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Connecting student learning and classroom teaching through the variation framework

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Learning and teaching

In a recent paper in *The Harvard Educational Review*, Graham Nuthall, has called attention to the lacking link between student learning and classroom teaching (2004). This lack is as evident in educational research as is in the minds of teachers. Lack in the first respect means that research does not offer any theoretical tools for teachers for learning from their own experiences, hence lack in the first respect contributes to lack in the second respect. Accordingly, "Teachers often feel that learning outcomes are unpredictable, mysterious and uncontrollable" (Kennedy, 1999 quoted by Nuthall, 2004 p 276).

The aim of our presentation is to describe a framework, the variation framework from phenomenography, which we believe to be useful in connecting student learning and classroom teaching. By using it, researchers can learn about the nature of the relationship between the two, and teachers can learn from their own experiences about how their students' learning relates to their teaching. This can be done by telling apart what is critical in teaching for the students' learning, from what is not critical. First, however, we have to point out the kind of learning outcomes in relation to which features of teaching might be critical.

We believe that one of the most important tasks of school is to enable students to handle novel situations in the future in powerful ways. "Powerful ways" refer to ways in which the students can achieve what they are trying to achieve. According to anthropological research (Goodwin, 1994) as well as research on expert-novice differences (see, for instance Chi, Feltovich and Glaser, 1981) powerful ways of handling novel situations presuppose powerful ways of seeing those situations. "Powerful ways of seeing novel situations" amounts to being able to discern what is critical in the situation in relation to a certain aim and to focus on all that is critical simultaneously. So, discernment and simultaneity are essential for this kind of learning. We can then inspect different classroom environments in order to find out to what extent this kind of learning is made possible or not and we can inspect the learning outcomes in order to find out to what extent there is a corresponding variation among students who have participated in different lessons dealing with the same content.

The principle which is suggested by our (variation) framework to account for the relationship is that learners can discern a feature only if that feature varies, while other features are invariant. Furthermore, simultaneous awareness between two or more features can only be developed if these features are discerned (separated) first and then they co-vary, while other features remain invariant. In a great number of empirical studies we have demonstrated that dramatic differences in student outcomes are frequently linked to seemingly subtle, but critical differences in how the content of learning is dealt with. Marton and Pang (2006) demonstrated, for instance, that the proportion of students achieving the learning target (to be able to deal with changes in price in terms of simultaneous changes in supply and demand) varied between 7 and 97% among five classes of highly motivated and able students taught by very well educated and experienced teachers, depending on subtle, but critical differences in how the content of learning was dealt with during a series of three lessons.

Now, let us illustrate how we have made use of our variation framework to connect student learning and classroom teaching by using an empirical study conducted in General Studies with Primary four students in Hong Kong.

The Study

This study reported here was a so-called "learning study". A "learning study" (e.g. Pang & Marton, 2003) is a hybrid between a design experiment, or design-based research as characterized by Brown (1992), Collins (1992) and Kelly (2004) on the one hand, and a Japanese lesson study as characterized by Yoshida (1999) and Stigler and Hiebert (1999) on the other. Theoretical grounding and systematic evaluation point to the former, while collaborative designing of the lesson(s) and teachers' ownership of the attempt to improve learning by means of successive revisions of the lesson design point to the latter.

In a learning study, a group of teachers comes together to develop instructional means for making it possible for students to appropriate a specific object of learning. An object of learning is a specific insight, skill, or capability that the students are expected to develop. The teachers, who work in collaboration with a researcher, choose a particular object of learning and plan the lesson(s) together. They draw upon their previous experiences in handling this object of learning; past research in relevant areas; the particular framework that they use to identify the necessary conditions for this object of learning; and above all the study of the extent to which the object of learning and/or its prerequisites have been appropriated by the students before the lesson.

A learning study could in principle have any theory of learning and teaching as its fundament, but in this study the "variation framework" has been used. The teachers pay great attention to the way in which the specific object of learning will be dealt with, in terms of the aspects of the object of learning that will vary and those that will remain invariant. The focus on what varies and what is invariant derives from the variation framework the teachers are making use of, and the choice of other factors such as teaching arrangements, forms of representation, etc. is subordinate to the teachers' design of the pattern of variation and invariance. One of the teachers first carries out the lesson, while the other members observe. Usually, the lesson is video-recorded, and what the students have learned is probed by written questions and interviews. The lesson is analyzed in terms of whether it is possible to appropriate the object of learning through the pattern of variation and invariance that is jointly constituted by the teacher and students. This is the basic tenet of the variation framework (see e.g. Marton and Booth, 1997) that underlies this study. From the student answers given before and after the lesson, the gains and the absence of gains can then be related to what has happened in the classroom.

