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# Inflation and late time acceleration in braneworld cosmological models with varying brane tension

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## Abstract

Braneworld models with variable brane tension  $\lambda$  introduce a new degree of freedom that allows for evolving gravitational and cosmological constants, the latter being a natural candidate for dark energy. We consider a thermodynamic interpretation of the varying brane tension models, by showing that the field equations with variable  $\lambda$  can be interpreted as describing matter creation in a cosmological framework. The particle creation rate is determined by the variation rate of the brane tension, as well as by the brane-bulk energy-matter transfer rate. We investigate the effect of a variable brane tension on the cosmological evolution of the Universe, in the framework of a particular model in which the brane tension is an exponentially dependent function of the scale factor. The resulting cosmology shows the presence of an initial inflationary expansion, followed by a decelerating phase, and by a smooth transition towards a late accelerated de Sitter type expansion. The varying brane tension is also responsible for the generation of the matter in the Universe (reheating period). The physical constraints on the model parameters, resulted from the observational cosmological data, are also investigated.

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## I. INTRODUCTION

The idea of embedding our Universe in a higher dimensional space has attracted a considerable interest recently, due to the proposal by Randall and Sundrum that our four-dimensional (4D) spacetime is a three-brane, embedded in a 5D spacetime (the bulk) [1, 2]. This proposal is based on early studies on superstring theory and M-theory, which have suggested that our four dimensional world is embedded into a higher dimensional spacetime. Particularly, the 10 dimensional  $E_8 \otimes E_8$  heterotic superstring theory is a low-energy limit of the 11 dimensional supergravity, under the compactification scheme  $M^{10} \times S_1/Z_2$  [3, 4]. Thus, the 10 dimensional spacetime is compactified as  $M^4 \times CY^6 \times S_1/Z_2$ , implying that our Universe (a brane) is embedded into a higher dimensional bulk. In this paradigm, the standard model particles are open strings, confined on the braneworld, whilst the gravitons and the closed strings can freely propagate into the bulk [5].

The Randall-Sundrum Type II model has the virtue of providing a new type of compactification of gravity [1, 2]. Standard 4D gravity can be recovered in the low-energy limit of the model, with a 3-brane of positive tension embedded in 5D anti-de Sitter bulk. The covariant formulation of the braneworld models has been formulated in [6], leading to the modification of the standard Friedmann equations on the brane. It turns out that the dynamics of the early Universe is altered by the quadratic terms in the energy density and by the contribution of the components of the bulk Weyl tensor, which both give a contribution in the energy momentum tensor. This implies a modification of the basic equations describing the cosmological and astrophysical dynamics, which has been extensively considered recently [7].

The recent observations of the CMB anisotropy by WMAP [10] have provided convincing evidence for the inflationary paradigm [11], according to which in its very early stages the Universe experienced an accelerated (de Sitter) expansionary phase (for recent reviews on inflation see [12]).

At the end of inflation, the Universe is in a cold and low-entropy phase, which is utterly different from the present hot high-entropy Universe. Therefore the Universe should be reheated, or defrosted, to a high enough temperature, in order to recover the standard Hot Big Bang [13]. The reheating process may be envisioned as follows: the energy density in zero-momentum mode of the scalar field decays into normal particles with decay rate  $\Gamma$ . The

decay products then scattered and thermalize to form a plasma [12].

Apart from the behavior of the inflaton field, the evolutions of dark energy and dark matter in reheating stage were also considered. In [14], dark energy and dark matter were originated from a scalar field in different stages of the inflation, according to a special form of potential. Meanwhile, the conditions for unifying the description of inflation, dark matter and dark energy were considered in [15]. A specific model was later proposed in [16], by using a modified quadratic scalar potential. The candidates of dark matter in [15] and [16] were oscillations of a scalar field. However, it may be possible that dark matter existed on its own without originating from the scalar field. This may pose less stringent constraint on the scalar field, so that dark matter can be included in inflation paradigm in a easier way. On the other hand, it was proposed that the decay products of scalar field acquired thermal mass [17].

The reheating in the braneworld models has also been considered recently. In the context of the braneworld inflation driven by a bulk scalar field, the energy dissipation from the bulk scalar field into the matter on the brane was studied in [18]. The obtained results supports the idea that the brane inflation model, caused by a bulk scalar field, may be a viable alternative scenario of the early Universe. The inflation and reheating in a braneworld model derived from Type IIA string theory was studied in [19]. In this model the inflaton can decay into scalar and spinor particles, thus reheating the Universe. A model in which high energy brane corrections allow a single scalar field to describe inflation at early epochs and quintessence at late times was discussed in [20]. The reheating mechanism in the model originates from Born-Infeld matter, whose energy density mimics cosmological constant at very early times and manifests itself as radiation subsequently. The particle production at the collision of two domain walls in a 5-dimensional Minkowski spacetime was studied in [21]. This may provide the reheating mechanism of an ekpyrotic (or cyclic) brane Universe, in which two BPS branes collide and evolve into a hot big bang Universe. The reheating temperature  $T_{\text{RH}}$  in models in which the Universe exits reheating at temperatures in the MeV regime was studied in [22], and a minimum bound on  $T_{\text{RH}}$  was obtained. The derived lower bound on the reheating temperature also leads to very stringent bounds on the compactification scale in models with  $n$  large extra dimensions. The dark matter problem in the Randall-Sundrum type II braneworld scenario was discussed in [23], by assuming that the lightest supersymmetric particle is the axino. The axinos can play the role of cold

dark matter in the Universe, due to the higher reheating temperatures in the braneworld model, as compared to the conventional four-dimensional cosmology. The impact of the non-conventional brane cosmology on the relic abundance of non-relativistic stable particles in high and low reheating scenarios was investigated in [24]. In the case of high reheating temperatures, the brane cosmology may enhance the dark matter relic density by many order of magnitudes, and a stringent lower bound on the five dimensional scale may be obtained. In the non-equilibrium case, the resulting relic density is very small. The curvaton dynamics in brane-world cosmologies was studied in [25].

Brane-worlds with non-constant tension, based on the analogy with fluid membranes, which exhibit a temperature-dependence according to the empirical law established by Eötvös, were introduced in [26]. This new degree of freedom allows for evolving gravitational and cosmological constants, the latter being a natural candidate for dark energy. The covariant dynamics on a brane with variable tension was studied in its full generality, by considering asymmetrically embedded branes, and allowing for non-standard model fields in the 5-dimensional space-time. This formalism was applied for a perfect fluid on a Friedmann brane, which is embedded in a 5-dimensional charged Vaidya-Anti de Sitter space-time. For cosmological branes a variable brane tension leads to several important consequences. A variable brane tension may remove the initial singularity of the Universe, since the brane Universe was created at a finite temperature  $T_c$  and scale factor  $a_{\min}$  [27]. Both the brane tension and the 4-dimensional gravitational coupling 'constant' increase with the scale factor from zero to asymptotic values. The 4-dimensional cosmological constant is dynamical, evolving with  $a$ , starting with a huge negative value, passing through zero, and finally reaching a small positive value. Such a scale-factor dependent cosmological constant has the potential to generate additional attraction at small  $a$  (as dark matter does) and late-time repulsion at large  $a$  (dark energy). The evolution of the brane tension is compensated by energy interchange between the brane and the fifth dimension, such that the continuity equation holds for the cosmological fluid [27]. The resulting cosmology closely mimics the standard model at late times, a decelerated phase being followed by an accelerated expansion. The energy absorption of the brane drives the 5D space-time towards maximal symmetry, thus becoming Anti de Sitter. Other physical and cosmological implications of a varying brane tension have been considered in [28].

