

The HKU Scholars Hub



Title	Quantity discount contract for supply chain coordination with false failure returns
Author(s)	Huang, X; Choi, SM; Ching, WK
Citation	The 6th International Conference on Natural Computation (ICNC 2010), Yantai, Shandong, China, 10-12 August 2010. In Proceedings of the International Conference on Natural Computation, 2010, v. 8, p. 4450-4454
Issued Date	2010
URL	http://hdl.handle.net/10722/129747
Rights	Proceedings of the International Conference on Natural Computation. Copyright © IEEE.

# Quantity Discount Contract for Supply Chain Coordination with False Failure Returns

Ximin Huang, Sin-Man Choi, Wai-Ki Ching

Advanced Modeling and Applied Computing Laboratory, Department of Mathematics, The University of Hong Kong, Pokfulam Road, Hong Kong. hehe1121@hkusua.hku.hk, kellyci@hkusua.hku.hk, wching@hkusua.hku.hk.

Abstract—Consumer return attracts more and more academic attention due to its rapidly expanding size, and a large portion of it falls into the category of false failure return, which refers to return without functional defect. In this paper, we exclusively consider profit results from exerting costly effort to reduce false failure returns in a reverse supply chain consisting of a retailer and a supplier. The supply chain as a whole has strong incentive to reduce false failure returns because it can avoid much reprocessing cost associated. But typically, retailers enjoy a full credit provided by suppliers in case of returns, and hence they may not have sufficient incentives to exert enough effort for supply chain profit maximization. In some scenarios they may even have the motivation to actually encourage such returns. We suggest using a coordination contract to resolve such profit conflicts. The contract we propose is a quantity discount contract specifying a payment to the retailer with an amount exponentially decreasing in the number of false failure returns. We give explicit forms of such contracts given different assumptions about distribution of the number of returns and we also prove that such contract is capable of increasing both retailer's and supplier's profit simultaneously. Besides, when the contract is used together with other forward supply chain coordination contracts in a closed-loop chain, it is shown that it can act to deter retailer's potential incentive to encourage false failure returns. Moreover, some modifications of the contract may lead to easy allocation of incremental profit within the supply chain.

Keywords—Consumer Returns; Closed-loop Supply Chains; Quantity Discount Contract; Supply Chain Coordination.

# I. INTRODUCTION

Many firms now offer liberal return policies allowing customers to return the products for any reasons within some time (typically 90 days) after the purchases. The volume of such consumer returns is rapidly increasing and has already exceeded \$100 billion per year in the U.S., see for instance [8]. However, a large portion (95% in the electronic industry in the U.S. by [7]) of them are results of reasons other than functional defects of the products. For example, the customer may regret over an impulse purchase, or find out later that the product is not suitable or too difficult to use. This kind of returns is referred to as "false failure returns" in [2]. This is the main subject we are going to study in this paper.

Another closely related concept here is the closed-loop supply chain and the reverse supply chain. According to [4], the process of material flowing from suppliers for manufacturing and then to retailers for selling and finally purchased by customers is referred to as a forward supply chain. Then, in case of returns, the process of retailers accepting the returned products from customers and transferring them back to suppliers for possible re-manufacturing is referred to as a reverse supply chain. The forward and the reverse together form a closed-loop supply chain. Usually, studies on reverse supply chain are motivated by two reasons. The first one concerns sustainable development of the earth because recollected products can be used for recycling. Besides the environmental consideration, the economical reason is that material value now involved in returns is too large to be ignored. Studies (for example, [4]) have shown that proper design of the reverse supply chain may even bring in extra profit to firms in addition to lowering its cost.

Although much existing literature focus on the planning or the impact of re-manufacturing, this paper studies the incentive conflicts in a reverse supply chain concerning exerting costly effort to reduce false failure returns. Reducing such returns is usually highly beneficial to suppliers. Suppliers who value their brand image typically provide a full credit to retailers and take the returned products back. And by [3], during the take-back process, suppliers not only bear goodwill cost but also have to pay for cost of possible test, refurbishment, remanufacture, recycle or loss in value of the products especially for innovative or fashionable ones, see for instance, a discussion in [5]. The sum of these costs could be substantial but if the amount of returns is reduced, such loss can be avoided. On the other hand, retailers do have to pay something in case of a false failure return, for example, goodwill cost or fees of reprocessing and transportation. But since they get full credit from suppliers, their incentives of reducing such returns is usually lower. In some circumstances, they may even want to actively push the level of such returns, for instance, if the product has poor demand.

