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Reputation and Competition for Information Intermediaries

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Abstract

This paper investigates the effect of competition on the reputation mechanism in the market for information intermediaries, such as rating agencies. I use a dynamic model to endogenize the value of reputation so as to enable comparison of equilibria under different market structures. In the model, behavior is determined by weighing the current rating fee against the future value the rating agency derives from having a higher reputation. I show that competition worsens the quality of ratings by reducing the value of high reputation but not the short-term gain of cheating.

1 Motivation

Information intermediaries can potentially alleviate the problem of asymmetric information between buyers and sellers of a product. Credit rating agencies convey information about credit-worthiness of an issuer of financial product; certifcation bodies such as standard-setting organizations, organic certifiers and accreditation agencies convey information about whether the product or production process satisfy certain properties. The value of these agencies rely on their ability to credibly convey information about the product they rate.

When the information intermediary is paid for by the sellers whose product they evaluate, there are potential conflicts of interests that can compromise the value of information delivered. A seller may threaten not to give the intermediary its current business unless a good rating is given. Alternatively, it may threaten not to give the intermediary its future business, or other types of business. The conflicts of interests are mitigated by reputation concern. If the intermediary delivers poor information, in the future their rating will not receive high value from the market participants and thus they can not charge a high fee for their service. Because the value of information delivered is determined by reputation mechanism, policy effect on this mechanism should be carefully examined.

There seems to be widely-held belief that competition enhances reputation mechanism. One goal of the Credit Rating Agency Reform Act of 2006 was to increase competition in the credit rating industry by making more explicity the requirements to become a Nationally Recognised Statistical Ratings Organisations. Richard A. Posner proposes eliminating the status of NRSRO in order to increase competition because market disciplines will mitigate potential conflicts of interests.[1] On the other hand, some empirical studies such as Becker et al. (2008) suggest that increasing competition in the rating industry causes more issuer-friendly ratings and worsens the value of information.

This project provides theoretical investigation on whether competition strengthens or weakens reputation mechanism when conflicts of interests arise from foregoing current rating business. Concern of reputation comes from the difference between the future value of a rating agency with a good track record and that of one with a bad track record. I will use a dynamic model of reputation to endogenize the value of reputation and compare equilibria under monopoly and duopoly.

Perfect monitoring implies that the agency loses its reputation once it lies. When an information intermediary cannot profit by adding no information to the market, Markov equilibrium structure implies that once an agency loses its reputation, its value drops to 0. Thus, the cost of cheating is simply the discounted value of future reputation from a good track record. Because a monopolist rating agency can extract all surplus from a seller, competition can only reduce the surplus left to a rating agency and hence the future value of good reputation. However, competition may also drive down the fee a rating agency can charge for its current business. I will show that competition increases the surplus left to a seller with a good product, and thus decreases value of reputation to a rating agency. However, competition does not increase surplus left to a seller with a bad product unless both agencies lie with probability 1. This is because the expected surplus a bad seller gets from a rating agency is not monotonic in the rating fee it charges. If rating fee is low, the agency will not lie for the bad seller. On the other hand, if rating fee is above the agency's discounted future value of reputation, the agency will give a good rating to the bad seller for sure. When a rating agency randomizes between lying and not lying for a seller with a bad product, it must charge a fee equal to its discounted future value of reputation. If it leaves positive surplus to a bad seller, then it can increase the expected surplus it gives to a bad seller by charging a slightly higher fee. Therefore, by charging a slightly higher fee, it will not lose its market share and thus will get a higher payoff. Thus in equilibrium, an agency must leave zero surplus to a bad seller if it randomizes. Because in equilibrium an agency charges a bad seller up to its willingness to pay for a good rating under both monopoly and duopoly, while the future value of reputation is lower under monopoly due to rent left to a good seller under competition, an agency lies with higher probability under duopoly and the value of a good rating is lower under duopoly.

Many papers in the literature on information intermediaries assume that intermediaries are able to commit to a disclosure policy and focus on the preci-

[1] See < http://www.finreg21.com/lombard-street/the-president's-blueprint-reforming-financial-regulation-a-critique-part-ii>

sion of information they provide. They aim to explain the crude grading scheme employed by intermediaries in reality. For example, Lizzeri (1999) shows that the optimal disclosure policy of an intermediary with perfect information facing risk-neutral buyers is a pass-fail system that only discloses whether the product gives non-negative value to the consumer. They show that with competition a full-disclosure equilibrium exists. Doherty et al. (2009) show that when buyers are risk-averse, competing rating agencies will each use a different rating scale.

In contrast this project studies the information provision problem when intermediaries cannot commit and rely on reputation mechanism to provide incentives to follow a rating rule. Strausz (2005) studies conditions under which intermediaries will not collude with the seller whom they rate via an enforceable side-contract with transfers. This project builds on Mathis et al (2009) who use a dynamic model of reputation and show conditions under which reputation concern is insufficient to prevent rating inflation. Bolten et al. (2008) use a static model with exogenous value of reputation and reach a similar conclusion that reputation costs may be insufficient to prevent rating inflation.

Several papers discuss the effect of competition on reputation mechanisms. Most address the moral hazard problem for the producer to provide quality. Horner (2002) shows that in such markets, competition enhances reputation concern and improves quality because it increases the future value of having a higher reputation by improving consumers' outside options and drastically lowering a firm's value of low reputation. In Horner (2002), current gain is equal to saving of the costly efforts and is indepdent of market structure, while in this project, current gain from lying is equal to current rating fee and may depend on market structure. For markets of information intermediaries, Strausz (2005) shows that the threat of collusion makes the market for rating agencies a natural monopoly. They find equilibria under competition where one agency gets all the business and remains honest. Bolten et al. (2008) is closest in spirit to this project. However, they don't provide conclusive answer to the effect of competition because of exogenous value of reputation.

2 Model Setup

In every period $t = 0, 1, 2, ...$, a seller arrives with a new product, which is either good or bad. The buyers do not know the quality of the product, but believe that a new product is good with probability λ. Let $w(p)$ denote the gains to the seller if the buyers share a common belief that the product is good with probability p.

Assumption $w(.)$ is increasing, $w(\lambda) = 0$ and $w(1) > w(p)$ for all $p < 1$.

Therefore, there is no value to revealing no information about a seller.

