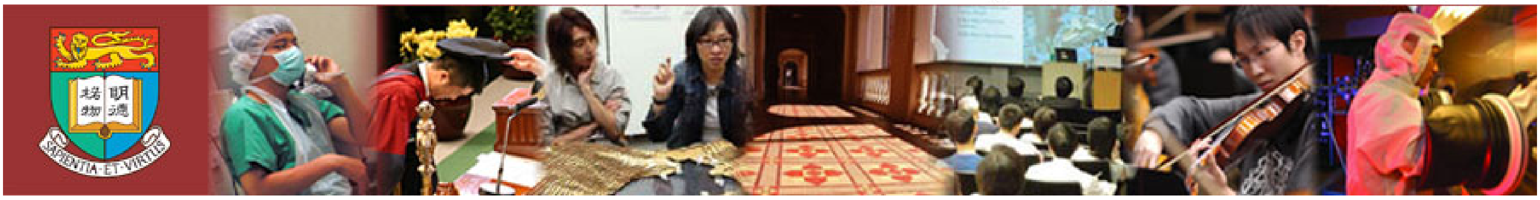

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Joint CFO and Channel Estimation for ZP-OFDM Modulated Two-Way Relay Networks

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***Abstract*—In this paper, we study the problem of joint carrier frequency offset (CFO) and channel estimation for two-way relay network (TWRN). We consider the frequency selective fading channels and adopt the zero padding (ZP) based orthogonal frequency division multiplexing (OFDM) as the modulation of the transmission. Due to the mixture of the first and the second transmission phases, the joint estimation problem becomes much challenging than that in the traditional point-to-point communication systems. By introducing some redundancy, we modify the structure of ZP-OFDM to cope with non-zero frequency synchronization errors. We then propose a nulling-based least square (NLS) method for joint CFO and channel estimation. A detailed performance analysis of NLS has been conducted, where we prove that the unbiasedness of NLS and derive the closed-form estimation mean-square-error (MSE) at high signal-to-noise ratio (SNR). Finally, simulations are provided to corroborate the proposed studies.**

***Index Terms*—Carrier frequency offset, channel estimation, two-way relay network, OFDM, performance analysis.**

I. Introduction

Research on two-way relay network (TWRN) has become popular since it has been recently reported in [1], [2] that the overall communication rate between two source terminals in TWRN is approximately twice of that achieved in one-way relay network (OWRN) [3]. In TWRN, the relay treat the received signals in a "network coding"-like manner [4], and the terminals can recover the signal collision since they know their own transmitted signals. As a result, TWRN can improve spectrum efficiency, which makes it particularly attractive to any bidirectional systems.

The capacity analysis and the achievable rate region for *amplify-and-forward* (AF) and *decode-and-forward* (DF) based TWRN are explored in [5], [6]. In [7] the optimal mapping function at the relay node that minimizes the transmission bit-error rate (BER) was proposed while in [8], the distributed space-time code (STC) was designed for both AF and DF TWRN. Moreover, the optimal beamforming at the multi-antenna relay that maximizes the capacity of AF-based TWRN was developed in [9] and the suboptimal resource allocation in an orthogonal frequency division multiplexing (OFDM) based TWRN was derived in [10].

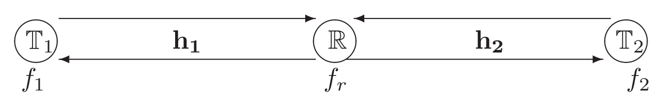

Fig. 1. System configuration for two-way relay network.

However, most existing works [5]– [10] assumed perfect synchronization and channel state information (CSI) at the relay node and/or the source terminals, which necessitates accuracy channel estimation and synchronization techniques. In this paper, we study the joint carrier frequency offset (CFO) and channel estimation for a classical TWRN with two source terminal nodes and one relay node. To make a general discussion, we consider frequency selective channels and adopt zero-padding (ZP) based OFDM to release the inter-block interference (IBI). Due to the mixture of the first and the second transmission phases, we introduce some redundancies and adapt ZP-OFDM into TWRN framework under the non-zero CFO values. This scheme can greatly facilitates the estimation process and we propose a nulling-based least square (NLS) method for the joint estimation. Moreover, we conduct the performance analysis for NLS and prove that NLS is an unbiased joint estimator at high signal-to-noise ratio (SNR). The closed-form estimation mean-square-error (MSE) is also derived.

II. ZP based OFDM for TWRN

Consider a two-way relay network (TWRN) with two terminal nodes $\mathbb{T}_1$ and $\mathbb{T}_2$, and one relay node $\mathbb{R}$, as shown in Fig. 1. Each node has one antenna that cannot transmit and receive simultaneously. The channels between $\mathbb{T}_j$ and $\mathbb{R}$ is denoted as $\mathbf{h}_j = [h_{j,0}, \ldots, h_{j,L}]^T$ and both lengths are assumed as $L+1$. The OFDM block length is set as N. Furthermore, denote the carrier frequency of $\mathbb{T}_j$ as f_j and that of $\mathbb{R}$ as f_r.