This paper is devoted to designing a coordination mechanism for a reverse supply chain in presence of false failure returns. Specifically, we consider a supply chain consisting of one retailer and one supplier. Attention is exclusively focused on the profit and cost associated with false failure returns. The operations in forward supply chain are thus not considered in the first part of discussion. We will then incorporate the two in a later section.

Consider the following events occurring in sequence: when a customer brings in a false failure return, typically a full money refund is provided by the retailer who later gets a full credit of the wholesale price from the supplier and returns



the product. Meanwhile, we assume effort can be exerted to reduce the volume of false failure returns. Given the above discussions, it is reasonable to add another assumption that the supplier has already exerted all possible effort and all possible remaining effort as well as the associated cost is to be taken by the retailers. We show that the profit-maximizing effort level chosen by the retailer is always less than the one chosen from a supply chain's point of view. This is suboptimal because the supply chain profit is eventually to be split between retailer and supplier. The origin of this problem is that retailer bears all the cost yet only enjoys partial benefit of reducing amount of false failure returns. Hence here we aim to design a contract specifying a way for the supplier to share the cost by making a payment to the retailer, so as to induce the retailer to exert global optimal level of effort. In addition, we prove under certain conditions, such contract is also capable of improving both parties' profit simultaneously. Specifically, the contract payment in our proposed coordination contract is exponentially decreasing in the returned amount. This is very similar to the quantity discount contract, which is a coordination contract in forward supply chain that specifies a wholesale price decreasing in order quantity. Therefore we will refer to our contract as a quantity discount contract throughout the paper. It is reasonable to have contract payment decreasing in the number of the false failure returns for two reasons. First, it is hard to directly contract on effort level since it is very difficult to be verified. Second, a lower returns level is rather credibly indicating that retailer is exerting effort, and thus should be compensated more in this case. In order to give the explicit formula for determining contract parameters, we consider two cases of distribution of the number of false failure returns. In the first case, we consider the number to be geometrically distributed. Since such return is a result of consumer's individual decision rather than because of the quality of the product itself, hence they are independent among themselves. So the geometric distribution, which is memoryless, would be a reasonable distribution for this random variable. In the second case, the number of returns is assumed to have a Poisson distribution, which is more general. The results obtained here would be more accurate if sales volume is large or the period under consideration is long.

The rest of this paper is organized as follows. In Section II, we set up the model of the problem, and optimal choices of effort level by supply chain and retailer are shown respectively. The design of coordination contracts given different distributions of the number of false failure returns are presented in Section III. Section IV is devoted to a discussion of the contract and a possible modification of the quantity discount contract is presented. Concluding remarks are then given in Section V.

## II. THE MODEL

We study the performance of a supply chain consisting of one retailer and one supplier in a single-period setting using the model similar with that appeared in [2]. For every unit of product, the supplier bears the manufacturing  $\cos c$  and then

sells it to the retailer at wholesale price w. Finally the product is sold in the market at retail price p. It is assumed that the consumers, after the purchases, have the right to return the products to the retailer anytime within the period. In case of a false failure return, consumer gets a full refund of p from the retailer, who later gets a full credit of w from the supplier. Now we proceed to formulate the profit function of reducing one unit of false failure return: first, the return comes at a cost, such as goodwill cost and the possible re-processing cost. We denote the sum of all these costs to the supplier as s and the costs to the retailer as r. Moreover, after the return, the consumer may choose to walk away, with certain probability  $\delta_s$ , or to make a repurchase from the same supplier. Hence the value of loss on sale to the supplier can be written as  $\delta_s(w-c)$ . Similarly, the value of loss on sale to the retailer is  $\delta_r(p-w)$ , with  $\delta_r$  being the probability that no repurchase from the same retailer is made after the return. Combining these two kinds of costs saved, the profit of avoiding one false failure return is  $S = \delta_s(w-c) + s$  to the supplier and  $R = \delta_r(p-w) + r$ to the retailer. On the other hand, as in [2], number of false failure returns, which denoted as  $X(\rho)$ , is modeled to be a non-negative random variable depending on the effort level  $\rho$ . Denote its Cumulative Distribution Function (CDF) as  $F(x|\rho)$ and Probability Mass Function (PMF) as  $f(x|\rho)$ . It is assumed that  $\rho \geq 1$  (at least a minimal level of effort has to be exerted), and effort cost is

