frac{\lambda_{\max}(\Delta)}{\lambda_1(L(0) + H)}.$$

Clearly, Q is a positive semi-definite function.

The nonnegative potential $\psi(\|q_{ij}\|)$ is defined to be a function of the distance $\|q_{ij}\|$ between agent i and agent j , differentiable with respect to $\|q_{ij}\| \in [0, r]$ such that

- i) $\frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} > 0$ for $\|q_{ij}\| \in (0, r)$;
- ii) $\lim_{\|q_{ij}\| \rightarrow 0} \left(\frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} \cdot \frac{1}{\|q_{ij}\|} \right)$ is nonnegative and bounded;
- iii) $\psi(r) = \hat{Q} \in [Q_{\max}, +\infty)$, where $Q_{\max} \triangleq \frac{1}{2} \sum_{i=1}^N (p_i(0) - p_\gamma(0))^\top (p_i(0) - p_\gamma(0)) + \frac{1}{2} \sum_{i=1}^N \left[\sum_{j \in \mathcal{N}_i(t)} \frac{(m_{ij}(0) - \theta)^2}{2k_{ij}} + h_i \frac{(c_{2i}(0) - \theta)^2}{k_i} \right] + \frac{N(N-1)}{2} \psi(\|r - \varepsilon\|)$.

Condition i) illustrates that the potential between the two agents is an increasing function of their distance, which makes two agents attract each other; Condition ii) requires that the magnitude of gradient between two agents is the same or a higher-order term of their distance when the two agents converge to the same position; Condition iii) states that the potential between two agents will be sufficiently large when the distance between the two agents reaches the sensing radius, which guarantees all existing edges not to be lost. One example of such a potential function is the following:

$$\psi(\|q_{ij}\|) = \frac{\|q_{ij}\|^2}{r - \|q_{ij}\| + \frac{r^2}{Q}}, \quad \|q_{ij}\| \in [0, r]. \quad (7)$$

Here, the positive constant c_1 is the weight on the position navigational feedback and can take any fixed value, the

adaptive parameters m_{ij} and c_{2i} represent the velocity coupling strengths and the weights on the velocity navigational feedbacks, respectively, and the positive constants $k_{ij} = k_{ji}$ and k_i are the corresponding weighting factors of their adaptation laws. If agent i is an informed agent, then $h_i > 0$; otherwise, $h_i = 0$. Without loss of generality, assume that the first M agents are informed, that is, $h_i > 0$ for $i = 1, 2, \dots, M$, and $h_i = 0$ for $i = M + 1, M + 2, \dots, N$. The adaptive strategy proposed here differs from those of [26] in that it is decentralized and does not require any knowledge of the nonlinear agent dynamics.

The adjacency matrix $A(t) = (a_{ij}(t))$ of system (1) on the above graph $G(t)$ is defined as

$$a_{ij}(t) = \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E}(t), \\ 0, & \text{otherwise.} \end{cases}$$

The corresponding Laplacian is $L(t) = D(A(t)) - A(t)$, where the degree matrix $D(A(t))$ is a diagonal matrix with the i th diagonal element equal to $\sum_{j=1, j \neq i}^N a_{ij}(t)$. Denote the minimal eigenvalue of a symmetric matrix as $\lambda_1(\cdot)$ and the eigenvalues of $L(t)$ as $\lambda_1(L(t)) \leq \dots \leq \lambda_N(L(t))$. Then, $\lambda_1(L(t)) = 0$ and $\mathbf{1} = [1, 1, \dots, 1]^T \in \mathbf{R}^N$ is its corresponding eigenvector. Moreover, if $G(t)$ is a connected graph, then $\lambda_2(L(t)) > 0$ [32]. The corresponding n -dimensional graph Laplacian is defined as $\widehat{L}(t) = L(t) \otimes I_n$, where I_n is the identity matrix of order n and \otimes stands for the Kronecker product.