A. OFDM modulation at terminals

Without loss of generality, we omit the block index and denote one OFDM block from $\mathbb{T}_i$ as $\tilde{\mathbf{s}}_i = [\tilde{s}_{i,0}, \ldots, \tilde{s}_{i,N-1}]^T$. The corresponding time-domain signal block is obtained from

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the normalized inverse discrete Fourier transformation (IDFT) as

$$\mathbf{s}_i = \mathbf{F}^H \tilde{\mathbf{s}}_i = [s_{i,0}, s_{i,1}, \ldots, s_{i,N-1}]^T, \tag{1}$$

where $\mathbf{F}$ is the normalized DFT matrix with the (p,q)-th entry given by $\frac{1}{\sqrt{N}} e^{-j2\pi(p-1)(q-1)/N}$. To avoid IBI in the first transmission phase, L zeros is padded at the end of $\mathbf{s}_i$.

In Phase I, $\mathbb{T}_1$ and $\mathbb{T}_2$ up-convert the baseband signals by the carriers $e^{j2\pi f_i t}$ and send them to $\mathbb{R}$ simultaneously.[1]

B. Relay processing

The relay $\mathbb{R}$ will down-convert the passband signal by $e^{-j2\pi f_r t}$ and obtain the baseband signal

$$\mathbf{r}_{zp} = \sum_{i=1}^{2} \mathbf{\Gamma}^{(N+L)}[f_i - f_r] \mathbf{H}_{zp}^{(N)}[\mathbf{h}_i]\mathbf{s}_i + \mathbf{n}_r, \tag{2}$$

where

$$\mathbf{\Gamma}^{(K)}[f] = \mathrm{diag}\{1, e^{j2\pi f T_s}, \ldots, e^{j2\pi f (K-1) T_s}\} \tag{3}$$

with T_s representing the sampling period, and

$$\mathbf{H}_{zp}^{(K)}[\mathbf{x}] \triangleq \underbrace{\begin{bmatrix} x_0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ x_P & \ddots & x_0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_P \end{bmatrix}}_{K \text{ columns}} \tag{4}$$

for any vector $\mathbf{x} = [x_0, x_1, \ldots, x_P]^T$. Moreover, $\mathbf{n}_r$ is the $(N+L) \times 1$ noise vector, each entry having the variance σ_n^2.

Next, $\mathbb{R}$ adds L zeros to the end of $\mathbf{r}$ and scales it by the factor of

$$\alpha_{zp} = \sqrt{\frac{(N+L)P_r}{\mathrm{E}\{\|\mathbf{r}_{zp}\|^2\}}} = \sqrt{\frac{P_r}{\sum_{i=1}^{2}\sum_{l=0}^{L} \sigma_{i,l}^2 P_i + \sigma_n^2}}$$

to keep the average power constraint. Then $\alpha_{zp}\mathbf{r}_{zp}$ will be up-converted to passband by $e^{j2\pi f_r t}$.

C. Signal reformulation at terminals

Due to symmetry, we only look into $\mathbb{T}_1$ during the second phase. After down-converting the passband signal by $e^{-j2\pi f_1 t}$, the $(N+2L) \times 1$ signal vector is expressed as:

$$\begin{aligned} \mathbf{y}_{zp} =& \alpha_{zp} \mathbf{\Gamma}^{(N+2L)}[f_r - f_1] \mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1]\mathbf{r}_{zp} + \mathbf{n}_1 \\ =& \alpha_{zp} \mathbf{\Gamma}^{(N+2L)}[f_r - f_1] \mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1] \\ & \times \left(\sum_{j=1}^{2} \mathbf{\Gamma}^{(N+L)}[f_i - f_r] \mathbf{H}_{zp}^{(N)}[\mathbf{h}_i]\mathbf{s}_i \right) \\ & \underbrace{+ \alpha_{zp} \mathbf{\Gamma}^{(N+2L)}[f_r - f_1] \mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1]\mathbf{n}_r + \mathbf{n}_1}_{\mathbf{n}_e}, \end{aligned} \tag{5}$$

[1]Note that the oscillator may have initial phase but it is omitted for brevity since the constant phase can be absorbed into the channel effects.

where $\mathbf{n}_1$ is the $(N+2L) \times 1$ noise vector at $\mathbb{T}_1$ with the variance σ_n^2, and $\mathbf{n}_e$ defines the overall noise component. The covariance of $\mathbf{n}_e$ is computed as

$$\begin{aligned} \mathbf{R}_{zp} =& \sigma_n^2 \Big(\alpha_{zp}^2 \mathbf{\Gamma}^{(N+2L)}[f_r - f_1] \mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1] (\mathbf{H}_{zp}^{(N+L)}[\mathbf{h}_1])^H \\ & \times (\mathbf{\Gamma}^{(N+2L)}[f_r - f_1])^H + \mathbf{I} \Big). \end{aligned} \tag{6}$$