$$c(\rho) = \frac{a}{2}\rho^2$$

because quadratic function can approximate two features of effort cost: it is strictly increasing and has decreasing marginal return. Assume that

$$E\{X(\rho)\} = \frac{\beta}{\rho}$$
 where  $\beta = E\{X(\rho = 1)\}$ 

i.e.  $\beta$  is the number of false failure returns when minimal level of effort is in force. As in [2], we confine our attention to the impact of false failure returns on profit of the supply chain. The profit of supply chain as a whole can be valued as

$$\Pi(\rho) = (S+R)(E\{X(1)\} - E\{X(\rho)\}) - \frac{a}{2}\rho^2 = (S+R)\beta(1-\frac{1}{\rho}) - \frac{a}{2}\rho^2$$
(1)

Note that the first term is the product of expected profit of avoiding one false failure return and expected number of reduced units with effort  $\rho$ , which is just the expected total profit. Then cost is subtracted to give the net profit.

We want to find an effort level  $\rho$  to maximize the supply chain profit. To this end, we set  $0 = \frac{\partial \Pi(\rho)}{\partial \rho} = \frac{(S+R)\beta}{\rho^2} - a\rho$ , solving it yields

ŀ

$$\rho_C = \left(\frac{S+R}{a}\beta\right)^{1/3}.$$
 (2)

Second order condition is checked by:  $\frac{\partial^2 \Pi(\rho)}{\partial \rho^2} = -\frac{2(S+R)\beta}{\rho^3} - a < 0$ . Hence  $\Pi(\rho)$  is actually concave in  $\rho$  and the  $\rho_C$  above is the effort level such that the supply chain profit is maximized.  $\rho_C \ge 1$  is always assumed.

Note that since supply chain profit is the sum of profit earned by both retailer and supplier. So with any transfer payment between the two, results in this section still hold.

## A. Decentralized Case

Now we investigate the supply chain in absence of any contracts between the the retailer and supplier concerning false failure returns. As in [2] and has been justified above, all effort and associated cost of reducing returns is assumed to be taken by the retailer. Like the supply chain profit, retailer's profit can be calculated as

$$\Pi_R(\rho) = R\beta(1 - \frac{1}{\rho}) - \frac{1}{2}a\rho^2.$$
 (3)

We can easily verify that  $\rho = (\frac{R\beta}{a})^{1/3}$  is the maximal solution by solving  $\frac{\partial \Pi_R(\rho)}{\partial \rho} = \frac{R\beta}{\rho^2} - a\rho = 0$ , and check that  $\Pi_R(\rho)$  is concave on  $\rho$  because  $\frac{\partial^2 \Pi_R(\rho)}{\partial \rho^2} = -\frac{2R\beta}{\rho^3} - a < 0$ . But since retailer has to exert at least minimal amount of effort, hence the effort level chosen by retailer to maximize his/her own profit would be

$$\rho_D = \max\left\{ \left(\frac{R\beta}{a}\right)^{1/3}, 1 \right\}$$

It is obvious that  $\rho_C \ge \rho_D$ , which means that if without any coordination, the retailer always chooses an effort level that is lower than the supply chain optimum. This disagreement arises because the retailer pays all the cost but only gets less expected profit than the supply chain does. Some wealth is transferred to the supplier who is paying nothing. So it would be expected that in order to induce the retailer to exert supply chain optimal effort, the supplier should compensate the retailer for part of the cost.

And in the following discussions, the condition  $\rho_C = \left(\frac{S+R}{a}\beta\right)^{1/3} > 1$  is always assumed, because otherwise,  $\rho_C = \rho_D = 1$ , the supply chain is already in coordination.