It is the purpose of the present paper to further investigate the cosmological implications

of a varying brane tension. As a first step in our study, we consider a thermodynamic interpretation of the varying brane tension models, by showing that the field equations with variable  $\lambda$  can be interpreted as describing matter creation in a cosmological framework. The particle creation rate is determined by the variation rate of the brane tension, as well as by the brane-bulk energy-matter transfer rate. In particular, by adopting a theoretical model in which the brane tension is a simple function of the scale factor of the Universe, we consider the possibility that the early inflationary era in the evolution of the brane Universe was driven by a varying brane tension. A varying brane tension may also be responsible for the generation of the matter after reheating, as well as for the late time acceleration of the Universe.

The present paper is organized as follows. In Section II we present the field equations of the brane world models with varying brane tension and we write down the basic equations describing the cosmological dynamics of a flat Friedmann-Robertson-Walker Universe. The thermodynamic interpretation of the brane-world models with varying brane tension and brane-bulk matter-energy exchange is considered in Section III. A power-law inflationary brane-world model with varying brane tension and non-zero bulk pressure is obtained in Section IV. The analytical behavior of the cosmological model with varying brane tension is considered in Section V, by using the small and large time approximations for the brane tension. The numerical analysis of the model is performed in Section VII. We discuss and conclude our results in Section VIII.

## II. GEOMETRY AND FIELD EQUATIONS IN THE VARIABLE BRANE TENSION MODELS

In the present Section we present the field equations for brane world models with varying brane tension, and the corresponding cosmological field equations for a flat Robertson-Walker space-time.

### A. Gravitational field equations

We start by considering a five dimensional ( $5D$ ) spacetime (the bulk), with a large negative  $5D$  cosmological constant  ${}^{(5)}\Lambda$  and a single four-dimensional ( $4D$ ) brane, on which

usual (baryonic) matter and physical fields are confined. The 4D braneworld  $(^{(4)}M, ^{(4)}g_{\mu\nu})$  is located at a hypersurface  $(B(X^A) = 0)$  in the 5D bulk spacetime  $(^{(5)}M, ^{(5)}g_{AB})$  with mirror symmetry, and with coordinates  $X^A, A = 0, 1, \dots, 4$ . The induced 4D coordinates on the brane are  $x^\mu, \mu = 0, 1, 2, 3$ . We choose normal Gaussian coordinates, and therefore the 5D metric is related to the 4D metric by the relation  $^{(5)}g_{MN} = ^{(4)}g_{MN} + n_M n_N$ , where  $n^M$  is the normal vector.

The induced 4D metric is  $g_{IJ} = ^{(5)}g_{IJ} - n_I n_J$ , where  $n_I$  is the space-like unit vector field normal to the brane hypersurface  $^{(4)}M$ . The basic equations on the brane are obtained by projections onto the brane world with Gauss equation, Codazzi equation and Israel junction condition, the projected Einstein equation are given by

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + k^2 T_{\mu\nu} + \bar{k}^4 S_{\mu\nu} - \bar{\varepsilon}_{\mu\nu} + \bar{L}_{\mu\nu}^{TF} + \bar{P}_{\mu\nu} + F_{\mu\nu}, \quad (1)$$

where

$$S_{\mu\nu} = \frac{1}{2} T T_{\mu\nu} - \frac{1}{4} T_{\mu\alpha} T_\nu^\alpha + \frac{3T_{\alpha\beta} T^{\alpha\beta} - T^2}{24} g_{\mu\nu}, \quad (2)$$

$$\varepsilon_{\mu\nu} = C_{ABCD} n^C n^D g_\mu^A g_\nu^B, \quad (3)$$

and

$$F_{\mu\nu} = ^{(5)} T_{AB} g_\mu^A g_\nu^B + \left( ^{(5)} T_{AB} n^A n^B - \frac{1}{4} ^{(5)} T \right) g_{\mu\nu}, \quad (4)$$

respectively.

Apart from the terms quadratic in the brane energy-momentum tensor, in the field equations on the brane there are two supplementary terms, corresponding to the projection of the 5D Weyl tensor  $\varepsilon_{\mu\nu}$  and of the projected tensor  $F_{\mu\nu}$ , which contains the bulk matter contribution. Both terms induce bulk effects on the brane.

Also, the possible asymmetric embedding is characterized by the tensor

$$\bar{L}_{\mu\nu} = \bar{K}_{\mu\nu} \bar{K} - \bar{K}_{\mu\sigma} \bar{K}_\nu^\sigma - \frac{g_{\mu\nu}}{2} (\bar{K}^2 - \bar{K}_{\alpha\beta} \bar{K}^{\alpha\beta}), \quad (5)$$

with trace  $\bar{L} = \bar{K}_{\alpha\beta} \bar{K}^{\alpha\beta} - \bar{K}^2$ , and trace-free part  $\bar{L}_{\mu\nu}^{TF} = \bar{K}_{\mu\nu} \bar{K} - \bar{K}_{\mu\sigma} \bar{K}_\nu^\sigma + \bar{L} g_{\mu\nu} / 4$ , respectively.

For a  $Z_2$  symmetric embedding  $\bar{K}_{\mu\nu} = 0$ , and thus  $\bar{L}_{\mu\nu} = 0$ .  $\bar{P}_{\mu\nu}$  is given by the pull-back to the brane of the energy-momentum tensor characterizing possible non-standard model fields (e. g. scalar, dilaton, moduli, radiation of quantum origin) living in 5D,

$$\bar{P}_{\mu\nu} = \frac{2\tilde{k}^2}{3} \overline{\left( g_\mu^\alpha g_\nu^\beta ^{(5)} T_{\alpha\beta} \right)^{TF}}, \quad (6)$$

which is traceless by definition. Another projection of the 5D sources appears in the brane cosmological constant  $\Lambda$ , which is defined as

$$\Lambda = \Lambda_0 - \frac{\bar{L}}{4} - \frac{2\tilde{k}^2}{3} \overline{(n^\alpha n^\beta)^{(5)} T_{\alpha\beta}}, \quad (7)$$

where  $2\Lambda_0 = k_5^2 \lambda + k_5^2 \Lambda_5$ .