In most practical communications, N is much larger than L. Then the following approximation can be made:

$$\mathbf{R}_{zp} \approx \sigma_n^2 \left(\alpha_{zp}^2 \sum_{l=0}^{L} \sigma_{h_{1,l}}^2 + 1 \right) \mathbf{I}. \tag{7}$$

Before we proceed, let us look at the following lemma:

Lemma 1: The following two equalities hold for any $\mathbf{\Gamma}^{(\cdot)}[f]$ in (3) and $\mathbf{H}_{zp}^{(\cdot)}[\mathbf{x}]$ in (4), where $(\cdot)$ represents the appropriate dimensions:

$$\mathbf{H}_{zp}^{(K)}[\mathbf{x}]\, \mathbf{\Gamma}^{(K)}[f] = \mathbf{\Gamma}^{(K+P)}[f] \mathbf{H}_{zp}^{(K)} \left[\mathbf{\Gamma}^{(K)}[-f]\mathbf{x} \right], \tag{8}$$

and reversely

$$\mathbf{\Gamma}^{(K+P)}[f] \mathbf{H}_{zp}^{(K)}[\mathbf{x}] = \mathbf{H}_{zp}^{(K)} \left[\mathbf{\Gamma}^{(P+1)}[f]\mathbf{x} \right] \mathbf{\Gamma}^{(K)}[f]. \tag{9}$$

Proof: Proved from the straightforward computation. ■

Lemma 1 says that, it is possible to switch $\mathbf{\Gamma}^{(\cdot)}[f]$ from the right (left) side of $\mathbf{H}_{zp}^{(\cdot)}[\mathbf{h}_i]$ to the left (right) side by changing the dimension of $\mathbf{\Gamma}^{(\cdot)}[f]$ and rotating $\mathbf{h}_i$.

From Lemma 1, $\mathbf{y}_{zp}$ can be rewritten as

$$\begin{aligned} \mathbf{y}_{zp} =& \alpha_{zp} \mathbf{H}_{zp}^{(N+L)}[\mathbf{\Gamma}^{(L+1)}[f_r - f_1]\mathbf{h}_1] \mathbf{H}_{zp}^{(N)}[\mathbf{h}_1]\mathbf{s}_1 + \mathbf{n}_e \\ & + \alpha_{zp} \mathbf{\Gamma}^{(N+2L)}[f_2 - f_1] \mathbf{H}_{zp}^{(N+L)}[\mathbf{\Gamma}^{(L+1)}[f_r - f_2]\mathbf{h}_1] \\ & \times \mathbf{H}_{zp}^{(N)}[\mathbf{h}_2]\mathbf{s}_2. \end{aligned} \tag{10}$$

We further note that

$$\mathbf{H}_{zp}^{(N+L)}[\mathbf{x}_1] \mathbf{H}_{zp}^{(N)}[\mathbf{x}_2] = \mathbf{H}_{zp}^{(N)}[\mathbf{x}_1 \otimes \mathbf{x}_2] \tag{11}$$

where $\otimes$ denotes the linear convolution between the two vectors. Hence $\mathbf{y}_{zp}$ is finally written as

$$\begin{aligned} \mathbf{y}_{zp} =& \alpha_{zp} \mathbf{H}_{zp}^{(N)} \Big[\underbrace{(\mathbf{\Gamma}^{(L+1)}[f_r - f_1]\mathbf{h}_1) \otimes \mathbf{h}_1}_{\mathbf{a}_{zp}} \Big] \mathbf{s}_1 + \mathbf{n}_e \\ & + \alpha_{zp} \mathbf{\Gamma}^{(N+2L)}[\underbrace{f_2 - f_1}_{v}] \mathbf{H}_{zp}^{(N)} \Big[\underbrace{(\mathbf{\Gamma}^{(L+1)}[f_r - f_2]\mathbf{h}_1) \otimes \mathbf{h}_2}_{\mathbf{b}_{zp}} \Big] \mathbf{s}_2, \end{aligned} \tag{12}$$

where $\mathbf{a}_{zp}$, $\mathbf{b}_{zp}$ are the $(2L+1) \times 1$ equivalent channel vectors and v is the equivalent CFO.

D. Data detection at terminals

If the cascaded channel $\mathbf{a}_{zp}$ is known to $\mathbb{T}_1$, then the first term on the right-hand side (RHS) of (12) can be removed since $\mathbb{T}_1$ knows its own signal $\mathbf{s}_1$. If the CFO v is also known, then $\mathbf{\Gamma}^{(N+2L)}[v]$ can be compensated and the remaining signal is

$$\mathbf{z}_{zp} = \alpha_{zp} \mathbf{H}_{zp}^{(N)}[\mathbf{b}_{zp}]\mathbf{s}_2 + \mathbf{n}_e. \tag{13}$$

Moreover, since $\mathbf{H}_{zp}^{(N)}[\mathbf{b}_{zp}]$ is the $(N+2L)\times N$ Toepliz matrix following the structure in (4), we can add the last $2L$ elements of $\mathbf{z}_{zp}$ to its first $2L$ elements [11] and obtain

$$\mathbf{w}_{zp} = \alpha_{zp}\mathbf{H}_{cp}^{(N)}[\mathbf{b}_{zp}]\mathbf{s}_2 + \tilde{\mathbf{n}}_e, \tag{14}$$

where $\mathbf{H}_{cp}^{(N)}[\mathbf{b}_{zp}]$ is the $N\times N$ circulant matrix with the first column $[\mathbf{b}_{zp}^T, \mathbf{0}_{1\times(N-2L-1)}]^T$, and $\tilde{\mathbf{n}}_e$ is the resultant noise vector. As long as $\mathbf{b}_{zp}$ is known, the regular OFDM detection can be efficiently performed from fast Fourier Transform (FFT).