Similar to the above, supplier's profit with effort level  $\rho$  would be

$$\Pi_S(\rho) = S\beta(1-\frac{1}{\rho}) \tag{4}$$

# III. THE QUANTITY DISCOUNT CONTRACT

In this section, we propose a quantity discount contract specifying a payment by supplier to retailer to induce the later to exert supply chain optimal level of effort and hence to achieve coordination. The payment amount is exponentially decreasing in the number of false failure returns. Specifically, the value of the payment is

$$P(x) = T\alpha^x \tag{5}$$

where x is the number of false failure returns; T is a contract parameter with value equals to the payment by supplier if there is no false failure return;  $\alpha$  is another contract parameter satisfying  $0 < \alpha < 1$ . Hence  $E\{P(x|\rho)\} = \sum_{x=0}^{\infty} (T\alpha^x) f(x|\rho)$ . With such contract, retailer's profit becomes

$$\Pi_R(\rho|(T,\alpha)) = R\beta(1-\frac{1}{\rho}) - \frac{1}{2}a\rho^2 + E\{P(x|\rho)\}$$
(6)

Similarly, supplier's profit changes to

$$\Pi_{S}(\rho|(T,\alpha)) = S\beta(1-\frac{1}{\rho}) - E\{P(x|\rho)\}$$
(7)

We examine how this contract works under two scenarios, one with  $X(\rho)$  following a geometric distribution, the other with  $X(\rho)$  following a Poisson distribution.

## A. The Geometric Distribution Case

In this section, the number of false failure returns is assumed to be following a geometric distribution. Specifically, consider

$$f(x|\rho) = \left(\frac{\rho}{\rho+\beta}\right) \left(\frac{\beta}{\rho+\beta}\right)^x$$
 with  $x = 0, 1, 2...$ 

It can be easily checked that  $E\{X(\rho)\} = \frac{\beta}{\rho}$  is satisfied here. The expected value of contract payment would be

$$E\{P(x|\rho)\} = \sum_{x=0}^{\infty} T\alpha^x \left(\frac{\rho}{\rho+\beta}\right) \left(\frac{\beta}{\rho+\beta}\right)^x = T\left(\frac{\rho}{\rho+k}\right)$$
(8)

with  $k = (1 - \alpha)\beta$ . The following proposition shows the form of the coordination contract.

Proposition 1: Assuming the number of false failure returns is geometrically distributed, then the quantity discount contract specifying a payment of  $T\alpha^x$  to the retailer with

$$T = \frac{aS(k+\rho_C)^2}{k(S+R)}\rho_C \quad \text{and} \quad 0 < \alpha < 1$$

can coordinate the supply chain. All proofs can be found in [6].

Now we give a qualitative analysis of  $T = \frac{aS(k+\rho_C)^2}{k(S+R)}\rho_C$ . Note that T increases in S, a and  $\beta$  yet decreases in R. The reason is that T measures the payment size paid by supplier to induce retailer to exert more effort to reach supply chain optimal level, hence if S is large, that means more benefit from the effort is transferred to supplier, so supplier should have to compensate the retailer more. But when R is large, retailers need less external incentive since they already benefit more from the reduction itself. However, more cost-sharing is needed when a is larger because it implies higher effort cost. T is also positively related to  $\beta$  because a larger  $\beta$  means that per unit effort exerted can bring down a larger number of false failure returns hence it becomes more urgent and supplier is willing to pay more for it.

*Proposition 2:* With the quantity discount contract described above, the retailer's profit is always increased while the supplier is better off if and only if

$$\frac{1}{\rho_D} - \frac{2}{\rho_C} > \frac{1}{\beta} \quad \text{and} \quad 0 < \alpha \le 1 - \frac{1}{\beta(\frac{1}{\rho_D} - \frac{2}{\rho_C})}.$$