In the post-lesson evaluation meeting, the group may come up with some suggestions to improve the design and implementation of the lesson(s). Another teacher then carries out the new version of the plan following the same procedures. The cycle will continue until all the group members have tried out the plan, and the study is always concluded by documentation.

The present learning study involved five Primary 4 General Studies teachers and 11 meetings were organized.

Ascertaining Students' Existing Understanding

The exploration of the students' understanding of price described above was actually the first phase of this intervention study carried out to inform the teachers about the students' ways of understanding in relevant respects. The distribution of the 162 Primary 4 students across the categories can be seen in Table 1 (For details of the categories, please refer to Pang & Marton, 2007).

Conception	Occurrence	Percentage
A: Object	12	7.4%
B1: Demand	92 56.8%	
B2: Supply	11	6.8%
C: Demand & Supply	11	6.8%
Unclassified ¹	36	22.2%

Table 1 Distribution of Conceptions for the Pre-test

¹ Because of the brevity of the written answers or the incoherent lines of thought expressed by the young students of Primary four, it was sometimes difficult to

classify their answers meaningfully. Moreover, although some of the answers in this category were quite reasonable, they did not look at the notion of price from an economic point of view, and thus they were not included in our system of categories.

The teachers found that it was prevalent for students to explain the phenomenon of price by focusing on the demand-side factors and ignored the supply side of the situation. Some students did the reverse and focused on supply, and a few others took both demand and supply into consideration.

After carefully examining students' qualitatively different ways of experiencing the phenomenon of price as shown in the pre-test, two critical differences were identified. The first was between seeing price in terms of features of the product itself and seeing price in terms of market conditions (such as demand and supply). The second was between seeing price in terms of either demand or supply and seeing price in terms of both demand and supply simultaneously. The pedagogical implication of this was that variation had to be introduced first in the dimension of demand, with supply and the product invariant, then in the dimension of supply, with demand and the product invariant, followed by simultaneous variation in both supply and demand (with invariant product). Finally, the last condition could then be repeated by different kinds of products.

Another major reason for setting the pre-test was to set a comparison baseline that the students' learning outcomes (measured by the same test) after the learning study could be compared with. As seen from Table 1, only 6.8% of the students came to class with the way of conceptualizing price that the lesson aimed at — that is, taking into account both the demand-side and the supply-side factors. All teachers involved agreed that it was a fairly advanced concept for students at this level.

Designing the lesson

When designing the lesson, the teachers drew on the pre-test results, their previous experiences in dealing with this object of learning and the research findings of relevant phenomenographic studies (for instance, Dahlgren, 1978, 1979; Pong, 2000). Based on their understanding of the theory of variation, the teachers made *conscious and systematic* use of variation and invariance, i.e. they planned to systematically vary one or two aspects while keeping the other aspects invariant. The uses of variation and invariance in the lesson were as follows:

The teacher starts off with the introduction of an auction to raise funds for the school's construction project. They explain the procedures and regulations of the auction, divide the class into groups of 6 students and allocate each group HK\$200 of auction money. The teacher then displays the auctioned items with the base prices shown, and this is followed by the actual auction. Each group of students is reminded to note down the auctioned prices and to discuss among themselves why some goods reach higher prices than others. Subsequently, some students are invited to share their views with the whole class.

Anticipating that some students would account for the higher prices by referring to keener competition amongst groups, the teacher introduces a new

scenario in which the quantity of each auctioned item is reduced by one while allocating the same amount of auction money to each group. *By varying the supply of the auctioned items and maintaining the demand and the features of the products invariant*, the teacher directs students' focal awareness to the dimension of supply.

In the next round, to bring students' focal awareness to bear upon the dimension of demand, *the teacher varies the demand (through varying the purchasing power of the students), while keeping the supply and the features of the products invariant.* The teacher deliberately reduces the auction money of each group by HK\$100, and thus their purchasing power and demand for goods are reduced.

According to the variation framework, in order to enable the students to focus on demand and supply at the same time, simultaneous variation in the dimensions of demand and supply should be introduced, *keeping the features of the products invariant*. However, the participating teachers thought that the lesson would be more interesting if a new trendy item could be brought in. So, it was decided that the teachers would ask every student to individually complete the following question which embodies the simultaneous variation in demand and supply *in a new context* (i.e. with another product).