E. Joint CFO and channel estimation

Clearly, the task of joint CFO and channel estimation is to estimate $\mathbf{a}_{zp}$, $\mathbf{b}_{zp}$, and v. Assume now $\mathbf{s}_1$ and $\mathbf{s}_2$ as the training blocks, we can rewrite (12) as :

$$\mathbf{y}_{zp} = \mathbf{S}_1^{(N+2L)}\mathbf{a}_{zp} + \mathbf{\Gamma}^{(N+2L)}[v]\mathbf{S}_2^{(N+2L)}\mathbf{b}_{zp} + \mathbf{n}_e, \tag{15}$$

where $\mathbf{S}_i^{(N+2L)}$ is the $(N+2L)\times(2L+1)$ circulant matrix with the first column $[\alpha_{zp}\mathbf{s}_i^T, \mathbf{0}_{1\times 2L}^T]^T$. Obviously, (15) is different from that in the conventional work [12] in that only part of the signal component is accompanied with the CFO matrix.

III. Nulling-based Least Square Estimation

For simplification, we can omit both the superscript and the subscript of (15). We then obtain

$$\mathbf{y} = \mathbf{S}_1\mathbf{a} + \mathbf{\Gamma}\mathbf{S}_2\mathbf{b} + \mathbf{n}_e. \tag{16}$$

Since $\mathbf{S}_1$ is a tall matrix, it is possible to find a matrix $\mathbf{J}$ such that $\mathbf{J}^H\mathbf{S}_1 = \mathbf{0}$. We propose to select $\mathbf{J}$ with the property that $\mathbf{J}^H\mathbf{J} = \mathbf{I}$, since it has the best condition number. A simple choice of $\mathbf{J}$ is the basis of the orthogonal complement space of $\mathbf{S}_1$.

Left-multiplying $\mathbf{y}$ by $\mathbf{J}^H$ gives

$$\mathbf{J}^H\mathbf{y} = \mathbf{0} + \underbrace{\mathbf{J}^H\mathbf{\Gamma}\mathbf{S}_2}_{\mathbf{G}}\mathbf{b} + \underbrace{\mathbf{J}^H\mathbf{n}_e}_{\mathbf{n}}, \tag{17}$$

where $\mathbf{G}$ and $\mathbf{n}$ are defined as the corresponding items. From the property of $\mathbf{J}$, we know the statistics of $\mathbf{n}$ remains the same as that of $\mathbf{n}_e$ if the latter is approximated as white Gaussian.

The least square (LS) estimate of $\mathbf{b}$ can be immediately found from (17) as:

$$\hat{\mathbf{b}} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{J}^H\mathbf{y}. \tag{18}$$

Similar as before, CFO is estimated from

$$\hat{v} = \arg\max_v \ \mathbf{y}^H\mathbf{J}\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{J}^H\mathbf{y}, \tag{19}$$

and $\hat{\mathbf{b}}$ is obtained from (18). Finally, the LS estimation of channel $\mathbf{a}$ is obtained from

$$\hat{\mathbf{a}} = (\mathbf{S}_1^H\mathbf{S}_1)^{-1}\mathbf{S}_1^H(\mathbf{y} - \hat{\mathbf{\Gamma}}\mathbf{S}_2\hat{\mathbf{b}}). \tag{20}$$

IV. Performance Analysis

Due to the nulling process, the estimation model (17) is complicated than that in [12] in the sense that $\mathbf{\Gamma}$ stays between two matrices. Hence, the performance analysis in [12] cannot be directly extended to our considered scenario. Based on the perturbation theory, we will prove that NLS is an unbiased estimator and derive the closed-form expression of MSEs at high SNR.

