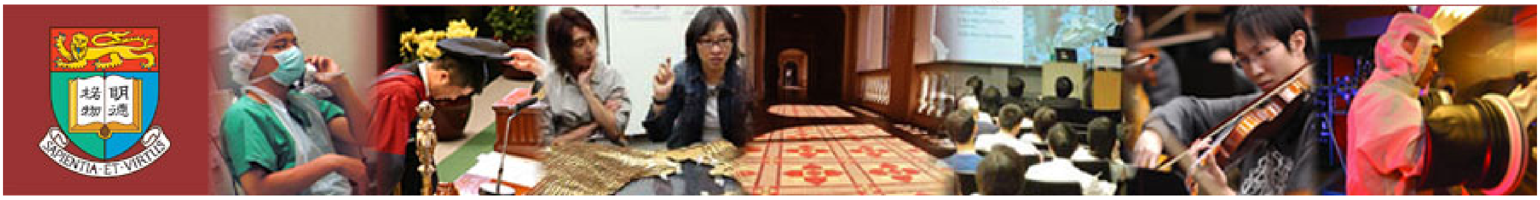

Improved Iterative Non-Cartesian SENSE Reconstruction Using Inner-regularization

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Introduction

The conjugate-gradient (CG)-based non-Cartesian SENSE reconstruction [1] in many cases exhibits unstable convergence behavior. This is because the generalized encoding matrix (GEM) is usually seriously ill-conditioned due to the large dimension and the mixed encoding scheme [2]. To overcome this difficulty, an improved iterative SENSE approach is presented. During a so-called Lanczos iteration process which is equally efficient as CG [3], the inversion of GEM can be approximated by calculating inversions of small tridiagonal matrices. In this fashion, inner-regularization can be incorporated into the reconstruction without touching the iteration process. With inner-regularization adaptively applied for every iteration loop, the convergence behavior of iterative SENSE can be significantly improved and noise booming can be avoided.

Theory

Generally parallel imaging reconstruction for arbitrary k-space trajectories can be simply formulated as solving a linear equation system $s = Em$, where *s* contains signal samples, *m* is the vector of the unknown image and *E* is the GEM composed of gradient encoding and coil sensitivity encoding. The least squares estimate can be obtained by iteratively solving the normal equation, which reads $(E^H E)m = E^H s$ (1).

The principle of Lanczos method is projecting a large Hermitian matrix onto a set of suitably chosen orthogonal vectors by an iteration process so that it is reduced to a much smaller matrix. In SENSE, Lanczos method is applied to Eq. (1). Let $A = E^H E$ is an *n*-by-*n* Hermitian matrix, the stepwise nature of the Lanczos process results in a (*j*+1)×(*j*+1) tridiagonal matrix T_j and in a unitary matrix $Q_j = [q_1, q_2, ...q_j], j = 1,...,n$ after *j* iteration loops, which are related as $AQ_j = Q_j T_j + b_j q_{j+1} e_j^T$ (2), where b_j decreases with *j* and $b_j \to 0$ when $j \to$ rank(*A*). In this work the Lanczos algorithm is adopted from Ref. [3]. Assuming that b_j becomes numerically negligible when $j \geq r$ (in practice usually $r << n$), such that Eq.(2) becomes $A = Q_j T_j Q_j^H$, and since Q_j is unitary, inverse of *A* can be simplified as $A^{-1} = Q_j (T_j)^{-1} Q_j^H$ (3).

Numerically the Lanczos process is equally efficient as a typical CG algorithm. Also, the Lanczos method holds a unique desirable property: the singular values (SV) of *A* are gradually captured in decreasing order by the small tridiagonal matrix T_j, i.e., the *j* SVs of T_j are approximations of the *j* largest SVs of *A*, with higher accuracy for larger SVs and more iterations. Taking advantage of the property of Lanczos process, regularization can be applied only for inversion of T_j in Eq. (3). Since the SVs of *A* are captured by T_j in decreasing order, regularization by directly manipulating the SV components in $(T_j)^{-1}$ is obvious. In this study we set a SV threshold as 1% of the largest SV, and simply disregard all the SV components below that threshold.

