RISK ASSESSMENT OF GENERATION INVESTMENT

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Abstract – This paper proposes an improved approach to risk assessment of generation investment in the new deregulated environment using the option pricing theory. A more realistic model for electricity price in the application of real option pricing method for generation asset valuation is proposed, which takes into account its fluctuation and uncertainties, and, more importantly, its daily, weekly and annual cyclic patterns, which is a unique characteristic of electricity price. Base on such a price process, the generation asset can be evaluated by the application of real option method. In order to manage the risks of the investment on generation expansion project, risk assessment tools such as Value at Risk (VaR) and Conditional Value at Risk (CVaR) may be used to provide the investors with tools for more informed decisions. A numerical example is given to illustrate the proposed method.

Keywords: Generation Expansion Planning, Real Option, Mean Reversion, Conditional Value at Risk, Capital Budgeting

1 INTRODUCTION

Unlike the situation in the regulated monopoly environment where there is a guaranteed rate of return, generation companies in the deregulated environment must take into account the market risks in their investment evaluation on generation expansion planning project [1], [2], especially the uncertainties caused by the fluctuation of electricity price in the market [3].

In recent years, various approaches to the evaluation of generation asset investment have been proposed [4-7]. In one of these approaches, the generation asset evaluation problem is treated as a problem of spread real option [8], [9]. The value of the asset is determined by the difference between the electricity price and the fuel price. In order to consider the uncertainties introduced by market competition, the mean reversion stochastic process is used to approximate the characteristic of electricity and fuel prices [10-13]. Base on such difference, the profit of generation asset during a short-term period can be obtained by option valuation method [14]. Net Present Value (NPV) analysis is applied to evaluate the investment project [15].

The methods employed in such approaches are mainly come from finance engineering area. Direct application of the standard method would treat electricity like any other commodity [11], [12]. This paper proposes a more realistic model for generation asset evaluation and risk assessment in the new environment, especially for the price-sensitive peaking generators. The daily, weekly and seasonally cyclic characteristics of electricity price, which is more important to reflect the specialty of electricity, are taken into account as a long-term mean in the mean reversion model for real option valuation approach. Since it is difficult to achieve an analytical description for such a complex process, Monte Carlo simulation method is used to solve the problem. Base on such an improved real option model, generation asset valuation during a long-term period will be obtained. By applying risk assessment tools such as Value at Risk and Conditional Value at Risk [16], the risks for generation asset investment can be assessed for making generation expansion planning decision.

The paper is organized as follows. In section 2 we present the modified mean reversion stochastic process with long-term periodic mean for electricity price. In section 3 a real option approach is applied to evaluate the generation investment. Section 4 shows how to assess the risks for generation investment project. And the algorithm of the approach can be found in section 5. In section 6, a numerical example is given to show the validity of the method.

2 IMPROVED MEAN REVERSION PROCESS FOR ELECTRICITY PRICE WITH LONG-TERM PERIODIC MEAN

In the new deregulated environment, the electricity price fluctuation reflects all the uncertainties of the market. In finance area, various stochastic models have been developed to describe the process of commodity price. From basic microeconomic theory point of view, the price of commodity ought to be tied to its marginal production cost in the long-run with short-term random fluctuations. The mean reversion process, which has been considered the natural choice for grasping such characteristic of commodity price [9], was applied to approximate electricity price process in [12] and [14].

The Mean Reversion stochastic process that used to describe the fluctuation of electricity price is as the following:

$$dP_t^E = k[\ln m - \ln(P_t^E)]P_t^E dt + sP_t^E dz \qquad (1)$$

where P_t^E is the electricity price at time t, m is the mean of electricity price that reflects the long-run value of price process, k is the mean-reverting coefficient, s is the volatility of P_t^E, and dz represents a wiener increment that reflects the short-term randomness.

For a mean reversion stochastic process in (1), the electricity price converges on the constant mean m in the long run, and k describes the converge speed to the mean price.

However, in a long-term scale, almost periodical characteristics of electricity price P_t^E – seasonally, weekly and daily – must be taken into account in the model. In other words, different from other commodities, the mean of electricity price shall be treated as a periodical function $m(t)$ which substitutes the constant mean m in the mean reversion model. The periodic function $m(t)$ is based on the daily price curve extend with weekly and seasonally fluctuate parameters if no price spike occurs.

The periodic characteristic of electricity price includes daily, weekly and seasonally fluctuations, which can be described as polynomial functions separately by historic data approximation. In order to incorporate the fluctuation characteristic and avoid the impacts of historic price level, per. unit value is used with respect to a given price base.