In the case of a variable brane tension, the projected gravitational field equations on the brane have a similar form to the general case,

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + k^2 T_{\mu\nu} + \bar{k}^4 S_{\mu\nu} - \bar{\epsilon}_{\mu\nu} + \bar{L}_{\mu\nu}^{TF} + \bar{P}_{\mu\nu} + F_{\mu\nu}. \quad (8)$$

However, the evolution of the brane tension appears in the Codazzi equation, and in the differential Bianchi identity. The Codazzi equation is

$$\nabla_\mu \bar{K}_\nu^\mu - \nabla_\nu \bar{K} = k_5^2 \overline{(g_\nu^\rho n^\sigma)^{(5)} T_{\rho\sigma}}, \quad (9)$$

and it gives the conservation equation of the matter on the brane as

$$\nabla_\mu T_\nu^\mu = \nabla_\nu \lambda - \Delta(g_\nu^\rho n^\sigma)^{(5)} T_{\rho\sigma}. \quad (10)$$

The differential Bianchi identity, written as  $\nabla^\mu R_{\rho\mu} = \frac{1}{2} \nabla_\rho R$ , gives

$$\begin{aligned} \nabla^\mu (\bar{\epsilon}_{\mu\nu} - \bar{L}_{\mu\nu}^{TF} - \bar{P}_{\mu\nu}) &= \frac{\nabla_\nu \bar{L}}{4} + \frac{k_5^2}{2} \nabla_\nu \overline{(n^\rho n^\sigma)^{(5)} T_{\rho\sigma}} - \frac{k_5^4 \lambda}{6} \Delta(g_\nu^\sigma n^\rho T_{\sigma\rho}) \\ &+ \frac{k_5^4}{4} (T_\nu^\mu - \frac{T}{3} g_\nu^\mu) \Delta(g_\mu^\sigma n^\rho)^{(5)} T_{\sigma\rho} + \frac{k_5^4}{4} [2T^{\mu\sigma} \nabla_{[\nu} T_{\mu]\sigma} \\ &+ \frac{1}{3} (T_{\mu\nu} \nabla^\mu T - T \nabla_\nu T)] - \frac{k_5^4}{12} (T_\nu^\mu - T g_\nu^\mu) \nabla_\mu \lambda. \end{aligned} \quad (11)$$

From Eq. (3), one can introduce an effective non-local energy density  $U$ , which can be obtained by assuming that  $\varepsilon_{\mu\nu}$  in the projected Einstein equation behaves as an effective radiation fluid,

$$-\varepsilon_{\mu\nu} = \frac{k_5^4}{6} \lambda U (u_\mu u_\nu + \frac{a^2}{3} h_{\mu\nu}), \quad (12)$$

where  $u_\mu$  is the matter four-velocity, and  $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ , respectively.

## B. Cosmological models with dynamic brane tension

We assume that the metric on the brane is given by the flat Robertson-Walker-Friedmann metric, with

$${}^{(4)}g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (13)$$



where  $a$  is the scale factor. The matter on the brane is assumed to consist of a perfect fluid, with energy density  $\rho$ , and pressure  $p$ , respectively. The gravitational field equations, governing the evolution of the brane Universe with variable brane tension, in the presence of brane-bulk energy transfer, and with a non-zero bulk pressure, are then given by [26, 27]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{{}^{(4)}\Lambda}{3} + \frac{k_5^4 \lambda}{18} \left[ \rho + \frac{\rho^2}{2\lambda} + U \right], \quad (14)$$

$$\frac{\ddot{a}}{a} = \frac{{}^{(4)}\Lambda}{3} - \frac{k_5^4 \lambda}{36} \left[ \rho \left(1 + \frac{2\rho}{\lambda}\right) + 3p \left(1 + \frac{\rho}{\lambda}\right) + 2U \right], \quad (15)$$

$$\dot{\rho} + 3H(\rho + p) = -\dot{\lambda} - 2P_5, \quad (16)$$

$$\frac{k_5^4 \lambda}{6} \left( \dot{U} + 4U \frac{\dot{a}}{a} + U \frac{\dot{\lambda}}{\lambda} \right) = \frac{k_5^2}{2} \dot{P}_B + \frac{k_5^4 \lambda}{3} \left(1 + \frac{\rho}{\lambda}\right) P_5, \quad (17)$$

$${}^{(4)}\Lambda = \frac{k_5^2}{2} \Lambda_5 + \frac{k_5^4}{12} \lambda^2 - \frac{k_5^2}{2} \bar{P}_B, \quad (18)$$

where  $P_5$  describes the bulk-brane matter-energy transfer, while  $P_B$  is the bulk pressure.

An important observational parameter, which is an indicator of the rate of expansion of the Universe, is the deceleration parameter  $q$ , defined as

$$q = \frac{d}{dt} \left( \frac{1}{H} \right) - 1 = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}/a}{(\dot{a}/a)^2}. \quad (19)$$

If  $q < 0$ , the expansion of the Universe is accelerating, while  $q > 0$  indicates a decelerating phase.

### III. THERMODYNAMIC INTERPRETATION OF THE VARYING TENSION IN BRANE-WORLD MODELS

For the sake of generality we also assume that there is an effective energy-matter transfer between the brane and the bulk, and the brane-bulk matter-energy exchange can be described as

$$P_5 = -\frac{\alpha_{bb}}{2} \rho_{cr} \left( \frac{a_0}{a} \right)^{3w} H, \quad (20)$$

where  $\alpha_{bb}$  is a constant,  $\rho_{cr}$  is the present day critical density of the Universe, and  $a_0$  is the present day value of the scale factor.

In the presence of a varying brane tension and of the bulk-brane matter and energy

exchange, the energy conservation equation on the brane can be written as

$$\dot{\rho} + 3(\rho + p)H = -\rho \left( \frac{\dot{\lambda}}{\rho} - \alpha_{bb}H \right), \quad (21)$$

where we have used Eq. (20) for the description of brane bulk energy transfer, by taking into account that  $\rho = \rho_{cr} (a_0/a)^{3w}$ . We suppose that the matter content of the early Universe is formed from  $m$  non-interacting comoving relativistic fluids with energy densities and thermodynamic pressures  $\rho_i(t)$  and  $p_i(t)$ ,  $i = 1, 2, \dots, m$ , respectively, with each fluid formed from particles having a particle number density  $n_i(t)$ ,  $i = 1, 2, \dots, m$ , and obeying equations of state of the form  $\rho_i(t) = k_i n_i^{\gamma_i}$ ,  $p_i(t) = (\gamma_i - 1)\rho_i$ ,  $i = 1, 2, \dots, m$ , where  $k_i = \rho_{0i}/n_{0i}^{\gamma_i} \geq 0$ ,  $i = 1, 2, \dots, m$ , are constants, and  $1 \leq \gamma_i \leq 2$ ,  $i = 1, 2, \dots, m$ . For example, we can consider that the particle content of the early Universe is determined by pure radiation (i. e., different types of massless particles, or massive matter (baryonic and dark) in equilibrium with electromagnetic radiation and decoupled massive particles. The total energy density and pressure of the cosmological fluid results from summing the contribution of the  $l$  simple fluid components, and are given by  $\rho(t) = \sum_{i=1}^m \rho_i(t)$  and  $p(t) = \sum_{i=1}^m p_i(t)$ , respectively. For a multicomponent comoving cosmological fluid and in the presence of variable brane tension and bulk-brane energy exchange, Eq. (21) becomes

$$\sum_{i=1}^l [\dot{\rho}_i + 3(\rho_i + p_i)H] = - \sum_{i=1}^m \rho_i(t) \left[ \frac{\dot{\lambda}}{\sum_{i=1}^m \rho_i(t)} - \alpha_{bb}H \right]. \quad (22)$$