To interpret the proposition, note that  $E\{P(x|\rho_C)\}$ , which is the payment born by the supplier in coordination, increases in  $\alpha$ . If  $\alpha$  is too large, the payment may outweigh the benefit of exerting effort, so it may actually damage the supplier's profit. Besides, it is of interest to compare the result here with the one in [2]. In that paper the condition for supplier to be better off is  $\frac{1}{\rho_D} - \frac{2}{\rho_C} > 0$ , which is equivalent to S > 7R. But here we require  $\frac{1}{\rho_C} - \frac{2}{\rho_C} > \frac{1}{\beta}$ , which implies S should be greater than an amount even larger than 7R. The reason is that, although [2] is applying a target rebate contract under the assumption that  $X(\rho)$  is normally distributed, which is different from our settings, it has been shown that in coordination  $E\{P(x|\rho_C)\} = \frac{S\beta}{\rho_C} + \frac{S\beta}{k}$ . Hence in comparison, we are actually asking the supplier to pay more to the retailer. So correspondingly, the profit of effort transferred to the supplier, which can be measured by S, should be larger here to guarantee the supplier's profit increment.

## B. The Poisson Distribution Case

In this section, the number of false failure returns is assumed to follow a Poisson distribution to model for a more general scenario. Specifically, we consider

$$f(x|\rho) = \frac{(\beta/\rho)^x}{x!} e^{-\beta/\rho} \quad \text{with} \quad x = 0, 1, 2 \dots$$

It can be easily checked that  $E\{X(\rho)\} = \frac{\beta}{\rho}$  holds as proposed. Expected value of contract payment is

$$E\{P(x|\rho)\} = \sum_{x=0}^{\infty} \frac{(\beta/\rho)^x}{x!} e^{-\beta/\rho} T \alpha^x = T e^{\frac{\beta}{\rho}(\alpha-1)}.$$
 (9)

We now aim to find a coordination contract under this distribution.

**Proposition 3:** When the number of false failure returns has a Poisson distribution, the quantity discount contract specifying a payment  $T\alpha^x$  by supplier can coordinate the supply chain with

$$T = \frac{S}{1-\alpha} e^{\frac{\beta}{\rho_C}(1-\alpha)} \quad \text{and} \quad \alpha \in (\max\{1 - \frac{3(S+R)}{S\beta}\rho_C, 0\}, 1).$$

Comparing the form of the quantity discount contract ensuring supply chain coordination in the last section with the one here, it can be seen that with geometrically distributed  $X(\rho)$ ,  $\alpha$  can be any number in (0, 1), while in the case of  $X(\rho)$  being a Poisson distributed random variable  $\alpha$  cannot be falling into too low a range under some conditions. Possible explanation for this may be that with the same mean  $\frac{\beta}{\rho_C}$ , geometrically distributed  $X(\rho)$  tends to cluster at a low range while Poisson distributed one is more dense around the mean, so given the payment amount is of the form  $T\alpha^x$ ,  $\alpha$  cannot be too small in the later case because the retailer gets small payments with higher probability than in the first case.

higher probability than in the first case. Take a close look into  $T = \frac{S}{1-\alpha} e^{\frac{\beta}{p_C}(1-\alpha)}$ , we conclude that the value of T increases in a and  $\beta$  while decreases in R. The analysis and the rationale are similar with the one in the last section.

**Proposition 4:** Under the assumption that  $X(\rho)$  has a Poisson distribution, if the contract specified in Proposition 3 is adopted, the retailer will be earning more if

$$\max\{0, 1 - \frac{2\rho_C(\rho_C^2 + \rho_C \rho_D + \rho_D^2)}{\beta(\rho_C^2 + \rho_C \rho_D - 2\rho_D^2)}\} < \alpha < 1 \quad \text{when} \quad \rho_D = (\frac{R\beta}{a})$$
$$\max\{0, 1 - \frac{2\rho_C}{\beta}\} < \alpha < 1 \quad \text{when} \quad \rho_D = 1$$

while the supplier will be better off if

$$0 < \alpha \le 1 - \frac{1}{\beta(\frac{1}{\rho_D} - \frac{1}{\rho_C})}$$

*Proposition 5:* To guarantee the contract in Proposition 3 is able to coordinate the supply chain and is also Pareto improving for supplier and retailer, it is required that