Lemma 1: [13] If G is a connected undirected graph, L is the symmetric Laplacian of the graph G and the matrix $E = \text{diag}(e_1, e_2, \dots, e_N)$ with $e_i \geq 0$ for $i = 1, 2, \dots, N$, and at least one element in E is positive, then all eigenvalues of the matrix $L + E$ are positive.

Lemma 2: [33] Suppose that the eigenvalues of symmetric matrices $A, B \in \mathbf{R}^{N \times N}$ satisfy

$$\lambda_1(A) \leq \lambda_2(A) \leq \dots \leq \lambda_N(A),$$

and

$$\lambda_1(B) \leq \lambda_2(B) \leq \dots \leq \lambda_N(B).$$

Then, the following inequalities hold:

$$\lambda_{i+j-1}(A+B) \geq \lambda_i(A) + \lambda_j(B), \\ i+j \leq N+1, 1 \leq i, j \leq N.$$

Lemma 3: [30] If G_1 is a connected undirected graph and G_2 is a graph generated by adding some edge(s) into the graph G_1 , then $\lambda_1(L_2 + E) \geq \lambda_1(L_1 + E) > 0$, where L_1 and L_2 are the symmetric Laplacians of graphs G_1 and G_2 , respectively.

B. Theoretical Analysis

Theorem 1: Consider a system of N mobile agents with dynamics (1), each steered by protocol (4), and a virtual leader with dynamics (2). Suppose that the initial network $G(0)$ is connected, Assumption 1 holds, and the initial energy $Q_0 := Q(q(0), p(0), q_\gamma(0), p_\gamma(0), m(0), c_2(0))$ is finite. Then, the following hold:

- i) $G(t)$ will remain to be connected for all $t \geq 0$;

- ii) the velocities and positions of all agents will converge to those of the virtual leader asymptotically.

Proof. Denote the position difference and the velocity difference between agent i and the virtual leader as $q_{i\gamma} = q_i - q_\gamma$ and $p_{i\gamma} = p_i - p_\gamma$, respectively. Then, one has

$$\begin{aligned} \dot{q}_{i\gamma} &= p_{i\gamma}, \\ \dot{p}_{i\gamma} &= f(p_i) - f(p_\gamma) - \sum_{j \in \mathcal{N}_i(t)} \nabla_{q_{i\gamma}} \psi(\|q_{i\gamma} - q_{j\gamma}\|) \\ &\quad - \sum_{j \in \mathcal{N}_i(t)} m_{ij} (p_{i\gamma} - p_{j\gamma}) - h_i c_{1\gamma} q_{i\gamma} - h_i c_{2i} p_{i\gamma}, \\ \dot{m}_{ij} &= k_{ij} (p_{i\gamma} - p_{j\gamma})^\top (p_{i\gamma} - p_{j\gamma}), \\ \dot{c}_{2i} &= k_i p_{i\gamma}^\top p_{i\gamma}. \end{aligned}$$

The energy function (5) can be rewritten as

$$\begin{aligned} Q(\tilde{q}, \tilde{p}, m, c_2) &= \frac{1}{2} \sum_{i=1}^N (U_i(\tilde{q}) + p_{i\gamma}^\top p_{i\gamma}) \\ &\quad + \frac{1}{2} \sum_{i=1}^N \left(\sum_{j \in \mathcal{N}_i(t)} \frac{(m_{ij} - \theta)^2}{2k_{ij}} + h_i \frac{(c_{2i} - \theta)^2}{k_i} \right), \end{aligned} \quad (8)$$

where

$$U_i(\tilde{q}) = \sum_{j \in \mathcal{N}_i(t)} \psi(\|q_{i\gamma} - q_{j\gamma}\|) + h_i c_{1\gamma} q_{i\gamma}^\top q_{i\gamma}, \quad (9)$$

and

$$\tilde{q} = \begin{bmatrix} q_{1\gamma} \\ q_{2\gamma} \\ \vdots \\ q_{N\gamma} \end{bmatrix}, \quad \tilde{p} = \begin{bmatrix} p_{1\gamma} \\ p_{2\gamma} \\ \vdots \\ p_{N\gamma} \end{bmatrix}.$$

Clearly, $Q(\tilde{q}, \tilde{p}, m, c_2)$ is a positive semi-definite function of $(\tilde{q}, \tilde{p}, m, c_2)$.