Eq. (22) can be recast into the form of  $m$  particle balance equations,

$$\dot{n}_i(t) + 3n_i(t)H = \Gamma_i(t)n_i(t), \quad i = 1, 2, \dots, m, \quad (23)$$

where  $\Gamma_i(t)$ ,  $i = 1, 2, \dots, m$ , are the particle production rates, given by

$$\Gamma_i(t) = -\frac{1}{\gamma_i} \left[ \frac{\dot{\lambda}}{m\rho_i(t)} - \alpha_{bb}H \right], \quad i = 1, 2, \dots, m. \quad (24)$$

In order for Eq. (23) to describe particle production the condition  $\Gamma_i(t) \geq 0$ ,  $i = 1, 2, \dots, m$ , is required to be satisfied, leading to the following restriction imposed to the time variation rate of the brane tension

$$\dot{\lambda} \leq \alpha_{bb}l\rho_i(t)H, \quad i = 1, 2, \dots, m. \quad (25)$$

Note that if  $\Gamma_i(t) = 0$ ,  $i = 1, 2, \dots, m$ , we obtain the usual particle conservation law of the standard cosmology. Of course, the casting of Eq. (22) is not unique. In Eqs. (23) and (24),

we consider the simultaneous creation of a multicomponent comoving cosmological fluid, but other possibilities can be formulated in the same way (for example, creation of a single component in a mixture of fluids).

The entropy  $S_i$ , generated during particle creation at temperatures  $T_i$ ,  $i = 1, 2, \dots, m$ , can be obtained from Eq. (23), and for each species of particles has the expression

$$T_i \frac{dS_i}{dt} = -\frac{1}{\gamma_i} \left[ \frac{\dot{\lambda}}{m\rho_i(t)} - \alpha_{bb}H \right] \rho_i(t) V, i = 1, 2, \dots, m, \quad (26)$$

where  $V$  is the volume of the Universe, or, equivalently,

$$\frac{dS_i}{dt} = \frac{\gamma_i \rho_i(t) V}{T_i} \Gamma_i(t), i = 1, 2, \dots, m. \quad (27)$$

In a cosmological fluid where the density and pressure are functions of the temperature only,  $\rho = \rho(T)$ ,  $p = p(T)$ , the entropy of the fluid is given by  $S = (\rho + p) V/T = \gamma \rho(t) V/T$ . Therefore we can express the total entropy  $S(t)$  of the multicomponent cosmological fluid filled brane Universe as a function of the particle production rate only,

$$S(t) = \sum_{i=1}^m S_{0i} \exp \left[ \int_{t_0}^t \Gamma_i(t') dt' \right], \quad (28)$$

where  $S_{0i} \geq 0$ ,  $i = 1, 2, \dots, m$ , are constants of integration. In the case of a general perfect comoving multicomponent cosmological fluid with two essential thermodynamical variables, the particle number densities  $n_i$ ,  $i = 1, 2, \dots, m$ , and the temperatures  $T_i$ ,  $i = 1, 2, \dots, m$ , it is conventional to express  $\rho_i$  and  $p_i$  in terms of  $n_i$  and  $T_i$  by means of the equilibrium equations of state  $\rho_i = \rho_i(n_i, T_i)$ ,  $p_i = p_i(n_i, T_i)$ ,  $i = 1, 2, \dots, m$ . By using the general thermodynamic relation

$$\frac{\partial \rho_i}{\partial n_i} = \frac{\rho_i + p_i}{n_i} - \frac{T_i}{n_i} \frac{\partial p_i}{\partial T_i}, i = 1, 2, \dots, m, \quad (29)$$

in the case of a general comoving multicomponent cosmological fluid Eq. (21) can also be rewritten in the form of  $m$  particle balance equations,

$$\dot{n}_i(t) + 3n_i(t)H = \Gamma_i(t)n_i(t), i = 1, 2, \dots, m, \quad (30)$$

with the particle production rates  $\Gamma_i(t)$  given by some complicated functions of the thermodynamical parameters, brane tension and brane-bulk energy exchange rate,

$$\Gamma_i(t) = -\frac{\rho_i}{\rho_i + p_i} \left[ \frac{\dot{\lambda}}{l\rho_i(t)} - \alpha_{bb}H + T_i \frac{\partial \ln \rho_i}{\partial T_i} \left( \frac{\dot{T}_i}{T_i} - C_i^2 \frac{\dot{n}_i}{n_i} \right) \right], i = 1, 2, \dots, m, \quad (31)$$

where  $C_i^2 = (\partial p_i / \partial T_i) / (\partial \rho_i / \partial T_i)$ . The requirement that the particle balance equation Eq. (30) describes particle production,  $\Gamma_i(t) \geq 0$ ,  $i = 1, 2, \dots, m$ , imposes in this case the following constraint on the time variation of the brane tension,

$$\dot{\lambda} < \alpha_{bb} l \rho_i(t) H - l \rho_i(t) T_i \frac{\partial \ln \rho_i}{\partial T_i} \left( \frac{\dot{T}_i}{T_i} - C_i^2 \frac{\dot{n}_i}{n_i} \right), i = 1, 2, \dots, m. \quad (32)$$

In the general case the entropy generated during the reheating period due to the variation of the brane tension and the bulk-brane energy exchange can be obtained for each component of the cosmological fluid from the equations

$$\frac{dS_i}{dt} = \frac{(\rho_i + p_i) V}{T_i} \left[ \frac{\dot{\lambda}}{l \rho_i(t)} - \alpha_{bb} H + T_i \frac{\partial \ln \rho_i}{\partial T_i} \left( \frac{\dot{T}_i}{T_i} - C_i^2 \frac{\dot{n}_i}{n_i} \right) \right], i = 1, 2, \dots, m, \quad (33)$$

while the total entropy of the Universe is given by  $S(t) = \sum_{i=1}^m S_i(t)$ .