Let v_0 and $\hat{v}_0$ be the true and the estimated CFO, respectively. For notation simplicity, we denote

$$\mathbf{y}_n = \mathbf{J}^H\mathbf{y}, \qquad \mathbf{P}_G = \mathbf{G}(\underbrace{\mathbf{G}^H\mathbf{G}}_{\mathbf{\Phi}})^{-1}\mathbf{G}^H, \tag{21}$$

where $\mathbf{\Phi}$ represents the corresponding item. The NLS estimator (19) can be written as:

$$\hat{v}_0 = \arg\max_v g(v) = \arg\max_v \mathbf{y}_n^H\mathbf{P}_G\mathbf{y}_n. \tag{22}$$

Lemma 2: At high SNR, the perturbation of the CFO estimated from (22) can be approximated by

$$\Delta v \triangleq \hat{v}_0 - v_0 \approx -\frac{\dot{g}(v_0)}{\mathrm{E}\{\ddot{g}(v_0)\}}. \tag{23}$$

Proof: From [13], we know that

$$\Delta v \approx -\frac{\frac{\partial g(v)}{\partial v}|_{v=v_0}}{\frac{\partial^2 g(v)}{\partial v^2}|_{v=v_0}} = -\frac{\dot{g}(v_0)}{\ddot{g}(v_0)}. \tag{24}$$

The first order derivative of $\mathbf{G}$ can be calculated as

$$\dot{\mathbf{G}} = \frac{\partial \mathbf{G}}{\partial v} = j\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2. \tag{25}$$

Applying the equation

$$\frac{\partial \mathbf{\Phi}^{-1}}{\partial v} = -\mathbf{\Phi}^{-1}(\dot{\mathbf{G}}^H\mathbf{G} + \mathbf{G}^H\dot{\mathbf{G}})\mathbf{\Phi}^{-1}, \tag{26}$$

we can get:

$$\begin{aligned}\dot{g}(v) =& \mathbf{y}_n^H\dot{\mathbf{P}}_G\mathbf{y}_n = \underbrace{\mathbf{y}_n^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n}_{M_1} + \underbrace{\mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{y}_n}_{M_2} \\ &-\underbrace{(\mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n + \mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n)}_{M_3}.\end{aligned} \tag{27}$$

The mean value of each item in (27) can be computed as

$$\begin{aligned}\mathrm{E}(M_1(v_0)) =& j\mathrm{E}[\mathbf{b}^H\mathbf{G}^H\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{b}] \\ &+ j\mathrm{E}[\mathbf{n}^H\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{n}]\end{aligned} \tag{28}$$

$$\begin{aligned}\mathrm{E}(M_2(v_0)) =& -j\mathrm{E}[\mathbf{b}^H\mathbf{S}_2^H\mathbf{D}\mathbf{\Gamma}^H\mathbf{J}\mathbf{G}\mathbf{b}] \\ &- j\mathrm{E}[\mathbf{n}^H\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{S}_2^H\mathbf{D}\mathbf{\Gamma}^H\mathbf{J}\mathbf{n}]\end{aligned} \tag{29}$$

$$\begin{aligned}\mathrm{E}(M_3(v_0)) =& \mathrm{E}[\mathbf{b}^H(-j\mathbf{S}_2\mathbf{D}\mathbf{\Gamma}^H\mathbf{J}\mathbf{G} + j\mathbf{G}^H\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2)\mathbf{b}] \\ &+ \mathrm{E}[\mathbf{n}^H\mathbf{G}\mathbf{\Phi}^{-1}(-j\mathbf{S}_2\mathbf{D}\mathbf{\Gamma}^H\mathbf{J}\mathbf{G} + j\mathbf{G}^H\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2)\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{n}]\end{aligned} \tag{30}$$

Then combining (28), (29), (30) and (27), we can get (31) shown on the top of the next page, where $\Im\{\cdot\}$ denote the imaginary component.

$$\begin{aligned}\mathrm{E}[\dot{g}(v_0)] =& j\mathrm{E}[\mathbf{n}^H(\mathbf{I}-\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H)\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{n}] - j\mathrm{E}[\mathbf{n}^H\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{S}_2^H\mathbf{D}\mathbf{\Gamma}^H\mathbf{J}(\mathbf{I}-\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H)\mathbf{n}] \\ =& -2\sigma_{ne}^2\Im\{tr(\underbrace{\mathbf{G}^H(\mathbf{I}-\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H)}_{\mathbf{0}}\mathbf{J}^H\mathbf{D}\mathbf{\Gamma}\mathbf{S}_2\mathbf{\Phi}^{-1})\} = 0. \end{aligned} \tag{31}$$

In order to get $\ddot{g}(v_0)$, we further compute the first-order derivative of M_1, M_2, and M_3 as :

$$\begin{aligned}\dot{M}_1 =& \mathbf{y}_n^H\ddot{\mathbf{G}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n - \mathbf{y}_n^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n \\ &+ \mathbf{y}_n^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{y}_n \end{aligned}\tag{32}$$

$$\begin{aligned}\dot{M}_2 =& \mathbf{y}_n^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{y}_n - \mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{y}_n \\ &+ \mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\ddot{\mathbf{G}}^H\mathbf{y}_n \end{aligned}\tag{33}$$

$$\begin{aligned}\dot{M}_3 =& \mathbf{y}_n^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n \qquad (34)\\ &- \mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n + \mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\ddot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n \\ &- \mathbf{y}_n^H\mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{y}_n + \mathbf{y}_n^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{y}_n. \end{aligned}$$