Assume that $G(t)$ switches at time t_k , $k = 1, 2, \dots$, and $G(t)$ is a fixed graph on each time-interval $[t_{k-1}, t_k)$. Note that Q_0 is finite and the time derivative of $Q(t)$ on $[t_0, t_1)$ satisfies

$$\begin{aligned} \dot{Q} &= \sum_{i=1}^N p_{i\gamma}^\top \left[f(p_i) - f(p_\gamma) \right] \\ &\quad - \sum_{i=1}^N p_{i\gamma}^\top \left[\sum_{j \in \mathcal{N}_i(t)} \theta (p_{i\gamma} - p_{j\gamma}) + h_i \theta p_{i\gamma} \right] \\ &\leq -\tilde{p}^\top [m(L(t) + H) \otimes I_n - I_n \otimes \Delta] \tilde{p}, \end{aligned} \quad (10)$$

where, by Lemma 1, $(L(0) + H) \otimes I_n > 0$. It then follows from (10) and $\theta \geq \frac{\lambda_{\max}(\Delta)}{\lambda_1(L(0) + H)}$ that

$$\dot{Q}(t) \leq 0, \quad \forall t \in [t_0, t_1),$$

which implies that

$$Q(t) \leq Q_0 < Q_{\max}, \quad \forall t \in [t_0, t_1).$$

By the definition of the potential function, one has $\psi(r) > Q_{\max} > Q_0$. Therefore, no distance of existing edges will tend to r for $t \in [t_0, t_1)$, implying that no existing edges will be lost before time t_1 . Hence, new edges must be added to the evolving network at the switching time t_1 . Note that the hysteresis ensures that if a finite number of edges are added

to $G(t)$, then the associated potential remains finite. Thus, $Q(t_1)$ is finite.

Similar to the above analysis, the time derivative of $Q(t)$ on every $[t_{k-1}, t_k)$ satisfies

$$\dot{Q}(t) \leq -\tilde{p}^T [\theta (L(t) + H) \otimes I_n - I_n \otimes \Delta] \tilde{p}. \quad (11)$$

By Lemma 3, it follows from $\theta \geq \frac{\lambda_{\max}(\Delta)}{\lambda_1(L(0)+H)}$ that

$$\theta \geq \frac{\lambda_{\max}(\Delta)}{\lambda_1(L(0)+H)} \geq \frac{\lambda_{\max}(\Delta)}{\lambda_1(L(t_{k-1})+H)}. \quad (12)$$

Consequently, it follows from Lemma 1, (11) and (12) that

$$Q(t) \leq Q(t_{k-1}) < Q_{\max}, \text{ for } t \in [t_{k-1}, t_k), k = 1, 2, \dots. \quad (13)$$

Therefore, no distance of existing edges will tend to r for $t \in [t_{k-1}, t_k)$, implying that no edge will be lost before time t_k and so $Q(t_k)$ is finite. Since $G(0)$ is connected and no edges in $E(0)$ were lost, $G(t)$ will remain connected for all $t \geq 0$.

Assume that there are m_k new edges being added to the evolving network at time t_k . Clearly, $0 < m_k \leq \frac{(N-1)(N-2)}{2} \triangleq \bar{M}$. From (8) and (13), one has

$$Q(t_k) \leq Q_0 + (m_1 + m_2 + \dots + m_k)\psi(\|r - \varepsilon\|) = Q_{\max}.$$

