



<b>Title</b>	<b>Lessons from a Small-scale Observational Study - An Example of the Teaching of Fractions</b>
<b>Author(s)</b>	<b>Runesson, U; Mok, IAC</b>
<b>Citation</b>	<b>The 28th Annual Meeting of the International Group for the Psychology of Mathematics Education (PME), Bergen, Norway, 14-18 July 2004</b>
<b>Issued Date</b>	<b>2004</b>
<b>URL</b>	<b><a href="http://hdl.handle.net/10722/109357">http://hdl.handle.net/10722/109357</a></b>
<b>Rights</b>	<b>This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.</b>

# LESSONS FROM A SMALL-SCALE OBSERVATIONAL STUDY: AN EXAMPLE OF THE TEACHING OF FRACTIONS

Ulla Runesson

Göteborg University, Sweden

Ida Ah Chee Mok

The University of Hong Kong

*The aim of this paper is to demonstrate what a small-scale project can tell about features of teaching and learning in two different cultures. We argue that some features, which may not be easily observed within one culture, can become more visible in the contrast in order to get a better understanding of the teaching practice per se, even from a small scale project. We have studied the mathematics teaching in one classroom in Hong Kong and four in Sweden. Based on the assumption, that how the content is taught has an important implication on what students may possibly learn, we compared how the teaching of the same topic (fraction) may differ between the two places. Some profound differences regarding how the same topic was dealt with in the two countries were found. In the Hong Kong data several things were handled in one lesson at the same time whereas in the Swedish data this happened in a sequence of lessons spreading over a substantial period.*

## INTRODUCTION

Being in an environment constantly, one usually takes things for granted and fails to see the characteristics of the environment as special or different from the others. To bring about a better understanding of mathematics teaching itself is one argument for comparative studies (Stigler & Hiebert, 1999; Lopez-Real & Mok, 2002). However, comparison can be made at different levels and with different focus. Mostly, these studies are to different extent grounded in data from more extensive data sets (e.g. TIMSS Video Study and in the PISA-project). But, are these very expensive and extensive studies the only way to bring about insights about cultural differences? The study we will report here captures a small number of mathematics lessons in Hong Kong and Sweden. In Sweden five consecutive lessons from four different classrooms and in Hong Kong only one lesson were studied. Compared to the extensive studies mentioned, our study can appear to be too thin and insufficient to generalize anything about mathematics teaching in the two countries. However, our aim was to some extent different from these studies the overall aim of which was to compare the teaching practices in different cultures. This was not the goal for our study. Instead we hoped that some features, which may not be easily observed within one culture, would become more visible in the contrast in order to get a better understanding of the teaching practice *per se*. The question whether this is possible even within the frame of a small-scale project will be discussed in this paper.

## **THEORETICAL FRAMEWORK: A THEORY OF VARIATION**

In this study, we approach classroom learning with a specific focus. Assuming learning is always learning of something – it has an object - we study how the object of learning is constituted in the classroom interaction, and with the particular interest in different possibilities to learn in different situations. What is possible to learn, has to do with those aspects of the object of learning that are possible for the learners to be aware of, or to discern. However, only that which is varying can be discerned (Bowden & Marton, 1998). So, the possibility to discern an aspect has to do with whether it is present as a dimension of variation or not (Marton & Booth, 1997; Marton, Runesson, & Tsui, 2004). If the particular aspect is present as a dimension of variation, it is likely discerned by the learner. And further, if the aspects are present as dimensions of variation at the same time, the learners likely discern them at the same time. So, what is studied is the pattern of simultaneous dimensions of variations related to the object of learning that are present to the learners in the situation (Runesson & Marton, 2002). And when studying the differences in possibilities to learn in different classrooms, it is the difference between the patterns of simultaneous dimensions of variations opened in the different classrooms that we describe.