We define the periodic function for long-term electricity price mean as the following:

$$m(t) = m_{daily}(t) \times b_{week}(t) \times b_{year}(t) \tag{2}$$

where $m_{daily}(t)$ describes the periodic characteristic of daily fluctuation of electricity price in the market, and $b_{week}(t)$, $b_{year}(t)$ are the polynomial functions that involve weekly and seasonally fluctuations.

Thus the mean reversion stochastic process for electricity price can be improved, considering the special characteristic of electricity price, by introducing a long-term periodic mean. With the $m(t)$ above, Equ. (1) can be rewrite as the following:

$$dP_t^E = k[\ln m(t) - \ln(P_t^E)]P_t^E dt + sP_t^E dz \tag{3}$$

With this change in the model, after introducing such a periodic function, it becomes impossible to achieve an analytical description for P_t^E . A simulation based approach is instead applied to solve the problem. Under the assumption of risk neutrality, the electricity price in the market can be derived as follows:

$$\begin{aligned} P_t^E(t) = \exp\{ & [\ln[P_t^E(t-1)]\exp[-k\Delta t]] \\ & + [[m(t) - \frac{(a-r)}{k}][1-\exp[-k\Delta t]]] \\ & - [(1-\exp(-2k\Delta t))\frac{s^2}{4k}] \\ & + s\sqrt{\frac{1-\exp[-k\Delta t]}{2k}}N(0,1)\} \end{aligned} \tag{4}$$

where a is the risk adjust discount rate, r is the risk free rate of return in the market, and $N(0,1)$ is the standard normal distribution.

Base on the above description, a long-term mean reversion stochastic process for electricity price with periodic characteristics is obtained for generation asset evaluation.

3 EVALUATION OF GENERATION ASSET

In the competitive electricity industry, the GENCOs shall dispatch their generation units based on prevailing spot electricity and fuel price [8]. The profit of GENCO is determined by the spread of the two price process. This characteristic can best be described by the “spread option” concept of finance area.

A generation asset, i.e., a generator, can be considered as a spread real option that transferring fuel to electricity. “Real option”, as it name implies, use option theory to evaluate physical or real assets, as opposed to financial assets like stocks and bonds. The value of the generation asset is determined by the difference between the electricity price and the fuel price. If the fuel price is higher than the electricity price (if neglect the maintenance cost and unit heat rate), the generator will stop generating electricity. Comparing with the traditional NPV method, real option theory can offer a more realistic value approximation to the investors due to its production flexibility.

The spread between the price of electricity and fuel determines the economic value of generation assets that can be used to transform the fuel to electricity [11], [12]. The amount of fuel that a generator requires to generate a given amount of electricity depends on the transform efficiency, which is defined as the generation asset *Heat Rate*. If we assume that the *Heat Rate* of the generator is a quadratic function of generator output q_t, the value of generation asset on time *t* can be written as:

$$V(t) = \max\{[P_t^E q_t - (a_0 + a_1 q_t + a_2 q_t^2)P^F],0\} \tag{5}$$

where P^F is the fuel price, q_t is the unit output at time *t*, a_0, a_1 and a_2 are the coefficients for unit heat rate.

For a given generator (or GENCO), the objective is to maximize its profit, i.e.,

$$\begin{aligned} & \max\{[P_t^E q_t - (a_0 + a_1 q_t + a_2 q_t^2)P^F]\} \\ & s.t. q_t \in [q_{\min}, q_{\max}] \end{aligned} \tag{6}$$

where q_{max} and q_{min} are the upper and lower limit for unit capacity.

Assume that each unit in the market bids with its true marginal cost. The optimal unit output q_t can be derived by maximizing the objective function in (6).

$$\begin{aligned} & q_t = (P_t^E - a_1 P^F)/2a_2 P^F \\ & s.t. q_t \in [q_{\min}, q_{\max}] \end{aligned} \tag{7}$$

Base on the output level q_t, the value *V(t)* of generation asset on time *t* can be obtained.

Besides unit capacity limits in (6), more physical constraints for generator operation can be considered in the model for the real power system, such as the response time for unit status switching between on-line and off-line [14]. These constraints will make the optimization problem more complex, but still can be solved by applying dynamic programming approach with no significant impacts on the whole structure of the evaluation approach. To make the approach clear, these physical constraints are not modeled with much detail in this paper.