The entropy flux vector of the  $k$ th component of the cosmological fluid is given by

$$S^{(k)\alpha} = n_k \sigma_k u^\alpha, k = 1, 2, \dots, m, \quad (34)$$

where  $\sigma_k$ ,  $k = 1, 2, \dots, m$ , is the specific entropy (per particle) of the corresponding cosmological fluid component and  $u^\alpha$  is the four-velocity of the fluid. By using the Gibbs equation  $n T d\sigma = d\rho - [(\rho + p) / n] dn$  for each component of the fluid, and assuming that the entropy density  $\sigma$  does not depend on the brane tension, we obtain

$$S_{;\alpha}^{(k)\alpha} = -\frac{1}{T_k} \left( \dot{\lambda} - \alpha_{bb} H \rho \right) - \frac{\mu_k \Gamma_k n_k}{T_k}, k = 1, 2, \dots, m, \quad (35)$$

where  $\mu_k$  is the chemical potential defined by  $\mu_k = [(\rho_k + p_k) / n_k] - T_k \sigma_k$ . The chemical potential is zero for radiation. For each component of the cosmological fluid the second law of thermodynamics requires that the condition

$$S_{;\alpha}^{(k)\alpha} \geq 0, k = 1, 2, \dots, m, \quad (36)$$

has to be satisfied.

#### IV. POWER LAW INFLATION IN BRANE WORLD MODELS WITH VARYING BRANE TENSION AND BULK PRESSURE

For a vacuum Universe with  $\rho = p = 0$ , in the presence of a non-zero bulk pressure and matter- energy exchange between the brane and the bulk, the field equations Eqs. (14) take

the form

$$3 \left( \frac{1}{a} \frac{da}{d\tau} \right)^2 = \frac{l^2}{2} - p_B + lu, \quad (37)$$

$$3 \frac{1}{a} \frac{d^2 a}{d\tau^2} = \frac{l^2}{2} - p_B - lu, \quad (38)$$

$$\frac{dl}{d\tau} = -2\sqrt{\frac{2}{3}}p_5, \quad (39)$$

and

$$\frac{d}{d\tau} (lua^4) = -a^4 \frac{d}{d\tau} \left( \frac{l^2}{2} - p_B \right), \quad (40)$$

where we have introduced a set of dimensionless variables  $(\tau, l, p_B, u, p_5)$  defined as

$$\tau = \sqrt{\frac{3}{2}}t, \lambda = \frac{3}{k_5^2}l, P_B = \frac{3}{k_5^2}p_B, U = \frac{3}{k_5^2}u, P_5 = \frac{3}{k_5^2}p_5. \quad (41)$$

Moreover, we consider that the five-dimensional cosmological constant  $\Lambda_5 = 0$ . We assume that the inflationary evolution is of the power law type, and therefore  $a = \tau^\alpha$ , where  $\alpha$  is a constant. Then Eqs. (37) and (38) give

$$2 \left( \frac{l^2}{2} - p_B \right) = \frac{3\alpha(2\alpha - 1)}{\tau^2}, \quad (42)$$

and

$$2lu = \frac{3\alpha}{\tau^2}. \quad (43)$$

Eq. (40) is identically satisfied. In order to completely solve the problem, we need to specify the form of the energy matter-transfer from the bulk to the brane. By assuming a functional form given by  $p_5 = p_{05}\tau^{-\beta}$ , where  $\beta > 0$  and  $p_{05} > 0$  are constants, we obtain immediately

$$l(\tau) = \sqrt{\frac{8}{3}} \frac{1}{\beta - 1} \tau^{-\beta+1}, p_B(\tau) = \frac{4}{3} \frac{1}{(\beta - 1)^2} \tau^{-2(\beta-1)} - \frac{3\alpha(2\alpha - 1)}{2\tau^2},$$

$$u(\tau) = \sqrt{\frac{27}{32}} \alpha (\beta - 1) \tau^{\beta-3}. \quad (44)$$

The Hubble parameter of the Universe during the inflationary phase is given by  $H = \alpha/t$ . The deceleration parameter is obtained as  $q = d(1/H)/dt - 1 = (1 - \alpha)/\alpha$ . Therefore, if  $\alpha > 1$ ,  $q < 0$ , and the brane world Universe experiences an inflationary expansion.

## V. SCALE FACTOR DEPENDENT BRANE TENSION MODELS

In the following we assume that there is no matter-energy exchange between the bulk and the brane,  $P_5 = 0$ , and that the bulk pressure is also zero,  $P_B = 0$ . For the matter on the brane we adopt as equation of state a linear barotropic relation between density and pressure, given by

$$p = (w - 1)\rho, \quad (45)$$

where  $w = \text{constant}$  and  $w \in [1, 2]$ . Therefore Eq. (16) gives

$$\dot{\rho} + 3Hw\rho = -\dot{\lambda}, \quad (46)$$

while Eq. (17) gives immediately

$$\lambda U = \frac{U_0}{a^4}, \quad (47)$$

where  $U_0$  is an arbitrary constant of integration. In the following, in order to simplify the analysis, we assume that  $U_0 = 0$ .

In order to explain the main observational features of modern cosmology (inflation, reheating, deceleration period and late time acceleration, respectively), we assume that the brane tension varies as a function of the scale factor  $a$  according to the equation

$$\lambda^2 = \lambda_0^2 e^{-2\beta a^2} - \frac{6^{(5)}\Lambda}{k_5^2} + \lambda_1^2, \quad (48)$$

where  $\beta$ ,  $\lambda_0$  and  $\lambda_1$  are constants.

Suppose  $t_{\text{in}}$ ,  $a_{\text{in}}$  and  $\rho_{\text{in}}$  are the values of the time, of the scale factor, and of the energy density before inflation. Generally, in the present paper we use the subscript “in” to denote the values of the cosmological parameters before the inflation, and the subscript “en” to denote values after inflation. Thus, for example,  $N = \ln(a_{\text{en}}/a_{\text{in}})$  is the e-folding number. The basic physical parameters of our model are  $t_{\text{in}}$ ,  $a_{\text{in}}$ ,  $\rho_{\text{in}}$ ,  $N$ ,  $k_5$ ,  ${}^{(5)}\Lambda$ ,  $\lambda_0$ ,  $\lambda_1$ , and  $\beta$ , respectively. The coupling constant  $k_5$  and the five-dimensional cosmological constant  ${}^{(5)}\Lambda$  are constrained by the present value of the gravitational constant,

$$\frac{k_5^4}{6} \sqrt{-\frac{6^{(5)}\Lambda}{k_5^2} + \lambda_1^2} \approx k_5^3 \sqrt{-\frac{{}^{(5)}\Lambda}{6}} = 8\pi G \approx 1.68 \times 10^{-55} \text{eV}^{-2}, \quad (49)$$

and by the constraints on the 5D cosmological constant [9]

$$\frac{k_5^2}{2} {}^{(5)}\Lambda \approx -\frac{6}{(0.1\text{mm})^2} \approx -2.3 \times 10^{-5} \text{eV}^2, \quad (50)$$

where we have used the natural system of units with  $\hbar = c = 1$ . From these two conditions, we obtain  $k_5^4 \approx 3.6 \times 10^{-105} \text{ eV}^{-6}$  and  ${}^{(5)}\Lambda \approx -7.7 \times 10^{46} \text{ eV}^5$ . Besides, the value of  $\lambda_1$  can be obtained from the value of the present day dark energy  $\rho_{\text{dark}} \approx 10^{-12} \text{ eV}^4$  [8],