Since there are at most \bar{M} new edges that can be added to $G(t)$, one has $k \leq \bar{M}$ and $Q(t) \leq Q_{\max}$ for all $t \geq 0$. Therefore, the number of switching times k of system (1) is finite, which implies that the evolving network $G(t)$ eventually becomes fixed. Thus, the remaining discussions can be restricted on the time interval (t_k, ∞) . Note that all the lengths of edges are not longer than $\psi^{-1}(Q_{\max})$. Hence, the set

$$\Omega = \left\{ \tilde{q} \in D_g, \tilde{p} \in \mathbf{R}^{Nn} \mid Q(\tilde{q}, \tilde{p}) \leq Q_{\max} \right\} \quad (14)$$

is positively invariant, where $D_g = \left\{ \tilde{q} \in \mathbf{R}^{N^2n} \mid \|q_{ij}\| \in [0, \psi^{-1}(Q_{\max})], \forall (i, j) \in E(t) \right\}$ and $\tilde{q} = [q_{11}^T, q_{21}^T, \dots, q_{1N}^T, \dots, q_{N1}^T, q_{N2}^T, \dots, q_{NN}^T]^T$.

Since $G(t)$ is connected for all $t \geq 0$, one has $\|q_{i\gamma} - q_{j\gamma}\| < (N-1)r$ for all i and j . Since $Q(t) \leq Q_{\max}$, one has $p_{i\gamma}^T p_{i\gamma} \leq 2Q_{\max}$, and thus $\|p_{i\gamma}\| \leq \sqrt{2Q_{\max}}$. Therefore, the set Ω satisfying $Q(t) \leq Q_{\max}$ is closed and bounded, hence compact. Note that system (1) with control input (4) is an autonomous system on the concerned time interval (t_k, ∞) . Therefore, the LaSalle Invariance Principle [34] can be applied to infer that if the initial conditions of the system lie in Ω , then the corresponding trajectories will converge to the largest invariant set inside the region

$$S = \left\{ \tilde{q} \in D_g, \tilde{p} \in \mathbf{R}^{Nn} \mid \dot{Q} = 0 \right\}.$$

From (11), $\dot{Q} = 0$ if and only if $p_1 = \dots = p_N = p_\gamma$, which implies that the velocities of all agents will converge to that of the virtual leader asymptotically.

Since $p_1 = \dots = p_N = p_\gamma$, one has

$$\begin{aligned} \dot{p}_{i\gamma} = & - \sum_{j \in \mathcal{N}_i(t)} \frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} \cdot \frac{1}{\|q_{ij}\|} (q_i - q_j) \\ & - h_i c_1 (q_i - q_\gamma) = 0. \end{aligned} \quad (15)$$

Rewrite (15) in a matrix form as

$$-[(\hat{L}(t) \otimes I_n) + (c_1 H \otimes I_n)] \tilde{q} = 0, \quad (16)$$

where $\hat{L}(t) = [\hat{l}_{ij}]$ is a matrix with

$$\hat{l}_{ij} = - \frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} \cdot \frac{1}{\|q_{ij}\|}, \quad i \neq j,$$

and

$$\hat{l}_{ii} = - \sum_{j=1, j \neq i}^N \hat{l}_{ij}.$$

By definition, $\frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} \cdot \frac{1}{\|q_{ij}\|}$ is positive for $\|q_{ij}\| \in (0, r)$ and $\lim_{\|q_{ij}\| \rightarrow 0} \left(\frac{\partial \psi(\|q_{ij}\|)}{\partial \|q_{ij}\|} \cdot \frac{1}{\|q_{ij}\|} \right)$ is nonnegative and bounded.

From [32] and Lemma 1, $\hat{L}(t) \otimes I_n + c_1 H \otimes I_n$ is a positive-definite matrix and hence is nonsingular. Therefore, it follows from (16) that $\tilde{q} = 0$, namely, $q_1 = q_2 = \dots = q_N = q_\gamma$. \square

IV. SIMULATION STUDY

In this section, one simulation example is given to illustrate the theoretical result. In the simulation, the intrinsic dynamics of each agent are governed by the chaotic Chua circuit,

$$\begin{cases} \dot{p}_x = 10(-0.32p_x + p_y + 0.295(|p_x + 1| - |p_x - 1|)), \\ \dot{p}_y = p_x - p_y + p_z, \\ \dot{p}_z = -14.87p_y. \end{cases}$$