In the beginning of period 0, one or two (depending on the market structure) rating agencies (CRA) are born. Denote by I the set of rating agencies. $I = \{1\}$ under monopoly and $I = \{1, 2\}$ under duopoly. Rating agencies are long-lived profit maximizers with discount factor $\delta \in (0, 1)$. They perfectly observe the quality of the new product when it arrives and can communicate to the market through issuing a rating. A rating agency may be "honest" or "opportunistic". An honest agency cannot lie, i.e. it can only issue a good rating to a good product and bad rating to a bad product. An opportunistic agency will lie if doing so increases its discounted sum of payoffs. The buyers, sellers and competitor rating agency all share a common prior that with probability q_i^0 rating agency i is honest and with probability $1 - q_i^0$ that he is opportunistic.

Within a period, the game proceeds as follows.

1. A seller arrives with a new product of quality $v \in \{g, b\}$. Both the seller and the rating agencies observe v. The buyers share a common prior that the new product is good with probability λ.

2. Bertrand competition in rating fee:

   (a) Rating agencies post simultaneously their rating fee ϕ_i.

   (b) The seller decides whether to, and if so, with which rating agency to initiate a rating deal.

3. Rating choice:

   (a) If the seller chooses to initiate a deal with agency i, agency i then chooses a rating $m \in \{G_i, B_i\}$.

   (b) After observing the report, the seller decides whether to purchase it.

   (c) If the seller decides to purchase the rating, the agency publishes it. If the seller decides not to purchase it, the agency can choose whether to and if so, which rating to publish. Publishing a rating has no cost.

4. Payoff realization and belief updates:

   (a) The market observes the rating or the lack thereof $m \in \cup_{i \in I} \{G_i, B_i\} \cup \{\emptyset\}$ where $\emptyset$ denotes a null rating, i.e. no published rating. The buyers then forms a posterior p^m about the quality of the product.

   (b) If $w(p^m) > 0$, the buyers try out the new product and learn perfectly its quality. If $w(p^m) = 0$, the buyers do not try the new product and do not learn its quality. After observing the market outcome o, the buyers update their belief about the rating agency χ^o

The solution concept we use is Markov Perfect Equilibrium. Therefore, every player's strategy within a period depends on the history only through $q = (q_i)_{i \in I}$, where q_i denotes the commonly held probability with which agency i is honest. Then the expected payoff from the beginning of a period onward, denoted by $V(.)$, depends only on the reputation q at the beginning of the period. We look at equilibria where $V(.)$ is strictly increasing in own reputation and $V(q) > 0$ for all $q \in (0, 1)^{|I|}$.

Strategy of an Agency Given the reputation q at the beginning of the period, a Markov strategy for an opportunistic agency i specifies the rating fee $\phi_i^v(q) \geq 0$ it offers when the new product is of quality v, the probability it issues a good rating to a good seller after charging fee ϕ_i, $y_i(q, \phi_i)$, the probability it issues a good rating to a bad seller after charging fee ϕ_i, denoted by $x_i(q, \phi_i)$, and the rating $d^v(q) \in \{G_i, B_i, \emptyset\}$ it publishes after the seller initiates a deal with itself but then rejects its rating. [2]Denote by $x_i^*(q)$ and $y_i^*(q)$ the equilibrium probability an opportunistic agency i issues a good rating to a bad seller and a good seller respectively.

A Markov strategy for an honest agency is similar except that it cannot choose a bad rating when the product is good nor can it choose a good rating when the product is bad. Its rating strategy is thus trivial.

Belief Revision Buyers update their belief about the seller to $p^m(q)$ after observing the rating m, and update the belief about the agency to $\chi^o(q)$.

Lemma 1 *$w\left(p^{B_i}(q)\right) = 0$ if $V(q) > 0$.*

Proof. Suppose $w\left(p^{B_i}(q)\right) > 0$.

Case 2 *Good seller pays with probability 1 on the equilibrium path. Then we need $y_i^*(q) < 1$ for $p^{B_i}(q) > 0$. Then $\chi_i^{G_i g}(q_i, q_j) > q_i > 0 = \chi_i^{B_i g}(q)$. But then agency i strictly prefers to issue a good rating because $V\left(\chi_i^{G_i g}(q_i, q_j)\right) \geq V(q) > 0 = V\left(\chi_i^{B_i g}(q)\right)$.*

Case 3 *Good seller refuses to pay with positive probability on the equilibrium path. If $V_i\left(\chi^{\emptyset}(q)\right) > 0 = V\left(\chi^{B_i g}(q)\right)$, then both types of agencies will issue a null rating to a good seller if it refuses to pay. But then $p^{B_i}(q) = 0$ contradiction. If $V_i\left(\chi^{\emptyset}(q)\right) = 0$, then $\chi_i^{\emptyset}(q) < q$ because $V(q) > 0$. But then $\chi_i^{G_i g}(q) > q_i$ because an honest agency either publishes a good rating or no rating for a good seller. So $V\left(\chi^{G_i g}(q)\right) \geq V(q) > 0$. But then both types strictly prefer to publish a good rating after a good seller refuses to pay. So $p^{B_i}(q) = 0$, contradiction.*

■

Lemma 4 *$w\left(p^{\emptyset}(q)\right) = 0$ in a weakly undominated equilibrium.*

[2] d^v does not depend on either the proposed fee ϕ_i or the proposed rating m_i because (ϕ_i, m_i) does not affect agency i's expected payoff when the seller rejects its proposed rating. This is because the buyers and future sellers do not observe agency i's fee offer ϕ_i and proposed rating m_i, and thus (ϕ_i, m_i) do not affect their belief about agency i and thus does not affect agency i's future payoff. In addition, once the seller rejects i's rating, it will not pay and thus (ϕ_i, m_i) does not affect i's current payoff either. Therefore, in a Markov equilibrium, the outside option for a seller after she initiates a deal does not depend on the rating the agency proposes.

Proof. W.l.o.g. assume that $w\left(p^{G_1}(q)\right) \geq w\left(p^{G_2}(q)\right)$. Assume to the contrary that $w\left(p^{\emptyset}(q)\right) > 0$. Then either a good seller does not initiate with probability 1 after equilibrium fee offer, or a good seller refuses to pay with positive probability.

1. If $w\left(p^{\emptyset}(q)\right) > w\left(p^{G_1}(q)\right)$, then no seller will initiate a deal with any agency. So $p^{\emptyset}(q) = \lambda$ but then $w\left(p^{\emptyset}(q)\right) = 0$, contradiction.

2. If $w\left(p^{\emptyset}(q)\right) = w\left(p^{G_1}(q)\right) > w\left(p^{B_1}(q)\right)$, then a bad seller will not initiate a deal with agency 1. But then $w\left(p^{G_1}(q)\right) = w(1) > 0$, contradiction.