## **THE STUDY**

The current study has its origin in a previous study of Swedish mathematics classrooms, which aimed at finding differences between the teachers as regards how the topic was handled (Runesson, 1999). To shed new light on this data, a similar study in Hong Kong was conducted. The aim was to find differences between how the same topic was taught by contrasting mathematics teaching in two different cultures. However, to be able to see critical features in our own classrooms and one's own culture, we chose the same mathematical topic in order to see how the same topic can be handled in different cultural context. Therefore, the selection of the Hong Kong data set was made on the basis of matching up with the existing Swedish data as much as possible. The Hong Kong lesson was a primary four (age 10, grade 4) lesson on the topic "Comparing fractions". The lesson was carried out in Cantonese and videotaped. The Swedish data is drawn from a larger data set consisting 20 lessons from four different classrooms in grade six and seven. These lessons were audio taped and transcribed verbatim. Our aim was to be as close as possible with regards to the content of teaching. That is, when sections of the Swedish data were selected, this was done at the level of sub-constructs of fractions. The sub-constructs of fractions, which were available in the Hong Kong data, did appear in four of Swedish teachers' teaching. The analysis is grounded on data from all of these classrooms. Due to differences between the Swedish and the Hong Kong curriculum, we could not match the age of the pupils in the two countries. The topic was taught in grade six and seven (age 12 and 13 respectively) in Sweden and in grade four in Hong Kong (age 10). And although, we tried to come as close as possible to study the same content, some differences occurred. In the Hong Kong lesson the students worked with finding the common denominators of two fractions.

In the Swedish lessons the tasks was slightly different; the task was to find another fraction with the same value (e.g.  $2/6=1/3$ ). However, in both the Hong Kong and the Swedish lessons, comparison of fractions with different denominators was found. Unlike the Hong Kong data, which is drawn from one single lesson, the Swedish data consist of several lessons.

## **SUMMARY OF THE RESULTS: TWO DIFFERENT EXAMPLES OF SIMULTANEITY AND VARIATION**

The analysis was with a particular focus on those aspects of the topic taught that were opened as dimensions of variation were identified. The Hong Kong lesson appears to have only one objective, i.e. comparing fractions with different denominators. Nevertheless, this objective was visited and revisited via several tasks, which were either in the form of questions in the worksheets or examples on the board. As a result of this, the Hong Kong lesson shows a pattern of variation, which consists many dimensions of variation. For example, some dimensions are: alternative representations of the method of amplification, the denominators, the fractional parts of different wholes and the contrast between the methods of comparison. Moreover, the intertwined relationship between these dimensions of variation forms a special arrangement or simultaneity of variation in a single lesson. Such experience is important because it provides a chance for “fusion” i.e., for the students to consider several aspects of the object of learning simultaneously (Marton, Runesson and Tsui, 2004). The Swedish lessons showed a very different pattern of variation. The most striking difference was perhaps that variation of methods was not opened. The students were presented to one method only, a diagrammatic method. Instead of varying the method, the teacher demonstrated a method on a couple of different examples. The other apparent difference was the sequential character identified in the Swedish lessons. We found that these sub-constructs were commonly never presented simultaneously in the Swedish lessons, but instead they were extended over time and presented as disjoint instances without any connection or reference to previous lessons. So, finding the bigger of two different fractions was taught in one lesson, and “fractions with different denominators but with the same value” was taught in another. The latter was taught with no reference to how this had been presented earlier although the two topics were indeed connected. In other words, since in the Hong Kong lesson several sub-constructs were presented and related to each other at the same time, the Hong Kong lessons were richer in terms of sub-constructs related at the same time.

Comparing to the Hong Kong lesson, the Swedish examples created a narrower space of variation, and in combination with the sequential character, accomplished a quite different space for learning in the Swedish lessons. From the theoretical position taken, we can assume that what was possible to discern of the same thing was different in Hong Kong and in Sweden. In other words, the students’ understanding of the two sub-constructs “comparison of fractions” and “fractions with the same value” are very likely to be different when the students from the two places

experience such different space of learning. So, what we can say is, it was possible to discern different things in Hong Kong and Sweden. But, what that means for what the students actually learned, we cannot say, since this has not been studied.

## **WHAT COULD COME OUT OF A SMALL-SCALE OBSERVATIONAL STUDY?**

The study presented here is in many respects a small-scale project, so what could possibly come out of such, as it seems, limited project?