$$\frac{k_5^4}{12} \lambda_1^2 = 8\pi G \rho_{\text{dark}} \approx 1.6 \times 10^{-67} \text{ eV}^2, \quad (51)$$

which gives  $\lambda_1 \approx 1.6 \times 10^{19} \text{ eV}^4$ . We can have a backward checking on Eq. (49), from which it follows that the condition  $\lambda_1^2 \ll -6{}^{(5)}\Lambda/k_5^2$  is indeed satisfied. We also choose  $\lambda_0$  to be of the same order of magnitude as the vacuum energy  $\rho_{\text{vac}} \sim 10^{100} \text{ eV}^4$  at GUT scale [8], [33],

$$\frac{k_5^4}{12} \lambda_0^2 = 8\pi G \rho_{\text{vac}} \sim 10^{45} \text{ eV}^2, \quad (52)$$

which gives  $\lambda_0 \sim 1.3 \times 10^{75} \text{ eV}$ . The scale difference between  $\lambda_0$  and  $\sqrt{-6{}^{(5)}\Lambda/k_5^2 + \lambda_1^2}$  is  $\lambda_0/\sqrt{-6{}^{(5)}\Lambda/k_5^2} \sim 10^{25}$ . The differences in the scales of  $\lambda_0$  and  $\lambda_1$  are of the order of  $\sim 56$ . For the remaining model parameters  $t_{\text{in}}, a_{\text{in}}, \rho_{\text{in}}, N, \beta$ , we constraint them in the next section.

When  $a$  is very small, the brane tension  $\lambda \approx \lambda_0$  dominates the early Universe at the time of inflation. Due to the exponential expansion of the Universe, the brane tension quickly decays to a constant just after the inflation. The decay of the brane tension will generate the matter content of the Universe, according to Eq. (16). This happens also during the accelerated expansion period of the Universe. Matter is created during all periods of the expansion of the Universe, but the most important epoch for matter creation is near the end of inflation. In the evolution of the Universe there is one moment when  $\ddot{a} = 0$ , which corresponds to the moment when the Universe switches from the accelerating expansion to a decelerating phase. After the matter (which is mainly in the form of radiation) energy density reaches its maximum, the Universe enters into a radiation dominated phase, and the quadratic term in Eq. (14) will become dominant first. The matter energy density continue to decrease due to the expansion. When the linear term in matter equals the quadratic term, the Universe switches back to the  $\Lambda$ CDM model. Therefore, the Universe enters in the matter dominated epoch at about  $4.7 \times 10^4 \text{ yr}$  [34]. Then the matter term equals the residue term in Eq. (8) at about  $10 \text{ Gyr}$  [34]. This is the second moment in the evolution of the Universe when  $\ddot{a} = 0$ . After this moment, the Universe enters in an accelerating expansionary phase again, and its dynamics is controlled by the term  $\lambda_1$ .

## VI. QUALITATIVE ANALYSIS OF THE MODEL

In the present Section we consider the approximate behavior of the cosmological model with varying brane tension in the different cosmological epochs.

### A. Early inflationary phase: $2\beta a^2 \ll 1$

When the scale factor  $a$  is very small, the exponential factor  $e^{-2\beta a^2}$  in Eq. (48) can be approximated by 1. Therefore, the brane tension is given by  $\lambda^2 \approx \lambda_0^2$ , and physically it corresponds to the vacuum energy necessary to give an exponential inflation. Since  $k_5^2 \lambda_0^2 / 6 \gg {}^{(5)}\Lambda$ , from Eq. (14) the scale factor evolves in time as an exponential function of time, given by

$$a = a_{\text{in}} e^{(k_5^2 \lambda_0 / 6) t}, \quad (53)$$

where  $a_{\text{in}}$  is the value of the scale factor prior to inflation. The e-folding number is given by  $N = \ln(a/a_{\text{in}})$ , which should be roughly of the order of  $N \gtrsim 60$  in order to solve the flatness, Horizon problem, etc. In the present paper we adopt for  $N$  the value  $N = 70$ . Since  $2\beta a_{\text{en}}^2 \sim 1$  at the end of inflation, we can roughly estimate the value of  $a_{\text{en}}$  to be

$$a_{\text{en}} \sim \frac{1}{\sqrt{2\beta}}. \quad (54)$$

From the value of  $N$  we adopted, we obtain the value of  $a_{\text{in}}$  as

$$a_{\text{in}} = a_{\text{en}} e^{-N}. \quad (55)$$

According to Eq. (53), the end time of the inflation  $t_{\text{en}}$  can be estimated as

$$k_5^2 \lambda_0 (t_{\text{en}} - t_{\text{in}}) / 6 \approx k_5^2 \lambda_0 t_{\text{en}} / 6 \approx N, \quad (56)$$

which implies that  $t_{\text{en}} \approx 10^{-36}$  s. The value of  $t_{\text{in}}$  is insensitive to the variation of the initial conditions, provided that  $t_{\text{in}}$  is at least one order smaller than  $t_{\text{en}}$ . With the adopted value of the e-folding  $N$  and  $\beta$ , we can fix the values of  $a_{\text{in}}$  and of  $t_{\text{in}}$ , respectively. Since at the beginning of the inflationary stage the matter is not yet generated, we have  $\rho_{\text{in}} = 0$ . For the deceleration parameter, from Eq. (19) we find

$$q \approx \frac{-\lambda_0^2 e^{-2\beta a^2} - \lambda_1^2}{\lambda_0^2 e^{-2\beta a^2} + \lambda_1^2} = -1. \quad (57)$$



By substituting the tension we can rewrite Eq. (46) as

$$\frac{d\rho}{dt} + 3w\frac{1}{a}\frac{da}{dt}\rho = -\frac{d\lambda}{dt} = \frac{2\beta\lambda_0^2 a \dot{a} e^{-2\beta a^2}}{\sqrt{\lambda_0^2 e^{-2\beta a^2} - \frac{6^{(5)}\Lambda}{k_5^2} + \lambda_1^2}}. \quad (58)$$

For  $2\beta a^2 \ll 1$ , the exponential factor does not change much as  $a$  increase. Therefore,

$$\frac{d\rho}{dt} + 3w\frac{1}{a}\frac{da}{dt}\rho = \frac{2\beta\lambda_0^2 a \dot{a}}{\sqrt{\lambda_0^2 - \frac{6^{(5)}\Lambda}{k_5^2} + \lambda_1^2}} \approx \frac{2\beta\lambda_0^2 a \dot{a}}{\sqrt{\lambda_0^2}} = \beta k_5^2 \lambda_0^2 a^2 / 3. \quad (59)$$

where we have also used Eq. (53). Conversion of the matter from the brane tension energy begins already during the inflationary stage, and the rate of the conversion is proportional to  $a^2$  during inflation. Therefore, the matter density also rises exponentially in the late stages of the inflationary phase. The matter generation rate becomes most important at the end of inflation.