Thus we can obtain

$$\begin{aligned}\ddot{g}(v_0) =& \mathbf{b}^H\mathbf{G}^H\ddot{\mathbf{G}}\mathbf{b} + \mathbf{n}^H\ddot{\mathbf{G}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{n} + \mathbf{b}^H\ddot{\mathbf{G}}\mathbf{G}^H\mathbf{b} \\ &+ \mathbf{n}^H\mathbf{G}\mathbf{\Phi}^{-1}\ddot{\mathbf{G}}^H\mathbf{n} + 2\mathbf{b}^H\mathbf{G}^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{G}\mathbf{b} \\ &+ 2\mathbf{n}^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{n} - 2\mathbf{b}^H\mathbf{G}^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{b} \\ &- 2\mathbf{n}^H\dot{\mathbf{G}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{n} - 2\mathbf{b}^H\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{G}\mathbf{b} \\ &- 2\mathbf{n}^H\mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H\mathbf{n} - \mathbf{b}^H\ddot{\mathbf{\Phi}}\mathbf{b} - \mathbf{n}^H\mathbf{G}\mathbf{\Phi}^{-1}\ddot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{n} \\ &+ 2\mathbf{b}^H\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{b} + 2\mathbf{n}^H\mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H\mathbf{n}. \end{aligned}\tag{35}$$

After some tedious simplification, it can be obtained that:

$$\mathrm{E}[\ddot{g}(v_0)] = 2\mathbf{b}^H\dot{\mathbf{G}}^H(\mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H - \mathbf{I})\dot{\mathbf{G}}\mathbf{b} \tag{36}$$

and $\ddot{g}(v_0)$ can be expressed as

$$\ddot{g}(v_0) = \mathrm{E}\{\ddot{g}(v_0)\} + \mathcal{O}_2(\mathbf{n}) + \mathcal{O}_2(\mathbf{n}^2), \tag{37}$$

where $\mathcal{O}_2(\mathbf{n})$ and $\mathcal{O}_2(\mathbf{n}^2)$ represent the linear and quadrature functions of $\mathbf{n}$ in $\ddot{g}(v_0)$, whose explicit forms are omitted for brevity.

Similarly, $\dot{g}(v_0)$ can be expressed as

$$\dot{g}(v_0) = \mathcal{O}_1(\mathbf{n}) + \mathcal{O}_1(\mathbf{n}^2), \tag{38}$$

where $\mathcal{O}_1(\mathbf{n})$ and $\mathcal{O}_1(\mathbf{n}^2)$ represent the linear and quadrature functions of $\mathbf{n}$ existing in $\dot{g}(v_0)$. Substituting (38) and (37) into (24) gives

$$\begin{aligned}\Delta v \approx& -\frac{\mathcal{O}_1(\mathbf{n}) + \mathcal{O}_1(\mathbf{n}^2)}{\mathrm{E}\{\ddot{g}(v_0)\} + \mathcal{O}_2(\mathbf{n}) + \mathcal{O}_2(\mathbf{n}^2)} \\ \approx& -\frac{\mathcal{O}_1(\mathbf{n}) + \mathcal{O}_1(\mathbf{n}^2)}{\mathrm{E}\{\ddot{g}(v_0)\}} = -\frac{\dot{g}(v_0)}{\mathrm{E}\{\ddot{g}(v_0)\}}. \end{aligned}\tag{39}$$

■

We then the following theorems can be derived.

Theorem 1: The NLS estimation of CFO is unbiased.

Proof: From (31) and Lemma 2, we can obtain $\mathrm{E}[\Delta v] = 0$. So NLS is an unbiased estimator. ■

Theorem 2: The MSE of the CFO estimation is

$$\mathrm{E}\{\Delta v^2\} = \frac{\sigma_{ne}^2}{2\mathbf{b}^H\dot{\mathbf{G}}^H[\mathbf{I} - \mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H]\dot{\mathbf{G}}\mathbf{b}}. \tag{40}$$

Proof: According to Lemma 2, the MSE of the CFO estimation is

$$\mathrm{E}\{\Delta v^2\} = \frac{\mathrm{E}\{\dot{g}(v_0)^2\}}{\mathrm{E}\{\ddot{g}(v_0)\}^2}. \tag{41}$$

The numerator can be computed as

$$\begin{aligned}\mathrm{E}[\dot{g}(v)^2] =& \sigma_{ne}^2\mathrm{E}[\mathbf{n}^H\dot{\mathbf{P}}_G\dot{\mathbf{P}}_G\mathbf{n}] + \sigma_{ne}^2\mathrm{E}[\mathbf{b}^H\mathbf{G}^H\dot{\mathbf{P}}_G\dot{\mathbf{P}}_G\mathbf{G}\mathbf{b}] \\ &+ \mathrm{E}[\mathbf{n}^H\dot{\mathbf{P}}_G\mathbf{G}\mathbf{b}\mathbf{b}^H\mathbf{G}^H\dot{\mathbf{P}}_G\mathbf{n}] \\ &+ \mathrm{E}[\mathbf{b}^H\mathbf{G}^H\dot{\mathbf{P}}_G\mathbf{G}\mathbf{b}\mathbf{b}^H\mathbf{G}^H\dot{\mathbf{P}}_G\mathbf{G}\mathbf{b}], \end{aligned}\tag{42}$$

where

$$\dot{\mathbf{P}}_G = \dot{\mathbf{G}}\mathbf{\Phi}^{-1}\mathbf{G}^H + \mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{G}}^H - \mathbf{G}\mathbf{\Phi}^{-1}\dot{\mathbf{\Phi}}\mathbf{\Phi}^{-1}\mathbf{G}^H. \tag{43}$$