3. If $w\left(p^{\emptyset}\right) < w\left(p^{G_1}(q)\right)$ and $w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right) > V\left(\chi^{B_i}(q)\right) - V\left(\chi^{G_i g}(q)\right)$, then after receiving $\phi_1 = w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}\right) - \varepsilon$, a good seller strictly prefers to initiate with agency 1 and strictly prefers to pay once a good rating is issued. But then agency 1 can strictly increase the probability that a good seller pays to 1 if it decreases its fee by any $\varepsilon > 0$, a profitable deviation. A contradiction.

4. If $w\left(p^{\emptyset}\right) < w\left(p^{G_1}(q)\right)$ and $w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right) \leq V\left(\chi^{B_i}(q)\right) - V\left(\chi^{G_i g}(q)\right)$. If $\phi_1^g(q) > w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right)$, or $\phi_1^g(q) = w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right)$ and $y_1^*(q) < 1$, then a good seller does not initiate with agency 1 because $w\left(p^{B_1}(q)\right) = 0$. But then $p^{G_1}(q) = 0$, contradiction. So $\phi_1^g(q) < w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right)$ and $y_i^*(q) = 0$ or $\phi_1^g(q) = w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right)$ and $y_i^*(q) = 1$. If $y_i^*(q) = 0$, then $\chi_i^{G_i g}(q) = 1$, so $w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right) \leq V\left(\chi^{B_i}(q)\right) - V\left(\chi^{G_i g}(q)\right) \leq 0$ because V is increasing in own reputation. Contradiction to $w\left(p^{G_1}(q)\right) > w\left(p^{\emptyset}(q)\right)$. If $y_i^*(q) = 1$ and $\phi_1^q(q) = w\left(p^{G_1}(q)\right) - w\left(p^{\emptyset}(q)\right)$, a good seller is indifferent between initiating and not initiating with 1, but initiating with 1 is weakly dominated.

■

It is useful to focus on symmetric equilibria in which both types of rating agencies offer the same rating fee, and the seller's belief about an agency does not change with the fee offerings. That is, signaling happens only to the extent of the rating chosen, not the rating fee. This further implies that the buyers' belief about an agency does not depend on the identity of the rater or the lack thereof. Therefore, buyers' belief about agency j does not change if j does not publish a rating. There are two reasons to focus on equilibria in which only the rating itself has signaling value. First, identity of rater has signaling value only under duopoly. Thus to better facilitate comparison of equilibria, we choose equilibria that have the same properties as those under monopoly. Second, this simplifies the analysis and we think delivers the intuition. In addition, I'll look at only equilbria in which a rating is published with probability 1 in equilibrium. Therefore, the probability that a seller rated is good is equal to the prior λ. Because a null rating $\emptyset$ is off-equilibrium-path, I will specify belief $\chi^{\emptyset}(q)$ such that if a bad seller refuses to pay, an agency will publish a bad rating;

if a good seller refuses to pay, an honest agency will publish no rating and an opportunistic agency will publish a bad rating. I will discuss the uniqueness of such equilibria later.

Strategy of a seller Because an agency does not publish a good rating when the seller refuses to pay, and the value of a bad rating or null rating is 0, it is a best response for a seller to pay for a good rating issued by agency i if and only if the fee agency i charges (ϕ_i) is no larger than the value of its good rating $w\left(p^{G_i}(q)\right)$; a seller will not pay for a bad rating. A seller initiates a deal with an agency who offers the highest non-negative expected payoff.

The core of the analysis is to establish the equilibrium value of a good rating from agency i, $w\left(p^{G_i}(q)\right)$, and reputation updates after agency i issues a good rating $\chi^{G_i g}(q)$ for a good seller and after i issues a bad rating $\chi^{B_i}(q)$, for every agency $i \in I$.

Agency's rating choice In equilibrium, a seller does not pay for a bad rating, and pays for a good rating if and only if $\phi_i \leq w\left(p^{G_i}(q)\right)$, after charging $\phi_i \leq w\left(p^{G_i}(q)\right)$, the current gain from issuing a good rating instead of a bad one is simply ϕ_i and while the future cost of issuing a good rating is $c_i^v(q) := \delta V\left(\chi^{B_i}(q)\right) - \delta V\left(\chi^{G_i v}(q)\right)$ if the seller is of type v.

If $\phi_i > w\left(p^{G_i}(q)\right)$, then it is weakly dominated for a seller to initiate a deal with agency i. Therefore, in the following discussion, $\phi_i \leq w\left(p^{G_i}(q)\right)$. Because $w\left(p^{B_i}(q)\right) = 0$, if i publishes a bad rating, the buyers will not try the product and its reputation will be updated to $\chi^{B_i}(q)$ even if the product is actually good. If $V_i(q) > 0$, then $w\left(p^{G_i}(q)\right) > w\left(p^{\emptyset}(q)\right) = 0$. Thus after i publishes a good rating the buyers will try the product, find out about the quality v and update i's reputation to $\chi^{G_i v}(q)$. If i issues a bad rating, the seller will not pay because $w\left(p^{B_i}(q)\right) = 0$. Agency i will then publish a bad rating because $\chi^{B_i}(q) \geq \chi^{G_i v}(q)$ for both $v \in \{g, b\}$ in equilibrium. Therefore, the cost of issuing a good rating is the discounted difference in the value of its future reputation, denoted by $c_i^v(q) = \delta V\left(\chi^{B_i}(q)\right) - \delta V\left(\chi^{G_i v}(q)\right)$, and the gain of issuing a good rating is the current rating fee ϕ_i because a seller pays for a good rating but not for a bad rating. It follows that

$$x(q, \phi_i) = \begin{cases} 1 & \text{if} \quad \phi_i > c_i^b(q) \\ \in [0,1] & \text{if} \quad \phi_i = c^b(q) \\ 0 & \text{if} \quad \phi_i < c^b(q) \end{cases}$$

and

$$y(q, \phi_i) = \begin{cases} 1 & \text{if} \quad \phi_i > c^g(q) \\ \in [0,1] & \text{if} \quad \phi_i = c^g(q) \\ 0 & \text{if} \quad \phi_i < c^g(q) \end{cases} .$$

Denote by $x_i^*(q)$ and $y_i^*(q)$ the probability on the equilibrium path.

Lemma 5 $\chi_i^{G_i g}(q) \geq q_i$.