### B. Reheating period: $2\beta a^2 \approx 1$

During the reheating period the evolution of the matter gradually changes from an exponential increase to a power law decrease,  $\rho \propto a^{-3w}$ . In our model the matter density is a smooth function, which is strictly increasing from the beginning of inflation, and then strictly decreases after the end of the reheating phase. Therefore there must be a maximum value of the density  $\rho_{\max}$  at a time  $t_{\max}$ . After  $t_{\max}$ , the Universe was dominated by matter which is in the form of radiation, and almost all the energy of the brane tension converted into matter. The temperature of matter, corresponding to a radiation dominated Universe at  $t_{\max}$ , is denoted  $T_{\text{RH}}$ , and is given by

$$\rho_{\max} = \frac{\pi^2}{15} (kT_{\text{RH}})^4, \quad (60)$$

where  $k$  is Boltzmann's constant. Current theory on gravitinos production constraints  $T_{\text{RH}}$  to be  $T_{\text{RH}} < 10^9 - 10^{10}$  GeV [30–32]. Therefore the maximum density of the Universe must satisfy the condition  $\rho_{\max} < 10^{36} - 10^{40}$  GeV<sup>4</sup>. The maximum density can be obtained from the condition  $\dot{\rho}|_{t=t_{\max}} = 0$ , and, with the use of Eq. (14) it is given as a solution of the equation

$$3w\frac{1}{a}\frac{da}{dt}\rho_{\max} = -\frac{d\lambda}{dt} = \frac{2\beta\lambda_0^2 a \dot{a} e^{-2\beta a^2}}{\sqrt{\lambda_0^2 e^{-2\beta a^2} - \frac{6^{(5)}\Lambda}{k_5^2} + \lambda_1^2}} \Bigg|_{t=t_{\max}}. \quad (61)$$

With the rough approximation  $\beta a^2|_{t=t_{\max}} \approx \beta a_{\text{en}}^2 \sim 1$ , Eq. (61) can be written as

$$3w\rho_{\max} = \frac{2\beta\lambda_0^2 a^2 e^{-2\beta a^2}}{\sqrt{\lambda_0^2 e^{-2\beta a^2} - \frac{6^{(5)}\Lambda}{k_5^2} + \lambda_1^2}} \Big|_{t=t_{\max}} \approx \frac{2\lambda_0^2 e^{-2\beta a^2}}{\sqrt{\lambda_0^2 e^{-2\beta a^2} - \frac{6^{(5)}\Lambda}{k_5^2} + \lambda_1^2}} \Big|_{t=t_{\max}} \approx 0.74\lambda_0. \quad (62)$$

This relation gives the maximum matter density of the Universe. And the value of  $\lambda_0 \sim 10^{39}\text{GeV}^4$  that we have chosen is consistent with the maximum value of the matter energy density.

### C. Matter Domination period: $2\beta a^2 \gg 1$ and $2\lambda\rho \gg \lambda_1^2$

At this stage, the Universe is dominated by matter and the brane tension is roughly a constant. During this period, the key difference with the conventional cosmological models is the presence of the quadratic term in matter density, which will dominate the dynamics of the Universe at the beginning of this period. The evolution equation of the scale factor is

$$\frac{da}{dt} = a \frac{k_5^2}{6} \left( \sqrt{2\lambda\rho + \rho^2} \right) \approx a \frac{k_5^2}{6} \rho, \quad (63)$$

and the evolution of the matter density is given by

$$\frac{d\rho}{dt} + 4\frac{1}{a}\frac{da}{dt}\rho = 0, \quad (64)$$

where we have assumed that immediately after the matter energy reaches its maximum the matter is in the form of radiation. The solution is  $\rho = \text{constant}/a^4 \equiv \rho_{\max} a^4(t_{\max})/a^4 \approx \lambda_0/\beta^2 a^4$ . Therefore, the scale factor and deceleration parameter evolve as

$$a(t) = \left( \frac{2k_5^2}{3} \rho_{\max} a^4(t_{\max}) t \right)^{\frac{1}{4}} \approx \frac{k_5^2 \lambda_0}{6\beta^2} t, \quad (65)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{\frac{k_5^4 \lambda}{36} \left[ \rho \left( 1 + \frac{2\rho}{\lambda} \right) + 3(w-1)\rho \left( 1 + \frac{\rho}{\lambda} \right) \right]}{\frac{k_5^4 \lambda}{18} \left( \rho + \frac{\rho^2}{\lambda} \right)} \approx 1 + 3w = 5. \quad (66)$$

After the quadratic density and linear density equality  $2\lambda\rho = \rho^2$ , the linear term in matter will take over, i.e.  $2\lambda\rho \gg \rho^2$ . The Universe enters in the  $\Lambda$ CDM model at this radiation dominated phase, and its dynamics is described by the equations

$$\frac{da}{dt} = a \left( \sqrt{\frac{k_5^4}{18} \lambda \rho} \right), \quad (67)$$

and

$$\frac{d\rho}{dt} + 4\frac{1}{a}\frac{da}{dt}\rho = 0, \quad (68)$$

respectively. The solution for the scale factor is

$$\frac{a^2(t)}{2} \approx \left( \frac{k_5^3 \sqrt{-6^{(5)}\Lambda\lambda_0}}{18\beta^2} \right)^{1/2} t, \quad (69)$$

with  $\rho = \rho_{\max} a^4(t_{\max})/a^4$ . During this period the deceleration parameter evolves as

$$q \approx \frac{\frac{k_5^4\lambda}{36} [\rho + 3(w-1)\rho]}{\frac{k_5^4}{18}\lambda\rho} = \frac{3w-2}{2} = 1. \quad (70)$$

After this stage, the Universe switches from radiation dominated to non-relativistic matter dominated at  $t_{\text{rm}} = 1.5 \times 10^{12}$  s [34]. To simulate the transition from the radiation dominated period to the baryonic matter dominated period one can introduce a time varying  $w$  given by [29]

$$w = \frac{4t_{\text{rm}}/3 + t}{t_{\text{rm}} + t}. \quad (71)$$

In the matter dominated era ( $w = 1$ ), the deceleration parameter shifts to  $q = 0.5$ , according to Eq. (70). Recall that  $\beta$  is still free, and we can use it to match the scale factor at the radiation-matter equality. With the use of Eq. (69), and by taking  $a_{\text{rm}} = 2.8 \times 10^{-4}$  [34], we obtain

$$2.6 \times 10^{-20} \text{s}^{-1} \sim \left( \frac{k_5^3 \sqrt{-6^{(5)}\Lambda\lambda_0}}{18\beta^2} \right)^{1/2}, \quad (72)$$

which gives  $\lambda_0 \approx 10^{-15} \times \beta^2$ . This gives the estimate of  $\beta \sim 10^{45}$ .