$$\max\{0, 1 - \frac{2\rho_C(\rho_C^2 + \rho_C \rho_D + \rho_D^2)}{\beta(\rho_C^2 + \rho_c \rho_D - 2\rho_D^2)}, 1 - \frac{3(S+R)}{S\beta}\rho_C\} < \alpha$$
$$\leq 1 - \frac{1}{\beta(\frac{1}{\rho_D} - \frac{1}{\rho_C})} \quad \text{when} \quad \rho_D = (\frac{R\beta}{a})^{1/3}.$$

$$\max\{0, 1 - \frac{2\rho_C}{\beta}, 1 - \frac{3(S+R)}{S\beta}\rho_C\} < \alpha \le 1 - \frac{1}{\beta(\frac{1}{\rho_D} - \frac{1}{\rho_C})}$$
  
when  $\rho_D = 1$ .

And to guarantee the existence of such  $\alpha$ , it is required that

$$\beta(\frac{1}{\rho_D} - \frac{1}{\rho_C}) > 1$$

holds in any case. Moreover, if  $\rho_D = 1$ , then the followings are also required

$$\rho_C \ge 3/2 \quad \text{and} \quad \frac{\beta}{\alpha} > \frac{(4S+3R)^3}{27(S+R)^4}.$$

# IV. DISCUSSION

In this section, we discuss the quantity discount contract designed above from the aspect of implementation as well as its impact on incentive when combined with a forward supply chain contract. We then discuss some modifications of the contract.

First, we believe that the quantity discount contract here is attractive to retailers because by the design of it, there will always be some payment to the retailer. And this payment is larger in size if the number of false failure returns is lower, hence it should be able to provide direct incentive for inducing effort.

#### A. Incentives to Encourage False Failure Returns

Now we proceed to see how the contract works in cooperation with forward supply chain contract in a closedloop supply chain context. As discussed in [2], when the supplier is providing a full credit, retailers sometimes have incentives to actually encourage sales even with awareness of the high probability that the products may end up being false failure returns. This actually works as a way to return their inventory back to the suppliers. For example, retailers may want to do so if they are selling something with poor demand. Note that when the contract is absent, profit of a false failure return includes the full credit of wholesale price w, and the cost would be the sum of goodwill cost, re-processing 1 cost of the returns, the salvage value of unsold inventory and possible refund from supplier for leftover inventory from forward supply chain contract (for example, buy-back contract or quantity flexibility contract). We sum these costs up and denote it as b. Retailer encourages returns whenever it is beneficial to do so, i.e., when w > b. However, since quantity discount contract gives the retailer a compensation which is higher for lower level of returns, returns is costing the retailer more, especially when returns level is high, denote this difference as d. Hence the condition of beneficially pushing false failure returns rises to w > b + d. Hence the contract reduces the incentive of encouraging false failure returns all the time.

## B. Modification of the Contract

It has been demonstrated that the quantity discount contract discussed above can coordinate a supply chain with false failure returns and is also Pareto improving in the sense that both retailer and supplier are better off with the contract. However, how the increased supply chain profit is eventually split between the two parties is not immediately clear. Here we introduce one possible way of modifying the original contract that results in arbitrary division of supply chain incremental profit between retailer and supplier by adding a fixed payment into the contract term.

To this end, define the followings:  $\Delta' \Pi_S = S\beta(\frac{1}{\rho_D} - \frac{1}{\rho_C}),$   $\Delta' \Pi_R = R\beta(\frac{1}{\rho_D} - \frac{1}{\rho_C}) - \frac{1}{2}a(\rho_c^2 - \rho_D^2)$  and  $\Delta \Pi = (R + S)\beta(\frac{1}{\rho_D} - \frac{1}{\rho_C}) - \frac{1}{2}a(\rho_c^2 - \rho_D^2).$ *Proposition 6:* A contract specifies a transfer payment with

Proposition 6: A contract specifies a transfer payment with an amount of  $T\alpha^x - Q$  by supplier to retailer with  $(T, \alpha)$ as in Proposition1 when  $X(\rho)$  is geometrically distributed or with  $(T, \alpha)$  as in Proposition 3 when  $X(\rho)$  has a Poisson distribution, and

$$Q = \theta \Delta' \Pi_R - (1 - \theta) \Delta' \Pi_S + E\{P(x|\rho_C)\}$$

is a coordination contract for the supply chain with  $0 \le \theta \le 1$ . Moreover, this contract helps to allocate a portion  $\theta$  of increased supply chain profit to supplier and  $(1 - \theta)\Delta\Pi$  to retailer and hence guarantees the existence of Pareto improvement for retailer and supplier.