In summary, Lanczos process provides possibility to apply regularization into the inversion without touching the iterations (so this method is called inner-regularization); meanwhile, it provides SV information to determine the degree of regularization.

Results

Radial and spiral experiments were performed with a homogenous sphere phantom. A full radial dataset of 128 projections and a full spiral dataset of 4 interleaves were acquired. These data were then decimated to simulate accelerated cases. Iterative SENSE reconstructions using inner-regularization and using conventional CG method were performed with the radial and spiral data, respectively. Matrix-vector multiplications were performed using the gridding/FFT procedure, where gridding were implemented using the LS-nuFFT method [4].

The reconstruction errors of phantom images from 8X accelerated radial data (16 projections) and 2X accelerated spiral data (2 interleaves) varying with iteration count are shown in Figs. 1a and 1b, respectively. The reconstruction error is measured by $err(j) = [\min_\alpha \sum_n (I_n(j) - \alpha I_n^{ref})^2]^{1/2}$, where $I_n(j)$ is the image resulted after *j* iterations with *n* denoting the pixel index; I_n^{ref} is the reference image reconstructed from the corresponding full datasets. The solid lines are the results of conventional CG-based method, and the dashed ones are their counterparts using inner-regularization. The curves clearly show that without regularization, the CG iteration does not converge stably. The image quality improves with the iterations in early stages but deteriorates in later stages. With the inner-regularization strategy, the convergence behavior of the reconstruction is significantly improved, as shown by the dashed lines.

As an example, Fig. 2 shows the radial phantom images after 30 iterations using the conventional CG method (Fig. 2a) and inner-regularization (Fig. 2b), respectively. Observe that the resulted image after 30 CG iterations is contaminated by noise, while the inner-regularization result exhibits excellent compromise between noise and artifacts.

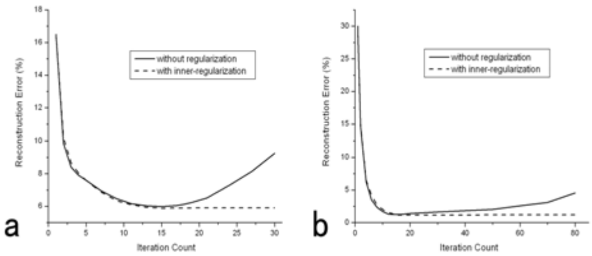

Fig. 1. Reconstruction errors vs. iteration count for SENSE phantom images with (**a**) radial trajectories; (**b**) spiral trajectories.

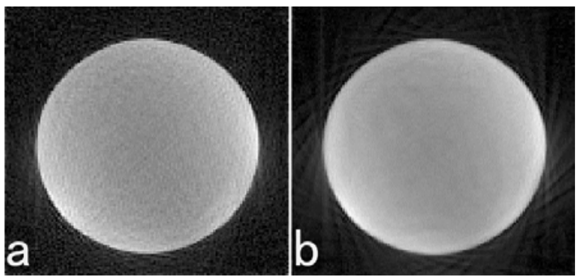

Fig. 2. 30-iteration reconstructions of radial SENSE images using (**a**)conventional CG-based method; (**b**)inner-regularization method.

Discussion

To handle the ill-conditioning issue, one may consider directly applying Tikhonov regularization to Eq. (1), which can be written as $(E^H E + \lambda I)m = E^H s$. However, we remark that this is not quite feasible. It is very difficult to choose the regularization parameter λ because each new λ entails a new iteration process. In contrast, in our method based on Lanczos iteration process, the degree of inner-regularization can be suitably determined by SV information.

Conclusion

An improved algorithm for iterative SENSE reconstruction has been proposed. Based on the Lanczos iteration process, inner-regularization can be applied adaptively to stabilize the reconstruction and avoid noise amplification.

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