At high SNR, the first term in (42) can be neglected, and the last term is 0 because

$$\mathbf{G}^H\dot{\mathbf{P}}_G\mathbf{G} = \mathbf{0}. \tag{44}$$

Moreover, the second and the third term are the same. The following equality can be proved after some tedious computation:

$$\mathbf{G}^H\dot{\mathbf{P}}_G\dot{\mathbf{P}}_G\mathbf{G} = \dot{\mathbf{G}}^H[\mathbf{I} - \mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H]\dot{\mathbf{G}}. \tag{45}$$

Therefore, (42) can be rewritten as

$$\mathrm{E}[\dot{g}(v)^2] = 2\sigma_{ne}^2\mathbf{b}^H\dot{\mathbf{G}}^H[\mathbf{I} - \mathbf{G}\mathbf{\Phi}^{-1}\mathbf{G}^H]\dot{\mathbf{G}}\mathbf{b}. \tag{46}$$

Substituting (36) and (46) into (41), we proved Theorem 2. ■

Theorem 3: The channel estimation $\hat{\mathbf{b}}$ is unbiased and its MSE is

$$\begin{aligned}\mathrm{MSE}\{\mathbf{b}\} =& (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\dot{\mathbf{G}}\mathbf{b}\mathbf{b}^H\dot{\mathbf{G}}^H\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathrm{E}\{\Delta v^2\} \\ &+ \sigma_{ne}^2(\mathbf{G}^H\mathbf{G})^{-1}. \end{aligned}\tag{47}$$

Proof: From (18), we know

$$\hat{\mathbf{b}} = (\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}\hat{\mathbf{G}}^H\mathbf{J}^H\mathbf{y} = (\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}\hat{\mathbf{G}}^H(\mathbf{G}\mathbf{b} + \mathbf{n}), \tag{48}$$

where $\hat{\mathbf{G}} = \mathbf{J}^H\hat{\mathbf{\Gamma}}\mathbf{S}_2$, and

$$\hat{\mathbf{\Gamma}} = \mathrm{diag}\{1, e^{j\hat{v}_0}, ..., e^{j(M-1)\hat{v}_0}\}. \tag{49}$$

From Taylor's expansion, we know

$$e^{jm\hat{v}_0} = e^{jmv_0} + jme^{jmv_0}\Delta v - m^2e^{jmv_0}\Delta v^2 + ...$$

and then (49) can be expressed as

$$\hat{\mathbf{\Gamma}} = \mathbf{\Gamma} + j\mathbf{D}\mathbf{\Gamma}\Delta v - \mathbf{D}^2\mathbf{\Gamma}\Delta v^2 + ... \tag{50}$$

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At high SNR, the higher order statistics can be omitted and $\hat{\mathbf{G}}$ can be rewritten as

$$\hat{\mathbf{G}} \approx \mathbf{J}^H(\boldsymbol{\Gamma} + j\mathbf{D}\boldsymbol{\Gamma}\Delta v)\mathbf{S}_2 = \mathbf{G} + \dot{\mathbf{G}}\Delta v. \quad (51)$$

Substituting (51) into (48), we obtain that

$$\hat{\mathbf{b}} = \mathbf{b} - (\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}\hat{\mathbf{G}}^H\dot{\mathbf{G}}\Delta v\mathbf{b} + (\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}\hat{\mathbf{G}}^H\mathbf{n}. \quad (52)$$

At high SNR, using the approximation $(\mathbf{I}+\Delta\mathbf{X})^{-1} \approx \mathbf{I}-\Delta\mathbf{X}$ for positive semi-definite matrix [14] and omit the higher order statistics we obtain

$$(\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1} \approx (\mathbf{G}^H\mathbf{G})^{-1} - \boldsymbol{\Phi}^{-1}\dot{\boldsymbol{\Phi}}\boldsymbol{\Phi}^{-1}\Delta v. \quad (53)$$

Then we can rewrite (52) as

$$\hat{\mathbf{b}} \approx \mathbf{b} - (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\dot{\mathbf{G}}\Delta v\mathbf{b} + (\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}\hat{\mathbf{G}}^H\mathbf{n}. \quad (54)$$