## V. CONCLUSION

In this paper, we design a quantity discount contract to resolve profit conflicts arising in a reverse supply chain. Here we focus exclusively on cost and revenue result from exerting effort to reduce false failure returns, which refers to customer returns with no functional defects of the products. Since from a supply chain's point of view, reducing such returns is highly beneficial because substantial goodwill cost and re-processing cost can be saved. But to the retailer, incentive of doing so may be distorted by having to pay all the cost of effort while enjoying a full credit provided by the supplier in case of a return. As a consequence, retailer always chooses an effort level that is lower than the supply chain's optimum. To tackle this problem, we propose to apply a quantity discount contract so as to induce retailer to exert more effort. Specifically, the payment specified by the contract is exponentially decreasing in the number of false failure returns. We give explicit formulations of the contracts under different assumptions about the distribution of the number of false failure returns. The distribution is assumed to be geometric in the first case and Poisson in the second case. It is proved that the contract is able to successfully coordinate the supply chain, inducing retailer to choose the desired effort level. We have also proved that with the contract parameters properly set, retailer's and supplier's profits are both increased at the same time. It is also demonstrated that the contract has the function to deter retailer from pushing false failure returns in the forward supply chain. In addition, we show how the contract may be modified so that it leads to easy allocation of the incremental profit within the supply chain.

The study of reverse supply chain is attracting more and more attention. Since coordination contracts in forward supply chain have already been studied in depth and there are a lot of available results, so in spite of the fact that most results from forward supply chain cannot be directly applied in the new context, they would serve as valuable references. The design of the quantity discount contract here is an example. We suggest this may be a further research direction. Some other research directions may also include to generalize our results in a more general setting, for example, to see how the quantity discount contract may work with different return policies offered by retailer to customers or to consider how to modify the contract to be applied in a multiple-period setting etc.

### ACKNOWLEDGMENT

Research supported in part by Hong Kong GRF Grant No. 7017/07P, HKU CRCG Grantsand HKU Strategic Research Theme Fund on Computational Sciences.

## REFERENCES

- G. P. Cachon, Supply Chain Coordination with Contracts. In: S. Graves, T. de Kok, eds. Handbooks in Operations Research and Management Science: Supply Chain Management. North-Holland, Amsterdam, The Netherlands, 2003.
- [2] M. Ferguson, V. D. R. Jr. Guide, G. Souza, *Supply Chain Coordination for False Failure Returns*. Manufacturing and Service Operations Management. Vol 4, pp376-393, 2006.
- [3] V. D. R. Jr. Guide, T. Harrison, and L. N. Van Wassenhove. *The Challenge of Closed-loop Supply Chains*. Interfaces Vol. 33, Issue 6, pp3-6, 2003.
- [4] V. D. R. Jr. Guide and L. N. Van Wassenhove, eds. Business Aspects of Closed-Loop Supply Chains. Carnegie Mellon University Press, Pittsburgh, PA. 2003.
- [5] V. D. R. Jr. Guide, G. Souza, L. Van Wassenhove and J. Blackburn. *Time Value of Commercial Product Returns*. Management Science Vol. 52, Issue 8, 2006.
- [6] X. Huang, S. Choi and W. Ching, Quantity Discount Contract for Supply Chain Coordination with False Failure Returns. Working paper, http://hkumath.hku.hk/~wkc/papers/false-failure-returns.pdf. 2010.
- [7] C. Lawton, *The War on Returns*. Wall Street Journal. Page D1, May 8, 2008.
- [8] J. Stock, T. Speh and H. Shear, *Many Happy(product) Returns*. Harvard Bus. Rev. Vol. 80, Issue 7, pp16-17, 2002.
- [9] X. Su, Consumer Returns Policies and Supply Chain Performance. Manufacturing and Service Operations Management, Vol 11, Issue 4, pp595-612, 2009.