Proof. Conditional on a good seller, a good rating is published if the seller initiates, the agency issues a good rating, and the seller agrees to pay. This is immediate because an honest agency always issues a good rating to a good seller. ■

Lemma 6 *If* $w(\lambda) = 0$*, then* $V(0, q_{-i}) = 0$*.*

Proof. If $q_i = 0$, then agency i's reputation will not change regardless of outcome. Therefore, an opportunistic agency i will issue the rating for which a seller has highest reservation price regardless of the quality of the product. Therefore, no information is revealed by agency i's rating and in equilibrium, $p^{G_i} = \lambda_i$. If $\lambda_i = \lambda$, then $w\left(p^{G_i}\right) = 0$ and agency i can never charge a positive fee for its rating and thus the discounted sum of payoff for agency i is 0. If $\lambda_i > \lambda$ and $w(\lambda_i) > 0$, then a seller initiates a deal with i with probability 1. Otherwise, a seller must get zero expected payoff from initiating with agency i. Then i has a profitable deviation because i lowers its fee to $w(\lambda_i) - \varepsilon$, a seller will initiate with i with probability 1, and i gets $w(\lambda_i) - \varepsilon > 0$ from the increased market share for any arbitrarily small price decrease ε. A contradiction. But then $\lambda_i = \lambda$ because seller of both types initate with i. ■

It follows that as long as $V(q) > 0$, $V\left(\chi^{G_i g}(q)\right) \geq V(q) > V\left(\chi^{G_i b}(q)\right)$ and the reputation costs from issuing a good rating is higher when the seller is bad.

3 Equilibrium Analysis

3.1 Monopoly

The optimality of rating fee implies that a monopolist leaves no surplus to a seller regardless of the quality of his product and charges him his reservation price for a good rating. That is, $\phi_1^g(q) = \phi_1^b(q) = w(q)$. Thus, even though in Mathis et al (2009) it is implicitly assumed that the rating fee is equal to the value of a good rating to the seller, the equilibrium in their game is a regular equilibrium in the aforementioned model.

Lemma 7 $y^*(q) = 1$*.*

Proof. Because $\phi_1^g(q) = \phi_1^b(q) = w(q)$, the gain from issuing a good rating does not depend on seller's type, but the costs are higher for a bad seller. It follows that either $y^*(q) = 1$ or $x^*(q) = 0$. Suppose $x^*(q) = 0$, then $p^G(q) = 1$ and $\chi^{Gg}(q) \geq \chi^B(q)$. But then giving a bad rating to a good seller implies forgoing current rating fee equal to $w\left(p^G(q)\right) = w(1) > 0$, without gaining higher reputation and higher future value. Therefore, the monopolist must strictly prefers to give a good rating to a good seller, a contradiction. ■

A seller initiates a deal for sure, and a rating is published for sure. Therefore, when there is only one rating agency, a seller rated by the monopolist is good with probability λ. Given the current reputation q and the buyers' belief that an opportunistic CRA lies with probability x, the buyers believe that a product receiving a good rating is good with probability $\tilde{p}(q,x) = \frac{\lambda}{\lambda+(1-\lambda)(1-q)x}$; the rating agency's reputation rises to $\chi^B(q,x) = \frac{q}{q+(1-q)x}$ after issuing a bad rating. Because the buyers observe quality perfectly after trying out the good, the rating agency's reputation remains q if it issues a good rating to a good seller, and drops to 0 if it issues a good rating to a bad seller.

Because a seller does not purchase a bad rating, when faced with a bad product, issuing a bad rating means losing the current rating fee $\phi^b = w(\tilde{p}(q,x))$ but enjoying a reputation of $\chi^B(q,x)$ as opposed to 0. They show that when $1+\lambda < \frac{1}{\delta}$, the monopolist rating agency lies with probability 1 when reputation is high. This is because when q is high, current gain of rating fee $w(\tilde{p}(q,x))$ at any level of x is higher than the value of a perfect reputation. It follows that the value of such high level of reputation $V(q)$ is equal to $\frac{w(\tilde{p}(q,1))}{1-\lambda\delta}$ because on the equilibrium path, the agency gets his rating fee equal to $w(\tilde{p}(q,1))$ and his reputation either remains the same or drops to 0. For lower reputation q, the equilibrium probability $x^*(q)$ is pinned down by the indifference condition

$$w(\tilde{p}(q,x^*)) = \delta V\left(\chi^B(q,x^*)\right),$$

that is

$$w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)x^*}\right) = \delta V\left(\frac{q}{q+(1-q)(1-x^*)}\right). \tag{1}$$

This equation has a unique solution as long because the value function V is increasing in reputation.

3.2 Duopoly

3.2.1 Equilibrium Probability of Lying

Now we consider competition between two long-lived rating agencies with identical discount factor $\delta \in (0,1)$.

Choice of Rating Fee Because a seller obtains positive payoff only if a good rating is published and an honest agency i always issues a good rating to a good seller, the expected payoff a good seller gets from initiating with agency i when i charges ϕ_i is

$$U_i^g(q,\phi_i) := (q_i + (1-q_i)\,y(q,\phi_i))\left(w\left(p^{G_i}\right) - \phi_i\right).$$

Because an honest agency i does not issue a good rating to a bad seller, the expected payoff a bad seller gets from initiating with agency i when i charges ϕ_i is

$$U_i^b(q,\phi_i) := (1-q_i)\,x(q,\phi_i)\left(w\left(p^{G_i}\right) - \phi_i\right).$$

By offering different fee ϕ_i, agency i changes the amount of surplus it leaves the seller if seller initiates a deal with i. Bertrand competition forces agencies to compete in leaving more surplus to a seller in order to get his business until one agency gets zero surplus. Denote by $\overline{U}_i^v(q)$ the maximal possible surplus agency i can leave to a seller of type v. Then seller type v initiates with i with probability 1 if $\overline{U}_i^v(q) > \overline{U}_{-i}^v(q)$. Generally the lower the fee is, the higher the surplus $U_i^v(q, \phi_i)$ for the seller. However, the function is discontinuous at $\phi_i = c_i^v(q)$, the reputation costs to issue a good rating.