As you know, the dinosaur machine is now selling for \$80 in toyshops. Suppose that you are the owner of a large toyshop, which is the sole supplier of the new model, which has not been publicly released and is issued as a limited version. At the same time, you observe that the Hong Kong economy has been recovering very well over this period of time. Given these conditions, what price will you set for this new model? Why?

The students are guided to consider the variation in the supply of the product as well as the variation in the purchasing power of people (demand for the product) at the same time. The pattern of variation and invariance is summarized in Table 2.

Supply	Demand	Features of the Product	Price
i	i	i	i
v	i	i	v
i	v	i	v
v	v	V	v

Table 2 Patterns of Variation and Invariance (Temporal Sequence)

Carrying out the lesson

Afterwards, all five teachers implemented the plan in a fairly faithful manner. The researcher and all other teachers involved observed each of the classes. Immediately after each trial, the teacher/researcher team conducted a meeting to evaluate the lessons. All lessons were video-recorded for later analysis.

Describing the outcomes of learning

Comparison between the pre- and post-test results

A post-lesson study was conducted to ascertain how well the students had developed the target capability. The same question was used in the post-test as in the pre-test, except that the product in question changed from hot dogs to biscuit sticks. The students' ways of experiencing the phenomenon were identified and then categorized in accordance with the categories of description found.

As shown in Table 3, considerable gains in students' learning outcomes were found. Around 26 % of the students' answers in the post-test fell into Category C, corresponding to the object of learning agreed upon by the teachers. This was noticeably higher than the percentage for the same category in the pre-test, which amounted to only 6.8%. The difference is statistically significant (Chi-square = 151.643, for df = 25, p < 0.001). It is noteworthy that compared with Dahlgren's study (1978), in which a majority of university undergraduates, after attending the first-year economics course, could not explain a daily-life economic phenomenon such as how the price of a bun is determined, using some fundamental economic concepts like supply and demand, the results obtained in this study with the 10-year-old Primary 4 students were fairly encouraging.

Conception	Pre-test	Post-test	
	(162 students)	(162 students)	
A: Object	7.4%	6.8%	
B1: Demand	56.8%	52.5%	
B2: Supply	6.8%	5.6%	
C: Demand & Supply	6.8%	25.9%	
Unclassified	22.2%	9.0%	

Table 3 Comparison of the Conceptions Displayed by Studentsin the Pre-test and Post-test

Chi-square = 151.643 (df = 25), p = 0.000

Considering the results, the theory of variation may be argued to be a powerful tool in promoting student learning of the notion of market price. Furthermore, as we will show in the next section, differences between the learning outcomes of different classes were linked to differences in how the object of learning was handled in terms of variation and invariance, i.e. how the teachers constituted patterns of variation and invariance to draw students' attention to the critical aspects of the object of learning.

Comparison between different classes

Table 4 shows that there were gains in all classes in terms of the percentage of students who demonstrated the target understanding of "price", i.e. Conception C. The outstanding performance of Class 4B (with about a 51.3% increase) compared to other classes caught our attention, and thus we tried to investigate what had actually taken place in Class 4B to account for this difference in results.

Class	Pre-test	Post-test
4A	3.0%	9.1%
4B	10.3%	61.5%
4C	3.2%	12.9%
4D	7.1%	14.3%
4E	9.7%	22.6%

Table 4 Students expressing Conception C in each class after each consecutive lesson in the pre-test and post-test

When scrutinizing the classroom teaching of 4B, we found that the teacher, together with the students, had constituted an unplanned pattern of variation and invariance during the lesson. According to the lesson plan, to help students to discern and focus on demand, the teacher deliberately reduced the auction money allocated to each group while keeping the supply and features of the product invariant. However, something unexpected happened. Instead of recording expected lower prices for the various items because of a fall in the purchasing power of the groups, the price of one of the items was found to be higher (\$190) than the price of the same item obtained in the first round (\$110). To deal with this anomaly, the teacher

spontaneously increased the supply, offering two more of the same item for auctioning and a group of students succeeded in getting it at the base price (\$20). The price of the auctioned item thus fell sharply from \$190 to \$20, as a function of the further drop in purchasing power due to the using up of auction money by most groups and the unexpected increase in supply of the item as initiated by the teacher.