For simplicity of presentation, simulation is performed on a group of 10 agents moving in a three-dimensional space under the influence of the control protocol (4). Initial positions and initial velocities of the 10 agents are chosen randomly from the cubes $[0, 10] \times [0, 10] \times [0, 10]$ and $[0, 3] \times [0, 3] \times [0, 3]$, respectively, and the initial position and velocity of the virtual leader are set at $q_\gamma(0) = [8, 8, 8]^T$ and $p_\gamma(0) = [3, 3, 3]^T$. The influencing/sensing radius is chosen as $r = 4$, with $\varepsilon = 0.5$, $m_{ij}(0) = 0$ for all i and j , $k_{ij} = 0.1$ for all i and j , $h_i = 1$ for the informed agent, $c_1 = 10$, $c_{2i}(0) = 0$ for all i and $k_i = 0.1$ for all i . Here, the initial interaction network is set to be connected. Potential function (7) is selected for the protocol (4). Then

$$\begin{aligned} Q_{\max} = & \frac{1}{2} \sum_{i=1}^N \tilde{p}_i^T(0) \tilde{p}_i(0) + \frac{N(N-1)}{2} \psi(\|r - \varepsilon\|) \\ & + \frac{1}{2} \sum_{i=1}^N \left[\sum_{j \in \mathcal{N}_i(t)} \frac{(m_{ij}(0) - \theta)^2}{2k_{ij}} + h_i \frac{(c_{2i}(0) - \theta)^2}{k_i} \right] \\ & \leq \frac{N(N-1)}{2} \psi(\|r - \varepsilon\|) + \frac{N}{2} \max \left(\tilde{p}_i^T(0) \tilde{p}_i(0) \right) \\ & + \frac{1}{2} \sum_{i=1}^N \left[\sum_{j \in \mathcal{N}_i(t)} \frac{(m_{ij}(0) - \theta)^2}{2k_{ij}} + h_i \frac{(c_{2i}(0) - \theta)^2}{k_i} \right] \\ & = \frac{N(N-1)}{2} \frac{(r - \varepsilon)^2}{\varepsilon + \frac{r^2}{Q_{\max}}} + \frac{N}{2} \max \left(\tilde{p}_i^T(0) \tilde{p}_i(0) \right) \end{aligned}$$

$$+ \frac{1}{2} \sum_{i=1}^N \left[\sum_{j \in \mathcal{N}_i(t)} \frac{(m_{ij}(0) - \theta)^2}{2k_{ij}} + h_i \frac{(c_{2i}(0) - \theta)^2}{k_i} \right]. \quad (17)$$

Note that we can evaluate Q_{\max} by using an estimate of the bound of the initial velocities of all agents. From (17), we choose $\hat{Q} = 80000$. Then, the potential function (7) can be explicitly described as

$$\psi(\|q_{ij}\|) = \frac{\|q_{ij}\|^2}{r - \|q_{ij}\| + \frac{1}{5000}}, \quad \|q_{ij}\| \in [0, 4]. \quad (18)$$

In Figure 1, there is one informed agent, which is chosen randomly from the group and marked with a star. Figure 1(a) shows the initial states of the agents, Figure 1(b) depicts the path and final states of the agents, Figures 1(c) and 1(d) show the differences in positions and velocities on the x -axis, y -axis and z -axis between each agent and the virtual leader, from which one can see that all agents eventually move with the same position and velocity as the virtual leader, and Figure 1(e) depicts the adaptive coupling strengths and the adaptive weights on the informed agents, where all converge to constants.

V. CONCLUSIONS

In this paper, we have investigated the rendezvous problem with connectivity preservation for nonlinear dynamics employing local adaptation strategies for both the weights on the velocity navigational feedback and the velocity coupling strengths and using the potential function method. We have constructed a class of bounded potential functions to guarantee the existing links not to be lost, and shown that all agents can asymptotically attain the desired velocity even if only one agent in the team has information on the virtual leader. Future work will consider the effects of time delay and disturbance on the new algorithm.