Considering the time value of cash flow, such *V(t)* is converted back to time *t=0*, and are integrated as the asset value during time period *T*.

$$V_T = \int_{t=0}^{T} e^{-rt} V(t) dt \tag{8}$$

The V_T achieved in (8) only considers the impacts of variable costs (fuel cost). For an investment project, capital investment on asset is depreciated at each period. Assume for period T, the equivalent investment is $Cost_T^I$, which has already been converted to $t=0$, the present value of profit for generation asset during period T can be written as the following:

$$V_T^{\Pr} = V_T - Cost_T^I \quad (9)$$

By applying Monte Carlo simulation approach, the probability distribution of the profit for generation asset during period T can be obtained for further risk assessment.

4 RISK ASSESSMENT FOR GENERATION INVESTMENT

Risk assessment is a tool to help the investor making the decision of whether to accept or to reject the project. Value at Risk and Conditional Value at Risk are widely used as the measures for risk assessment.

Value at Risk is a measurement of risk in terms of maximum likely loss. It can be defined as the following: we are *x%* certain that our loss on the investment project during period T in the future will not larger than y dollar. This y is the VaR of the investment.

Define $V_T^{\Pr}$ and $-V_T^{\Pr}$ as the profit and loss of the investment project respectively, thus VaR can be defined mathematically as

$$\Pr\{-V_T^{\Pr} > VaR\} = \Pr\{V_T^{\Pr} < -VaR\} = (1 - x\%) \quad (10)$$

It can be described as the following explicit formula:

$$VaR = -CDF_{V_T^{\Pr}}^{-1}(1 - x\%) \quad (11)$$

where $CDF_{V_T^{\Pr}}^{-1}$ is the inversed cumulative distribution function for investment profit $V_T^{\Pr}$ (and the negative profit means the loss), *x%* is the confidence level that the investor is willing to accept on the project.

Although VaR is a very popular measure of risk, it has undesirable characteristics such as lack of sub-additivity and convexity. VaR is coherent only when it is based on the standard deviation of normal distributions. As an improved measure of risk, CVaR can overcome the shortcomings of VaR [17]. CVaR is a coherent risk measure having the properties of positively homogeneous, convex, *etc*. It can tell us how bad the investment could be, i.e. when the loss is greater than VaR, what is the expected loss.

$$CVaR = E\{Loss_{V_T^{\Pr}} \mid Loss_{V_T^{\Pr}} > VaR\} \quad (12)$$

Using the probabilistic density function and cumulative distribution function for generation investment project obtained from Monte Carlo simulation in section 3, these risk indexes VaR and CVaR can be calculated for long-term generation asset risk assessment.

For different payoff confidence levels for the investment, series of VaR and CVaR are achieved to support the decision of generation expansion investment.

5 ALGORITHM

The algorithm for generation asset valuation and risk assessment as proposed above can be summarized in the following steps:

Step-1. Approximate periodic function from historic market data of daily electricity price curve and weekly and seasonally fluctuation factors to construct the long-term periodic mean for electricity price stochastic process.

Step-2. Solve mean reversion stochastic process with the long-term periodic mean by the simulation based method to achieve the process of electricity price with uncertainties.

Step-3. Apply real option approach on generation asset valuation to estimate the value and profit during period T of the investment.

Step-4. Use Monte Carlo simulation method to get the probabilistic distribution of profit for generation asset investment during period T.

Step-5. Evaluate the risk of the investment on generation expansion project by applying VaR and CVaR approaches to support the decision.

6 NUMERICAL EXAMPLES

A numerical example is given in this section.

6.1 Long-term Periodic Mean for Electricity Price

The test data comes from California electricity market. A period of one year from 1998 to 1999 is used to derive the periodic characteristic of electricity price. By approximating data in different time scale, the daily, weekly and seasonally periodic functions for the long-term periodic mean of electricity price can be obtained.

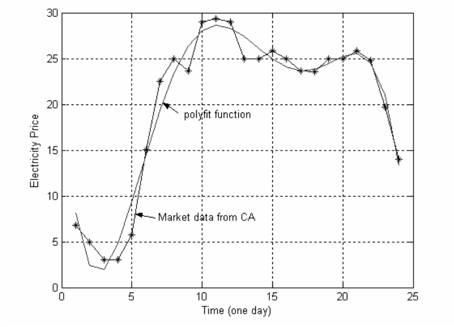

Figure 1: Daily electricity price polynomial approximation.

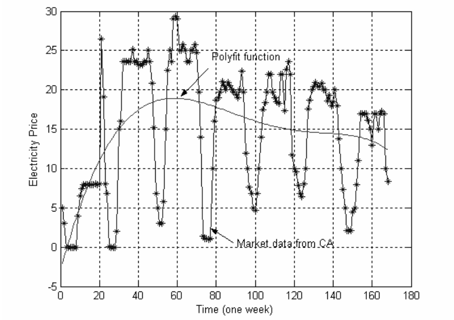

Figure 2: Polynomial approximation for weekly fluctuation function.