If on the equilibrium path at some state (q_i, q_j), CRA$_i$ randomizes between giving a good and a bad rating, CRA$_i$ must be indifferent between the current gain of rating fee and the discounted future value of reputation difference. Therefore, if CRA$_i$ increases his rating fee, subsequently he will have a strict incentive to give a good rating. Therefore, if seller type v obtains a positive expected surplus from CRA$_i$ on the equilibrium path, the seller must obtain a greater expected surplus if CRA$_i$ increases its rating fee by a small amount. Because the rating agencies engage in price competition, if CRA$_i$ leaves a positive surplus in equilibrium, both agencies must leave the same surplus. Therefore, by increasing its rating fee by $\varepsilon > 0$, CRA$_i$ can increase its payoff, a contradiction. Thus in equilibrium either CRA$_i$ leaves zero surplus to a seller or CRA$_i$ gives a good rating for sure. Lemma 8 summarizes this.

Lemma 8 *If $U_i^v(q_i, q_j, \phi^v(q_i, q_j)) > 0$, then $y(q_i, q_j, \phi^g(q_i, q_j)) = 1$ if $v = g$, and $x\left(q_i, q_j, \phi^b(q_i, q_j)\right) = 1$ if $v = b$.*

Proof. Otherwise, conditional on the arrival of a type-v seller, CRA$_i$ can earn ε more by charging $\phi^v(q_i, q_j) + \varepsilon$ if CRA$_i$ weakly prefers rating a type-v seller. If CRA$_i$ strictly prefers letting the opponent rate type-v seller, charging $\phi^v(q_i, q_j)$ and leaving a positive surplus to type-v seller is weakly dominated by raising the rating fee and leaving a negative surplus to type-v seller. ■

If CRA$_i$ does not give a good rating for sure to a good seller, then it must be giving zero expected surplus to a good seller and thus the value of CRA$_i$'s good rating is no greater than the discounted value of future reputation difference. Since giving a good rating to a bad seller has worse consequence than giving a good rating to a good seller, CRA$_i$ must give a bad rating to a bad seller for sure. But then giving a good rating is more indicative of a rating agency being the honest type. Thus CRA$_i$ must prefer to give a good rating to a good seller. Let $V(q_i, q_j)$ denote the discounted sum of payoffs for rating agency i when own reputation is q_i and the opponent's reputation is q_j.

Because $y_i(q) > 0$, lemma 8 says that either $y_i(q) = 1$ or agency i leaves zero surplus to a good seller. The following lemma says that in a regular equilibrium under duopoly, an opportunistic agency i issues a good rating to a good seller with probability 1, just like under monopoly.

Lemma 9 *$y_i^*(q) = 1$ if $w\left(p^{G_i}\right) > 0$*

Proof. First we show that $y_i > 0$. If not, then $\chi^{G_i g}(q_i, q_j) = (1, q_j)$ and $\chi^{B_i}(q_i) < 1$. It follows that $V\left(\chi^{G_i g}(q_i, q_j)\right) \geq V\left(\chi^{B_i}(q_i, q_j)\right)$. If strict inequality holds, then it is optimal for i to give a good rating to a good seller for any $\phi \geq 0$. A contradiction. If equality holds, then $y_i = 0$ is optimal only if $\phi_i^g = 0$. Suppose in equilibrium, i rates a good seller with positive probability, then by raising rating fee to $\varepsilon > 0$, i will issue a good rating to a good seller for sure, thus increasing the surplus left to a good seller and current payoff to himself. Therefore, i will not lose any good seller and will increase its own payoff, a contradiction. Therefore, i must rate a good seller with zero probability, but then $\lambda_i = 0$ and $w\left(p^{G_i}\right) = 0$, contradiction.

Suppose $y_i \in (0,1)$. If $U_i^g > 0$, then $y_i = 1$ by lemma 8. If $U_i^g = 0$, then

$$w(p_i) = \phi_i(q_i, q_j) = \delta V\left(\chi^{B_i}(q_i, q_j)\right) - \delta V\left(\chi^{G_i g}(q_i, q_j)\right).$$

Because $\chi_i^{G_i g}(q) > q_i$, $V\left(\chi^{G_i g}(q_i, q_j)\right) \geq V(q) > 0$. Therefore $w(p_i) < \delta V\left(\chi^{B_i}(q_i, q_j)\right)$ and thus $x_i(q_i, q_j) = 0$, in which case $\chi^{B_i}(q_i) < q_i$ (because $x_i = 0$ and $y_i < 1$) and $V\left(\chi^{B_i}(q_i, q_j)\right) < V\left(\chi^{G_i g}(q_i, q_j)\right)$ because $\chi^{G_i g}(q_i) \geq q_i$ and V increasing in own reputation. It follows that $w(p_i) < 0$, a contradiction. ■

Because $y^*(q) = 1$, $p^{G_i}(q) = \frac{\lambda_i}{\lambda_i + (1-\lambda_i)(1-q_i)x_i}$ where λ_i is the probability that a seller rated by agency i is good.

Lemma 8 and the nature of Bertrand competition between agencies imply that a bad seller is left with zero surplus unless both agencies give a bad seller a good rating for sure. Thus when CRA_i lies with probability between $(0,1)$, the probability of lying x_i is determined by

$$w\left(\frac{\lambda_i}{\lambda_i + (1-\lambda_i)(1-q_i)x_i}\right) = \delta V\left(\frac{q_i}{q_i + (1-q_i)(1-x_i)}, q_j\right).$$

We will now show that whenever two rating agencies have different initial reputations, the leading rating agency is the only active one if the value of being a leading rating agency is greater than that of being a trailing rating agency. Therefore, if i is active, then $\lambda_i = \lambda$.

We first establish this statement for boundaries on the top.

Lemma 10 *In equilibrium, a seller initiates a deal with agency* 1 *with probability 1 if* $q_1 = 1 > q_2$.

Proof.

$$\begin{aligned}\overline{U_1^g}(1, q_2) \quad &: \quad = w(1) - \left(\delta V\left(\chi^{B_i}(q)\right) - \delta V(q)\right) \\ &= \quad w(1) \\ &\geq \quad w\left(p^{G_2}(q)\right) - \left(\delta V\left(\chi^{B_2}(q)\right) - \delta V(q)\right)\end{aligned}$$

and strictly inequality holds unless $x_2^*(q) = 0$. If strictly inequality holds, then in equilibrium under Bertrand competition for good sellers, agency 1 will charge a fee so that a good seller initiates with agency 1 with probability 1. If $x_2^*(q) = 0$ and $\overline{U_1^g}(1, q_2) = \overline{U_2^g}(1, q_2)$, Bertrand competition implies that $\phi_1^g = \phi_2^g = 0$. But then agency 1 does not get positive fee payment from a seller of any type, and its reputation never changes and thus $V(q_2, 1) = 0 + \delta V(q_2, 1)$ and $V(1, q_2) = 0$ for all $q_2 < 1$. But then $V(q_1, q_2) = 0$ for all $q \in [0,1] \times [0,1]$. Contradiction to our assumption that $V(q) \neq 0$. ■

Lemma 11 $V(1,1) = \frac{\frac{1}{2}(1-\lambda)w(1)}{1-\lambda\delta}$.