Although the activity might not have been in line with the original intention, it allowed simultaneous variation in both the dimension of demand (a decrease in purchasing power when compared with the first round as the auction money was reduced by HK\$100, and a further decrease in purchasing power because most groups had used up their auction money) and the dimension of supply (an increase in supply as two more of the same item were offered for auctioning) to come into play, *while the product was invariant*. As we pointed out above, in the original lesson design the product was unfortunately changed when simultaneous variation in demand and supply was introduced. Such a simultaneous variation with the same product resulted in drawing the students' attention to the effect of the demand-supply interaction on the price of a product, which was, however, not found in other classes, in which no additional auction was introduced with any of the auctioned products. This was probably the reason why students in Class 4B were better at discerning and simultaneously focusing on demand and supply when determining the price of a product.

Conclusions from the study

In this study, we could see how the teachers determined the critical differences between different meanings of the direct object of learning (price) before it was taught, and how they made use of this variation, alongside the theoretical framework, in planning the lesson. The same theoretical framework was also used when interpreting differences in learning outcomes due to differences between the lesson plan and what actually happened when the lesson plan was carried out.

The relationship between student learning and classroom teaching

One of the most important aspects of the study just described is that we were using a theoretical framework to interpret changes and differences in student learning in terms of the teaching and classroom interaction that students had participated in. Nuthall (2004) discussed the failure of educational research to bridge the theory-practice gap, as reflected in the missing link between student learning and classroom teaching. He quotes Prawat (1992) who claims that most teachers believe that "student interest and involvement constitutes both necessary and sufficient condition for worthwhile learning" (p.389). Educational research does not contribute much towards counteracting such beliefs as it does not provide teachers with tools for interpreting success and failure in teaching, nor for improving their practice. In order to do the latter, we would need better ideas of the consequences of their actions, Nuthall argues. He points to the need for an explanatory theory specifying "an underlying mechanism or process that connects, in a direct and unavoidable way, a cause with its effect" (p.277).

There is nothing that the teacher does, however, that can "cause" learning in an "unavoidable way". Having a certain experience may imply a certain learning effect, but even if you can say that having a certain experience is a necessary and sufficient condition for a certain kind of learning, you can never claim that something you (as a teacher) do makes a necessary and sufficient condition for a third party (a student, for instance) to have a certain experience.

On the other hand, we can specify the conditions which are necessary for a certain experience (even if they are not sufficient). If life does not provide one with an opportunity to experience an elephant, one cannot possibly experience an elephant. If elephants are the only animals that have crossed one's path, one cannot possibly separate the category "elephant" from the category "animal". A student who has only come across Pythagoras's theorem cannot possibly separate "mathematical proof" from Pythagoras's theorem. Nobody can discern "demand" and "supply" without having encountered different prices of the same product.

The point we are making here is that what varies and what is invariant in the classroom is of decisive importance for what students can possibly experience and hence learn. What varies and what is invariant have to do with how the content – or rather the object of learning – is dealt with in the classroom. They constrain what can possibly be learned. The way in which the object of learning is dealt with may or may not provide students with the necessary conditions for appropriating that specific object of learning.

A certain pattern of variation and invariance is, of course, not the only necessary condition for a certain kind of learning to take place. There are actually countless others (and here is one of the main differences between research in education and research in science). Above all, there are material conditions that have to be met (visibility, hearability, temperature, etc); there are necessary conditions which have to do with the teachers, or with the students; there are necessary conditions which have to do with how learning is organized (there must be a certain space for learners' active involvement). However, even when the necessary conditions are met in classrooms, there are still differences in learning outcomes. In our studies carried out in materially well-developed societies such as Hong Kong (e.g. Lo, Pong & Chik, 2005), Sweden (e.g. Holmqvist et al., in press), and the U.K. (e.g. Thabit, 2006), we found that what varied critically between classrooms - in the sense of being related to variation in learning outcomes, was the way in which a particular object of learning is dealt with. In order to discover this empirical fact, you need to compare how the very same object of learning is dealt with in different classrooms. This is rarely done, but this is exactly what we did, and this is why we managed to find the actual relationship between learning and teaching.

Furthermore, we found that differences in how the object of learning is dealt with can be described in terms of what varies and what is invariant, and it is differences in this respect which are essential when it comes to understanding differences in learning outcomes.

The object of learning

Our points of departure are the questions: What are the students expected to learn? What have they learned before being taught? What is made possible for them to learn in the classroom? What have they learned by being taught? There is an obvious focus on the "what" of learning. This "what" corresponds to "the object of learning". What is then "the object of learning"? Let us briefly elaborate.