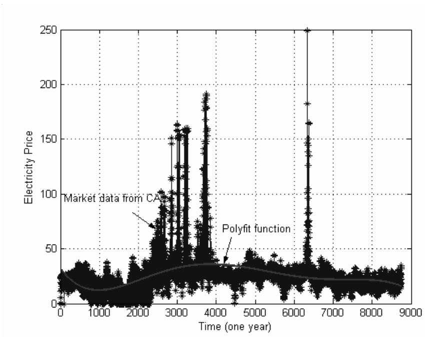

Figure 3: Polynomial approximation for seasonally fluctuation function.

Fig. 1 shows the daily polynomial approximation curve for electricity. Fig. 2 and 3 give the fluctuate functions of weekly and seasonally electricity price respectively. The lines with stars are the historic data from the CA market, and the smooth ones are the periodic functions obtained from polynomial fitness.

In order to construct the long-term periodic mean, daily, weekly and seasonally fluctuations are converted to per. unit with respect to the selected base price level. Thus the periodic function for long-term electricity price mean can be achieved, which is shown in Fig. 4.

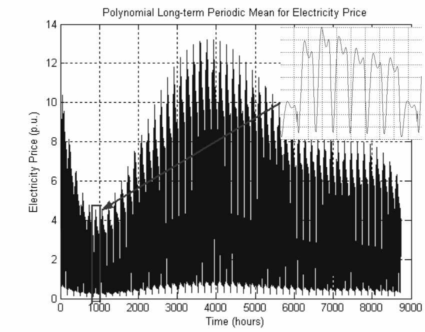

Figure 4: Long-term periodic mean for electricity price.

6.2 Mean Reversion Stochastic Process for Electricity Price

Base on the periodic mean for electricity price obtained in subsection 6.1, a simulation based method is applied to solve the mean reversion stochastic process to obtain the solution of the stochastic electricity process.

Assume that the base price of electricity in the market is 1.0($/MWh), the volatility of electricity price is $s = 0.1$, the mean-reverting coefficient $k = 0.5$, market risk free rate for one year $r = 0.15$, and the risk adjust discount rate for one year $a = 0.2$. The long-team mean reversion stochastic process for electricity price with periodical characteristic can be obtained by (4).

Fig. 5 shows the stochastic process for electricity price with consideration of periodic characteristics. It can be clearly found that the stochastic process of price follows the long-term periodic mean in Fig. 4. The uncertainties in the market and system operation are reflected by the randomness of the electricity price around mean price.

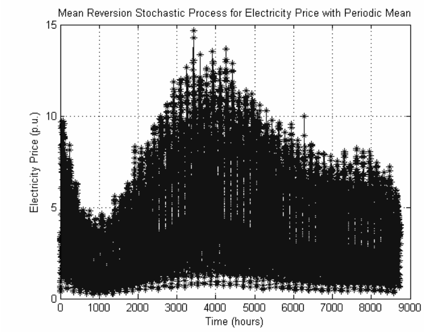

Figure 5: Mean reversion stochastic process for electricity price with periodic mean.

6.3 Evaluation for Generation Asset using Real Option Approach

In order to obtain the value of generation asset, real option approach with the spread of electricity and fuel price is used to derive the long-term value of the investment. For the given parameters in Table 1, the generator output *q(t)* and value *V(t)* on time *t* can be calculated. Fig. 6 and 7 shows the simulation results respectively.

Assumed Parameters	Value
P^F	0.6 ($/MMBtu)
a_0 (equivalent unit)	800
a_1 (equivalent unit)	9.00
a_2 (equivalent unit)	0.0011
q_{max} (equivalent unit)	3000 (MW)
q_{min} (equivalent unit)	600 (MW)

Table 1: Assumed parameters used in the example.

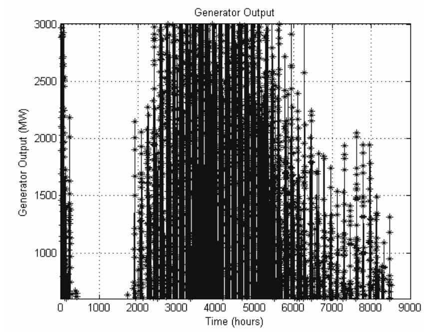

Figure 6: Generator output with capacity constraints.