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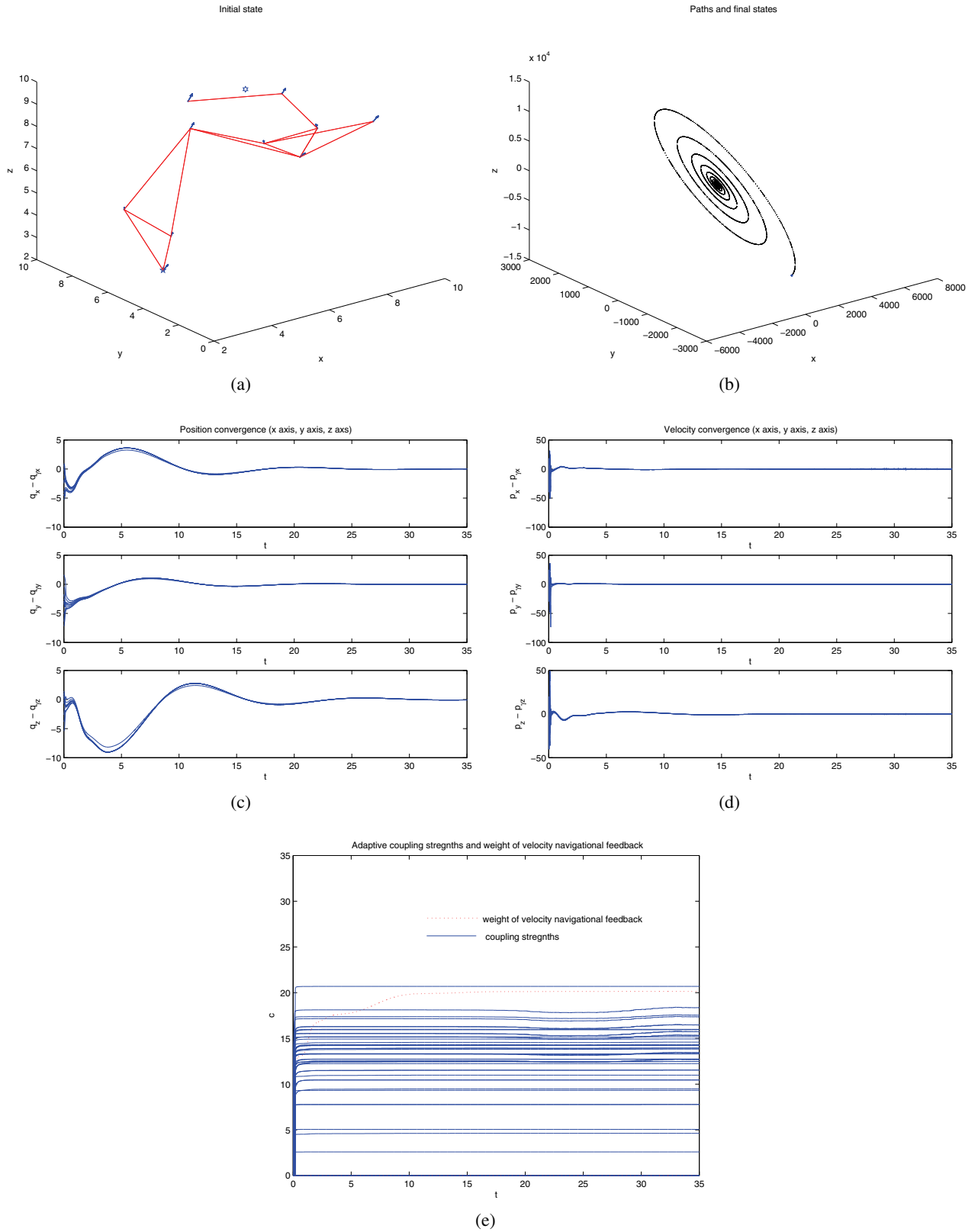


Fig. 1. Rendezvous of 10 agents with 1 virtual leader and 1 informed agent under the algorithm (4).