There is something we might refer to as "the content of learning". In our study above, this content was "price as a function of demand and supply". This is what the students were supposed to focus on during the lesson. We call this the direct object (that is, content) of learning. But what were the students supposed to be able to do with the content? Were they supposed to be able to recall the statement "price is a function of demand and supply"? Or were they supposed to be able to determine the correct price of a product for a given supply and a given demand curve (cf. the textbook example above)? Therefore, in addition to the content, *the direct object of learning*, the object of learning has another component which has to do with the nature of capability, what the students are supposed to be able to see a situation in terms of ...". The direct object was something like "being able to see a situation in terms of ...". The direct and indirect objects of learning together define the object of learning, which in this case is: "being able to see price-related situations in terms of demand and supply".

So this was the object of learning in the study. But it answers only the first of several questions "What are the students expected to learn?" The answer – which we have just given – (being able to see price-related situations ...) is the <u>intended</u> object of learning, corresponding to the far more common expression "learning objective". Our intended object of learning is one kind of – and one way of describing – the learning objective. In this special case, we imagine the learner encountering a "price-related situation" (for instance, a sale, where the price of a mountain-bike is reduced from \$1000 to \$500) and making sense of it in terms of demand and supply (for instance, as "There are too many bikes of this old model left and there are not many people willing to buy it for \$1000"), instead of seeing price as an inherent feature of the bike (for instance, as "Now I can earn \$500 if I buy that bike"). The idea is that the student will see variation in price in terms of variation (possible differences) in demand and supply.

The intended object of learning in this particular case can thus be characterized:

Product	Demand	Supply	Price
i	V	v	V

So this is what the students should learn (the intended object of learning), but what about *what they actually learn*? Both what they have actually learnt before the lesson and what they actually learn during the lesson, we call *"the lived object of learning"*,

and in order to distinguish between these two lived objects of learning, we add (1) to the former and (2) to the latter.

As far as the "lived object of learning (1)" in the above study was concerned, there were actually 4 different answers to the question "what have they learned already?" They might think of price as a function of features of the product, of demand, of supply or of demand and supply at the same time. In accordance with Table 5, we can characterize these four ways of seeing price-related situations:

Product	Supply	Demand	Price
V	i	i	V
i	i	V	V
i	V	i	V
i	V	V	V

As we can see, concerning the fourth category, which was identical with the intended object of learning, a few students had reached it already before being taught.

Next, we want to characterize what was possible to learn. The way the direct object of learning was dealt with in the classroom was also characterized in terms of what varied and what was invariant. This was also a form of the object of learning, namely the *enacted object of learning*, referring to what was possible to learn. Here we did not have four different patterns but a succession of four patterns which together constituted the enacted object of learning. According to the design developed by the group of teachers, after introducing the scenario, variation would be introduced in demand, then in supply and finally in both demand and supply. The products were supposed to remain invariant during the whole series:

Product	Supply	Demand	Price
i	i	i	V
i	i	V	V
i	V	i	V
i	V	V	V

As we pointed out earlier, this pattern was brought out only in the class with the highest results – by accident. In all the classes, the products were unfortunately changed in the last stage, and thus the pattern was:

Product	Supply	Demand	Price
i	i	i	V
i	i	V	V
i	V	i	V
V	V	V	V

When the teaching was completed, the students were probed again in order to get an answer to the question "What have they learned by being taught?" The answer to this question is "The lived object of learning (2)" described in terms of the same four categories as were used to characterize the "lived object of learning (1)" (cf. Table 5).

 Table 5 Patterns of experienced variation and invariance that correspond

 to different ways of seeing price

Category	Features of product	Demand	Supply	Price
A	V	i	i	V
B1	i	V	i	V
B2	i	i	V	v
С	i	V	V	V

The most remarkable feature of this model of description is – in our view – that goal (intended object of learning), teaching (enacted object of learning) and results (lived object of learning) are described in the same terms, in commensurable ways. This enables us to examine the relationship between the differences in the intended and enacted objects of learning and the relationship between the differences in the intended and enacted object of learning and student learning in a more systematic and consistent manner, when the intended/enacted objects of learning are described in terms of the different planned/observable patterns of variation and invariance co-constituted by the teachers and students in the different classrooms, and the lived object of learning are described in terms of different patterns of variation and invariance experienced and expressed by the students through their ways of dealing with the problems used as criteria of learning.

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