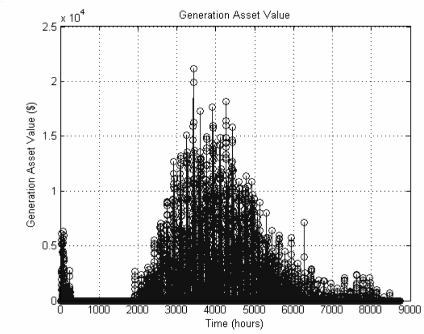

Figure 7: Generation asset value by real option approach.

Thus the present value of the generation asset during the period T can be obtained by (8):

$$V_T = 7.4494 \times 10^6 (\$)$$

By applying Monte Carlo simulation, the distribution of V_T can be achieved. Choose 1000 as the size of the sample set, the probability distribution of generation asset value V_T is obtained. Fig. 8 shows the empirical probabilistic density function curve of V_T.

Assume that the investment of the generation project (for one year) is 7.2×10^6 (\$), thus the *profit* of the investment in this year can be calculated by (9). Fig. 9 shows the empirical probabilistic distribution curve of profit for investment during period T.

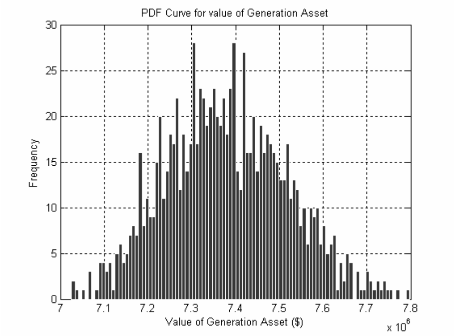

Figure 8: Empirical probabilitistic density function for value of generation asset.

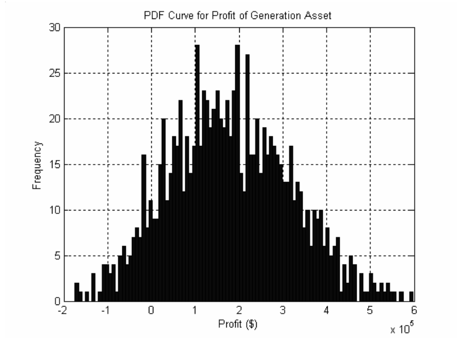

Figure 9: Empirical probabilitistic density function for profit of generation asset.

6.4 Risk Assessment of Generation Investment

In order to manage the risk on the investment of generation asset, risk assessment tool VaR and CVaR are used. Suppose that GENCO's payoff confidence level for the investment is 95%. According to the definition in (11), the risk evaluation index VaR of the investment on this expansion project can be obtained based on the CDF curve. Negative profit on the curve means the loss for the investment project.

$$VaR = 0.32901\times10^5 \$$$

The index CVaR can be calculated as the following:

$$CVaR = 0.75622\times10^5 \$,$$

which is the expectation when loss is larger than VaR.

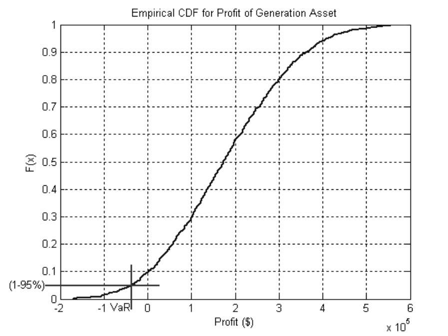

Figure 10: Empirical cumulative distribution function for profit of generation asset.

With respect to different payoff confidence levels for the GENCO, a series of CVaR are achieved to support the decision of generation expansion investment. Fig. 11 shows the risk assessment results.

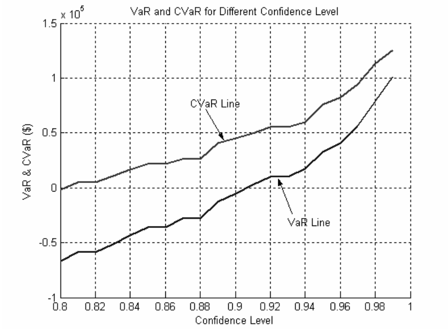

Figure 11: VaR and CVaR lines for different confidence level for generation expansion investment.

7 CONCLUSIONS

This paper presents an improved approach to generation expansion planning under the new deregulated environment. The periodic characteristic of electricity price is taken into account in the proposed model. A mean reversion model of with periodic mean is used in the spread real option approach for generation asset evaluation. In order to manage the risks of the investment on generation expansion project, risk assessment tools VaR and CVaR are used to help the investors making informed investment decision. A numerical example shows the validity of the method.

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