Therefore, we can obtain

$$\begin{aligned} \mathrm{E}\{\Delta\mathbf{b}\} &= \mathrm{E}\{\hat{\mathbf{b}} - \mathbf{b}\} \\ &= \mathrm{E}\{-(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\dot{\mathbf{G}}\Delta v\mathbf{b} + (\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}\hat{\mathbf{G}}^H\mathbf{n}\} \\ &= 0, \end{aligned} \quad (55)$$

and

$$\begin{aligned} &\mathrm{E}\{\Delta\mathbf{b}\Delta\mathbf{b}^H\} = \mathrm{E}\{(\hat{\mathbf{b}} - \mathbf{b})(\hat{\mathbf{b}} - \mathbf{b})^H\} \\ =&\mathrm{E}\{(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\dot{\mathbf{G}}\mathbf{b}\mathbf{b}^H\dot{\mathbf{G}}^H\mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}(\Delta v)^2\} \\ &+ \sigma_{ne}^2\mathrm{E}\{(\hat{\mathbf{G}}^H\hat{\mathbf{G}})^{-1}\}. \end{aligned} \quad (56)$$

Using (53), we can prove (47) from (56). ■

V. Numerical Results

In this section we numerically study the performance of our proposed NLS estimation algorithm. Three-tap model for both $\mathbf{h}_i$ is assumed, while each tap is Gaussian with unit variance. The variance of the noise is taken as $\sigma_n^2 = 1$. The normalized frequencies f_1, f_r, and f_2 are set as 0.94, 1 and 1.06, respectively. The MSE is chosen as the figure of merit, defined by

$$\mathrm{MSE}(v) = \frac{1}{10000}\sum_{i=1}^{10000}(\hat{v}_i - v)^2,$$

$$\mathrm{MSE}(\mathbf{x}) = \frac{1}{10000}\sum_{i=1}^{10000}\frac{1}{3}(\hat{\mathbf{x}}_i - \mathbf{x})^2,$$

where $\mathbf{x}$ represents $\mathbf{a}$ or $\mathbf{b}$, and 10000 is the number of the Monte-Carlo trials used for average.

First we examine the performance of CFO estimation and the corresponding MSEs versus SNR curves are shown in Fig. 2 for $N = 16$ and $N = 32$, respectively. The theoretical MSEs are also displayed for comparison. It is seen that for both values of N, CFO estimation MSEs approach their theoretical values in high SNR region. The mismatch at the low SNR region is generally known as *outlier* [13], that happens because of the estimation ambiguity in several Monte-Carlo runs, which ruins the average performance.

We then demonstrate the corresponding channel estimation results, as well as the theoretical MSEs for $\mathbf{b}$ in Fig.3. We see that the estimation MSEs of $\mathbf{b}$ approach their corresponding theoretical values much faster than that of CFO estimation. This is because the errors in the estimated phase have less effect on the channel estimation but have severe effect on the CFO estimation. However, when CFO errors are too large at low SNR region, the channel estimation results still deviate from its theoretical values.

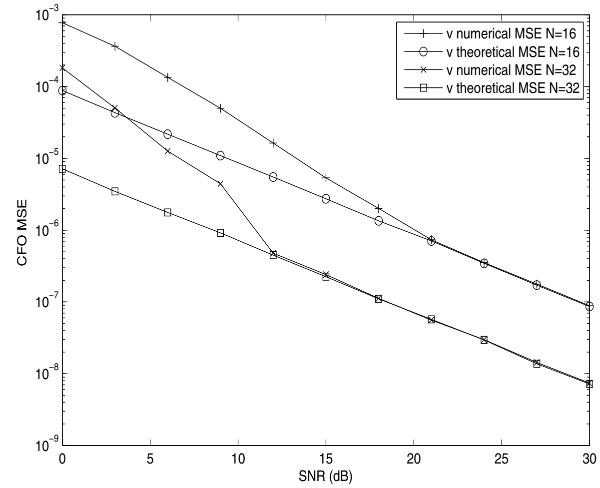

Fig. 2. Numerical and Theoretical MSEs of CFO versus SNR

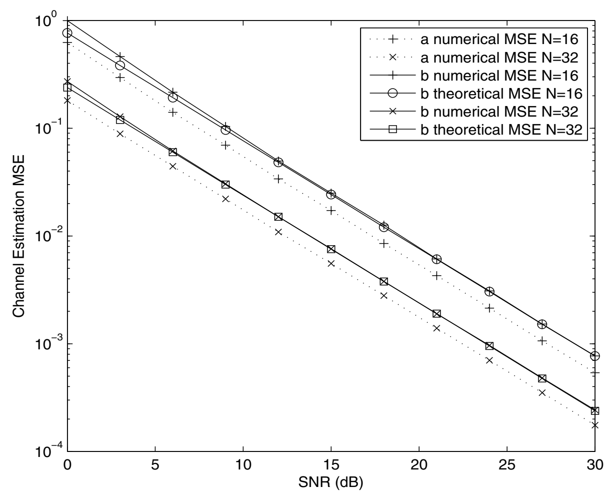

Fig. 3. Numerical and Theoretical MSEs of Channel Estimation versus SNR

VI. Conclusions

In this paper, we adapted ZP-based OFDM transmission scheme for TWRN under the non-perfect frequency synchronization errors. The scheme introduces a little redundancy but greatly facilitates the joint estimation of CFO and channels. We then designed a joint NLS estimator and provided a detailed performance analysis. Finally, our numerical results verify the effectiveness of the proposed study.

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