Proof. Agency i charges a bad seller $w(1)$ because a bad seller expects zero surplus no matter what the rating fee is from being given a bad rating by the honest agency i. Bertrand competition implies $\phi_i^g = 0$. ■

It is easy to see that $V(r,1) = 0$ for all $r < 1$ because the trailing agency believes that the opponent is honest and thus will never lose its reputation. Hence, the trailing agency will never get any payment from a seller and thus obtains zero value. The following lemma shows that the trailing agency gives zero surplus to both a bad seller and a good seller.

Lemma 12 *$\lambda(r,1) = 0$ and $p^{G_2} = 0$.*

Proof. Because leading agency has perfect reputation, it is going to give a bad seller a good rating with zero probability. Therefore, it leaves zero surplus to a bad seller. Therefore, Bertrand competition implies that in equilibrium, the trailing agency leaves zero surplus to a bad seller too. But because $V(r,1) = 0$ for $r < 1$, the surplus a trailing agency leaves a good seller is $V(r,1) = 0$. Therefore, either $\lambda(r,1) = 0$ and $p^{G_2} = 0$ or

$$w\left(\frac{\lambda(q_2,1)}{\lambda(q_2,1) + (1-\lambda(q_2,1))(1-q_2)}\right) = \delta V(1,1).$$

But if

$$w\left(\frac{\lambda(q_2,1)}{\lambda(q_2,1) + (1-\lambda(q_2,1))(1-q_2)}\right) = \delta V(1,1) > 0,$$

If $\phi_2^g \geq \delta V(1,1)$, then in equilibrium a good seller gets zero surplus from CRA_1. CRA_2 can lower its price to below $\delta V(1,1)$ and gets a good seller and thus a strictly positive value. Therefore, $\phi_2^g < \delta V(1,1)$ and $y_2 = 0$, a contradiction. ■

Because $\lambda(r,1) = 0$ for all $r < 1$, we have $\phi^v(1,r) = w(1)$ and $\phi^v(r,1) = 0$ for $v \in \{g,b\}$. It follows that $V(1,r) = \frac{w(1)}{1-\lambda\delta} = V(1)$.

We now establish that an agency weakly prefers to rate a seller to letting the opponent rate it. Therefore, the costs of publishing a good rating are the difference between giving a bad rating and a good rating. Consider a good seller. Because $y^*(q) = 1$ in equilibrium, $\chi^{G_i g}(q) = q$. Payoff if the opponent rates it is $\delta V\left(\chi^{G_j g}(q)\right) = \delta V(q)$. Payoff if i rates it is $\phi_i^g(q) + \delta V\left(\chi^{G_i g}(q)\right) \geq \delta V(q)$. So i weakly prefers to rate a good seller. Consider a bad seller. If $V\left(\chi^{B_i}(q)\right) < x_j^*(q)(1-q_j)V(q_i,0) + \left(q_j + (1-q_j)\left(1-x_j^*(q)\right)\right)V\left(\chi^{B_j}(q)\right)$, then if i does rate a bad seller, the fee i charges must be greater than or equal to $x_j^*(q)(1-q_j)V(q_i,0) + \left(q_j + (1-q_j)\left(1-x_j^*(q)\right)\right)V\left(\chi^{B_j}(q)\right)$, thus greater than $V\left(\chi^{B_i}(q)\right)$. So $x_i^*(q) = 1$ and thus $\chi^{B_i}(q) = (1,q_j)$. Because $V(1,q_j) = V(1) > V(q)$ for all $q_i < 1$ and $q_j < 1$. Then we get a contradiction.

Now we will show that the leading agency gets all the rating business.

Lemma 13 *Let V be the value function of a regular equilibrium. If $V(q_1,q_2) > V(q_2,q_1)$ where $q_1 > q_2$, then a seller initiates with the leading agency, agency 1, with probability 1 at q.*

Proof. $y^*(q) = 1$. So $\phi_i^g(q) \geq \delta V\left(\chi^{B_i}(q)\right) - \delta V\left(\chi^{G_i g}(q)\right)$. Maximal surplus agency i can leave to a good seller is

$$\overline{U_i^g}(q) := w\left(p^{G_i}(q)\right) - \left(\delta V\left(\chi^{B_i}(q)\right) - \delta V(q)\right).$$

If $x_i^*(q) \in (0,1)$ for $i \in (1,2)$, then $U_i^b(q) = 0$ and $w\left(p^{G_i}(q)\right) = \delta V\left(\chi^{B_i}(q)\right)$, and $\overline{U_i^g}(q) = \delta V(q)$.

If $x^*(q_1, q_2) = 1$, then

$$\begin{aligned}\overline{U_1^g}(q) &= w\left(\frac{\lambda_1}{\lambda_1 + (1-\lambda_1)(1-q_1)}\right) - \delta V(1, q_2) + \delta V(q_1, q_2) \\ &= \frac{\overline{U_1^b}(q)}{1-q_1} + \delta V(q_1, q_2).\end{aligned}$$

If $\overline{U_1^g}(q) \leq \overline{U_1^g}(q)$, then $\lambda_1 < \lambda_2$, $x_2^*(q) = 1$ and $\frac{\overline{U_1^b}(q)}{1-q_1} < \frac{\overline{U_2^b}(q)}{1-q_2}$. But then price competition in the bad seller market implies that a bad seller initiates with 2 with probability 1. Either $\lambda_1 = 1 > \lambda_2$ or $\lambda_2 = \lambda$ and a seller initiates with 1 with probability 0. But in the latter case $V(q_1, q_2) < V(q_2, q_1)$, contradiction. ■

Therefore, when CRA$_i$ is active, $\lambda_i = \lambda$. So the current gain from lying is the same function of x as that under monopoly, while the future discounted value of reputation as a function of x is weakly lower than that under monopoly. As a result, the equilibrium probability of lying in monopoly is lower. Because whenever $x < 1$, future reputation is not perfect. We will show that value of reputation is strictly lower under monopoly as long as the reputation is not perfect. Thus equilibrium probability of lying is strictly higher under duopoly.