The original purpose of this study was to shed new light on a study conducted earlier in Swedish classrooms. In line with the theoretical framework taken, discovering something new when revisiting the data would be easier if it was contrasted against something different, e.g. by contrasting mathematics classrooms and possibilities to learn in different school systems and educational traditions. The object of research in this study was not possibilities to learn in a general sense, but possibilities to learn *the same thing*. Therefore, it was important to study how the same topic was dealt with, i.e. to keep the content constant. This design has been used in a number of studies (Runesson, 1999; Marton & Morris, 2002; Marton, Tsui et al., 2004) However, it was in many ways a bit problematic to match up with a data set from Hong Kong to the existing Swedish data. From our point of view we wanted to delimit our definition of “the same topic” as much as possible. “The same topic” was defined in terms of how it appears in classroom practice, and on the level of tasks, so we asked the teacher to invite us to study a lesson when fractions with different denominators would be the topic taught. Although, we tried to come as close as possible to study the same content, some differences occurred. Being restrictive to having the same topic, it was not possible to study pupils of the same age, due to different curricula in the two countries. However, from this point of view the result is interesting. In the Hong Kong classroom the pupils were about three years younger than their counterparts, however a space of learning consisting of many simultaneous dimensions of variation was afforded to the learners, whereas for the older Swedish pupils dimensions of variation were brought out in sequence.

It could be argued that this sequential pattern of variation was a result of the longer period of observed lessons in the Swedish data, that the likelihood of such a finding is bigger if several consecutive lessons are observed. It could not be excluded that the sequential character of handling the object of learning, which was found in the Swedish data, would not appear in Hong Kong. This has never been claimed, and it was never the purpose of the study either, i.e. to say anything about the general in the two cultures. What we have described is two different ways of handling the same topic, or two different patterns of variation and simultaneity when teaching the same topic. This was found by comparing two different school cultures.

The way we worked in this study, implied doing a close and narrow analysis, but without the aim of finding more overall patterns or a more general character in the different classrooms. A main difference between, for instance, the TIMSS Video

Study and ours is what we were studying. To us the TIMSS Video Study was a study of teaching, whereas ours is a study of possibilities to learn the same thing. In our study we identified and described how the same object of learning could be dealt with differently by means of examples from different cultures. This was possible to do, even if only one single lesson from one teacher from each country was studied.

Necessarily a small-scale project like this touches the issue of representatives. Our aim was not to come up with something that could tell us something about the possibilities to learn about fractions with different denominators in Swedish and Hong Kong classrooms *in general*, or to explain differences in learning outcomes between the two countries. Instead we wanted this study to open our eyes to that, which is not easily seen within our own culture, so it would become visible, but without saying anything about the typical Swedish or the typical Hong Kong classroom. The most prominent coming out of this study is, that by seeing what could be done differently, what *could be the case*, new light has shed light on what *is* done and *what is the case* in some classrooms our own countries. When the characteristics identified from the two different data sets are used as a mirror, it gives us a better understanding of the practice in our countries. Surprisingly, such understanding could be achieved from a small-scale study like this.

## REFERENCES

- Bowden, J. & Marton, F. (1998). *The University of Learning. Beyond quality and competence*. London: Kogan Page.
- Lopez-Real, F.J. & Mok, I.A.C. (2002). Is there a Chinese Pedagogy of Mathematics Teaching. *Perspectives in Education*, 18(2), pp. 125-128.
- Marton, F. & Booth, S. (1997). *Learning and Awareness*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Marton, F. & Morris, P. (2002). *What matters? Discovering critical conditions of learning*. Göteborg: Acta Universitatis Gothoburgensis.
- Marton, F., Runesson, U., and Tsui, A.B.M. (2004). The space of learning. In Marton, F. and Tsui, A.B.M. with P. Chik, P.Y. Ko, M.L. Lo, I.A.C. Mok, D. Ng, M.F. Pang, W.Y. Pong and U. Runesson, *Classroom Discourse and the Space of Learning*, N.J.: Lawrence Erlbaum Associates. Chapter 1.
- Runesson, U. (1999). *Variationens pedagogik. Skilda sätt att behandla ett matematiskt innehåll*. [The pedagogy of variation] Göteborg: Acta Universitatis Gothoburgensis.
- Runesson, U., & Marton, F. (2002). The object of learning and the space of variation. In F. Marton & P. Morris (Eds.). *What matters? Discovering critical conditions of classroom learning*. Göteborg Studies in Educational Sciences, nr 181 Göteborg: Acta Universitatis Gothoburgensis.
- Stigler, J. W. & Hiebert, J. (1999). *The Teaching Gap*. NY: Free Press.