3.2.2 Construction of equilibrium

I will construct an equilibrium in which when reputations differ, a seller initiates a deal with the leading agency for sure, and randomizes with equal probability between the two when they have the same reputations. By symmetry, it is w.l.o.g. to assume that CRA$_1$ is the leading agency. For clarity, denote the reputation of the leading CRA by q and that of the trailing CRA by r. I will first establish equilibrium pricing behavior, choice of ratings and value for $q = 1$. I will then work backwards for $q \in \left[q_1^D, 1\right]$. Having done so for all $q \in \left[q_k^D, 1\right]$, I will construct behavior and value for $q \in \left[q_{k+1}^D, q_1^D\right]$ for some $q_{k+1}^D < q^D$.

We have shown that $V(r, 1) = 0$ and $V(1, r) = \frac{w(1)}{1-\lambda\delta} = V(1)$ for all $r < 1$. It follows that if $\frac{\delta}{1-\lambda\delta} < 1$, $x(1, r) = 1$ and $x(1, r) = 0$ if $\frac{\delta}{1-\lambda\delta} > 1$.

It follows immediately that when $\frac{\delta}{1-\lambda\delta} > 1$, it is an equilibrium in which the leading CRA never lies. As a result, the value for a trailing CRA is 0. Therefore, when lying does not happen in equilibrium under monopoly, industry structure has no effect on the value of ratings. For the rest of the discussion, we will assume that $\frac{\delta}{1-\lambda\delta} < 1$.

When $\frac{\delta}{1-\lambda\delta} < 1$, $w(1) > \delta V(1,r)$ and $x(1,r) = 1$. Let $q_1^D(r)$ be the solution to

$$w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-q_1^D(r)\right)}\right) = \delta V(1,r).$$

Because $V(1,r) = V(1)$, this equation is exactly the same as the one that determines q_1^M. Thus $q_1^D(r) = q_1^M$ for all $r < 1$. For $q \geq q_1^D$, $x(q,r) = 1$ for all $r < q$. Thus when the leading CRA's reputation is sufficiently high, the value of a good rating does not depend on market structure. However, the value to a CRA does. For $q \in \left(q_1^D, 1\right)$, if a good seller arrives, reputations will not change. When a bad seller arrives, the opportunistic leading CRA will lie for sure and its reputation will drop to 0. If the leading CRA does not lie, its reputation will rise to $\chi^{B_1}(q,r) = 1$ and CRA$_2$'s value conditional on that event is $V(r,1) = 0$. It follows that

$$V(r,q) = \lambda\delta V(r,q) + (1-\lambda)\delta\left((1-q)V(r,0) + qV(r,1)\right).$$

Because $V(r,0) = V(r)$ and $V(r,1) = 0$, rearranging the expression we get

$$V(r,q) = \frac{(1-\lambda)\delta}{1-\lambda\delta}(1-q)V(r).$$

This is strictly positive for any $0 < r < q < 1$. In addition, it is strictly increasing in r and decreasing in q. By lemma 8, either CRA$_2$ leaves zero surplus to a bad seller, or it lies with probability 1 conditional on getting a deal from a bad seller, in which case the expected surplus a bad seller gets from CRA$_2$ is

$$(1-r)\left(w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)(1-r)}\right) - \delta V(1,q)\right). \tag{2}$$

The largest expected surplus CRA$_1$ can leave a bad seller in equilibrium is

$$(1-q)\left(w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right) - \delta V(1,r)\right) \tag{3}$$

because in equilibrium $x(q,r) = 1$ and thus $\phi^b(q,r) \geq \delta V(1,r)$. Because $V(1,q) = V(1,r)$ and $q > r$, for CRA$_1$ to win the Bertrand competition over a bad seller, $\lambda_2 < \lambda$. Thus, expression 2 is positive only if $r > q_1^M$. Given λ_2 such that expression 2 is non-negative and no greater than expression 3, $\phi^b(r,q) = \delta V(1,q) = \delta V(1)$ and $\phi^b(q,r) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right) - \frac{1-r}{1-q}\left(w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)(1-r)}\right) - \delta V(1,q)\right)$. The surplus CRA$_2$ can give a good seller can then be no greater than

$$w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)(1-r)}\right) - \delta V(1,q) + \delta V(r,q).$$

Thus in equilibrium $\phi^g(r,q) = \delta V(1,q) - \delta V(r,q)$ and $\phi^g(q,r) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right) - \left(w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)(1-r)}\right) - \delta V(1,q) + \delta V(r,q)\right)$. It follows that the value to the leading CRA is

$$\begin{aligned} & \frac{1}{1-\lambda\delta} w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)-\frac{\lambda}{1-\lambda\delta}\delta V(r,q) \\ & -\frac{1}{1-\lambda\delta}\left(\left(\lambda+(1-\lambda)\frac{1-r}{1-q}\right)\left(w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)(1-r)}\right)-\delta V(1,q)\right)\right) \\ < \; & \frac{1}{1-\lambda\delta}\left(\begin{array}{c} w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)-w\left(\frac{\lambda_2}{\lambda_2+(1-\lambda_2)(1-r)}\right) \\ +(\delta V(1,q)-\lambda\delta V(r,q)) \end{array}\right). \end{aligned}$$

Given λ_2 such that expression 2 is negative, $x(r,q)<1$ and by lemma 8, the expected surplus a bad seller obtains from CRA_2 is zero. Thus CRA_1 leaves zero surplus to a bad seller in equilibrium as well and $\phi^b(q,r)=w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)$. The largest surplus CRA_2 can give a good seller is then $\delta V(r,q)$. Thus in equilibrium CRA_1 also leaves $\delta V(r,q)$ to a good seller. It follows that the value to the leading CRA is

$$\begin{aligned} V(q,r) &= \frac{\lambda\phi^g(q,r)+(1-\lambda)\phi^b(q,r)}{1-\lambda\delta} \\ &= \frac{1}{1-\lambda\delta} w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)-\frac{\lambda}{1-\lambda\delta}\delta V(r,q). \end{aligned} \tag{4}$$

Because $V(q)=\frac{1}{1-\lambda\delta} w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)}\right)$, the value to the leading CRA under duopoly is strictly less than that under monopoly.

Notice that when $q\in\left(q_1^D,1\right)$, the value to the trailing agency, $V(r,q)$, depends only on the reputations of the agencies but not on the off equilibrium belief λ_2. The value to the leading agency, on the other hand, may be different in different equilibria. However, in any equilibrium, this value is always strictly less than that under monopoly and the upper bound is attained when off equilibrium path belief that a seller rated by the trailing agency is good with sufficiently small probability. In addition, $V(q,r)$ is the same across all equilibria in which $x(r,q)<1$. Thus for the purpose of finding equilibrium values, we do not need to worry about the value of $x(r,q)$ if it is less than 1.

Given $V(q,r)$ increasing in q, $V(r,q)$ decreasing in q for all $q\geq q_k^D(r)$, let $q_{k+1}^D(r)$ the solution of q to

$$w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-\frac{q}{q_k^D(r)}\right)}\right)=\delta V\left(q_k^D(r),r\right).$$

A solution $q_{k+1}^D(r)<q_k^D(r)$ exists because when the left hand side is increasing in q and is greater than the right hand side when $q=q_k^D(r)$. For $q\in\left(q_{k+1}^D(r),q_k^D(r)\right)$, define $\chi^{B_1}(q,r)$ to be the solution of χ to

$$w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-\frac{q}{\chi}\right)}\right)=\delta V(\chi,r). \tag{5}$$

A solution $\chi \in \left(q_k^D, q_{k-1}^D\right)$ exists because the left hand side is decreasing in χ while the right hand side is increasing in χ by construction, and the left hand side is greater than the right hand side when $\chi = q_k^D$ and smaller than the right hand side when $\chi = q_{k-1}^D$. $\chi^{B_1}(q,r)$ is increasing in q and decreasing in r. Define $x(q,r)$ to be such that $1-(1-q)x = \frac{q}{\chi^{B_1}(q,r)}$. In equilibrium, CRA$_1$ lies with probability $x(q,r)$. Thus the value of its good rating is equal to $w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-\frac{q}{\chi^{B_1}(q,r)}\right)}\right)$ and its reputation will become $\chi^{B_1}(q,r)$ after issuing a bad rating while the opponent's reputation remains r. Equation 5 implies that CRA$_1$ is indifferent between giving a good and a bad rating to a bad seller.

$$
\begin{aligned}
V(r,q) &= \frac{(1-\lambda)}{1-\lambda\delta}\delta\left((1-q)\,x(q,r)\,V(r) + \frac{q}{\chi^{B_1}(q,r)}V\left(r,\chi^{B_1}(q,r)\right)\right)\\
&= \frac{(1-\lambda)}{1-\lambda\delta}\delta\left(V(r) - \frac{q}{\chi^{B_1}(q,r)}\left(V(r) - V\left(r,\chi^{B_1}(q,r)\right)\right)\right).
\end{aligned}
$$

Because $\chi^{B_1}(q,r) > q_k^D$, by construction, $V\left(r,\chi^{B_1}(q,r)\right)$ is decreasing in the second argument. Therefore, $V(r,q)$ is decreasing in q because $\chi^{B_1}(q,r)$ is increasing in q and $\frac{q}{\chi^{B_1}(q,r)}$ is increasing in q. In equilibrium, both agencies leave zero surplus to a bad seller. Therefore, $\phi^b(q,r) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)x(q,r)}\right)$ and $\phi^g(q,r) = w\left(\frac{\lambda}{\lambda+(1-\lambda)(1-q)x(q,r)}\right) - \delta V(r,q)$. Thus $V(q,r) = \frac{1}{1-\lambda\delta}w\left(\frac{\lambda}{\lambda+(1-\lambda)\left(1-\frac{q}{\chi^{B_1}(q,r)}\right)}\right) - \frac{\lambda\delta}{1-\lambda\delta}V(r,q) = \frac{\delta}{1-\lambda\delta}V\left(\chi^{B_1}(q,r),r\right) - \frac{\lambda\delta}{1-\lambda\delta}V(r,q)$. Because $\chi(q,r)$ is increasing in q and $V(r,q)$ is decreasing in q, $V(q,r)$ is increasing in q. We have thus found $V(q,r)$ increasing in q and $V(r,q)$ decreasing in q for all $q \in \left(q_{k+1}^D(r), q_k^D(r)\right)$.

4 Discussion of assumptions

A seller can walk away from a bad rating It is believed that the issuer-pay model creates pressure from sellers for the rating agency to give a good rating.[3] In the model the pressure comes from threats to forego current rating fee. This assumption is made in both Bolten et al (2008) and Mathis et al (2009). It seems consistent with industry practice where a seller invites some rating agencies to look over its product and conduct preliminary work before deciding which rating agencies to select to rate the product afterwards. Stephen W. Joynt, the CEO of Fitch Ratings, also stated that Fitch is frequently requested by issuers to informally comment on the ratings effect of big corporate events

[3] Richard Posner argued that "This puts the NRSROs under greater pressure to give the sellers of securities a high rating, and thus weakens market discipline". See < http://www.finreg21.com/lombard-street/the-president's-blueprint-reforming-financial-regulation-a-critique-part-ii>

or multiple different scenarios. In an article about a reform in the rating fee structure, Reuters cites New York Attorney General Andrew Cuomo saying that "under the old fee system, the agencies had a financial incentive to assign high ratings because they only received fees if a deal was completed."[4] Both [12] and [?, ?] also assume that sellers can walk away from a bad rating and thus does not pay.

A seller obtains one or no rating According to Joynt, in structured finance, Fitch is "frequently one of two rating agencies rating a security chosen by the issuer from among the three agencies." The market behavior seems to change dramatically only after Fitch became an important third player.[5] The significant effect of increasing the number of rating agencies from two to three may be driven by regulational requirements for some products to have at least two ratings. As a simplified model to address this phenomenon, I assume that a seller obtains at most one rating and compare equilibria when the number of agencies increases from one to two. This setup enables comparison of equilibria when a seller has a degree of freedom in choosing rating agencies.

Agencies observe the quality of the product before deciding on its rating fee This assumption is equivalent to assuming that a rating agency can obtain information about the product costlessly. It is a simplifying assumption reflecting the fact that an agency has all the public information about the product. According to Joynt, "an objective opinion about the creditworthiness of an issuer can be formed based solely on public information in many jurisdictions."[6] In addition, a rating agency often follows an important firm even if it is not selected to rate its issuances. Therefore, a rating agency knows a good deal about a product before it makes an offer. Moreover, Joynt admits that "structured finance analysts may be involved in fee discussions"[7] and the analyst may learn even more about the product during negotiation. This assumption also seems consistent with the case-by-case nature of rating fees.[8]

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[5] See Becker et al (forthcoming).

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[7] The full quote "Although structured finance analysts may be involved in fee discussions, they are typically senior analysts who understand the need to manage the potential conflict of interest." See <http://www.sec.gov/news/extra/credrate/fitchratings1.htm>

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