#### D. Dark Energy Domination era ( $2\lambda\rho < \lambda_1^2$ )

With the increase of the cosmological time, the constant term  $\lambda_1$  in Eq. (14) will dominate over matter. Thus this term plays the role of the dark energy of the standard  $\Lambda$ CDM models. In its late stages of evolution the Universe becomes “dark energy” dominated. The Universe turn to an exponential acceleration again, with

$$a \propto e^{(k_5^2/6)\lambda_1 t}, \quad (73)$$

but with a time scale much longer than the inflationary time scale. The deceleration parameter will converge to

$$q \rightarrow \frac{-k_5^4\lambda_1^2}{k_5^4\lambda_1^2} = -1, \quad (74)$$

showing that the expansion of the universe is accelerating.

## VII. NUMERICAL ANALYSIS OF THE MODEL

The field equation can be rewritten in a simple form by introducing a set of dimensionless variables  $\tau$ ,  $l$ , and  $r$ , defined as

$$\tau = k_5 \sqrt{\frac{(-\Lambda_5)}{2}} t, \rho = k_5^{-1} \sqrt{3 \times (-\Lambda_5)} r, \lambda = k_5^{-1} \sqrt{3 \times (-\Lambda_5)} l, \quad (75)$$

respectively. Then the field equations Eqs. (14)-(18) can be written in a dimensionless form as

$$3 \left( \frac{1}{a} \frac{da}{d\tau} \right)^2 = -1 + \frac{l^2}{2} + lr + \frac{r^2}{2}, \quad (76)$$

$$\frac{1}{a} \frac{d^2 a}{d\tau^2} = -\frac{1}{3} + \frac{l^2}{6} - \frac{1}{6} l \left[ r \left( 1 + \frac{2r}{l} \right) + 3(w-1)r \left( 1 + \frac{r}{l} \right) \right], \quad (77)$$

$$\frac{dr}{d\tau} + 3w \frac{1}{a} \frac{da}{d\tau} r = -\frac{dl}{d\tau}. \quad (78)$$

By rescaling the four-dimensional cosmological constant so that  ${}^{(4)}\Lambda = [k_5^2(-\Lambda_5)/2] l_{eff}$ , it follows that  $l_{eff} = -1 + l^2/2$ . In the dimensionless variables, the deceleration parameter is given by  $q = -(ad^2a/d\tau^2) / (da/d\tau)^2$ , and can be explicitly expressed as a function of the physical parameters of the model as

$$q = \frac{1 - l^2/2 + l[r(1 + 2r/l) + 3(w-1)r(1 + r/l)]/2}{-1 + l^2/2 + lr + r^2/2}. \quad (79)$$

To obtain the numerical solution, one should also consider the rescaled form of  $\lambda$ ,

$$l^2 = 2(l_0^2 e^{-2\beta a^2} + 1 + l_1^2), \quad (80)$$

where  $l_0$  and  $l_1$  are constants, with  $l_0 \approx 10^{25} \gg l_1 \approx 10^{-31}$ . Eq. (71) becomes

$$w = \frac{4\tau_{rm}/3 + \tau}{\tau_{rm} + \tau}, \quad (81)$$

with  $\tau_{rm} = (4.8 \times 10^{-3} \text{ eV}) t_{rm} = 10^{25}$ . Note that the conversion in Eq. (75) can be written out numerically

$$t = (208 \text{ eV}^{-1}) \tau = (1.4 \times 10^{-13} \text{ s}) \tau, \rho = (2.0 \times 10^{50} \text{ eV}) r, \lambda = (2.0 \times 10^{50} \text{ eV}) l. \quad (82)$$

The time variations of the scale factor, of the energy density, of the brane tension, and of the deceleration parameter of the Universe are presented, for different scales of vacuum

energy  $l_0$ , in Figs. 1 respectively. From the plot of  $a$  and  $q$ , we find that there are 5 stages in the evolution of the Universe. Namely, the inflation and reheating stage, the quadratic density stage, the radiation domination stage, the non-relativistic matter stage, and the late time acceleration stage. Comparing plots with different  $l_0$ 's, we see the effects of different vacuum energy scales on the evolution of the Universe. According to the plot with different  $l_0$ 's, we find that a larger vacuum energy would provide a faster inflation, and it also generates more matter. The matter energy density reaches its maximum at earlier times. Although there are many changes in the evolution of the Universe caused by changing  $l_0$ , these characteristics are hard to be constraint by observations. On the other hand,  $l_0$  cannot be arbitrary due to the theoretical constraint, e.g. vacuum energy density, gravitinos production, etc.

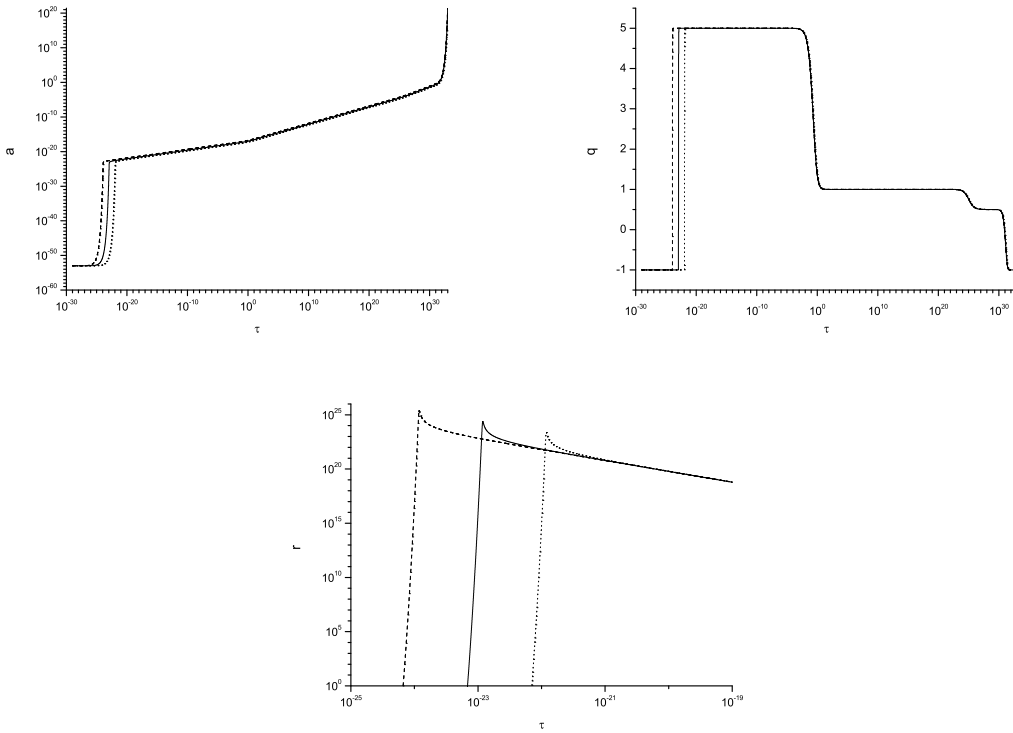


FIG. 1: Time variations of the scale factor  $a$ , deceleration parameter  $q$ , and energy density  $r$  of the Universe in braneworld models with varying scale factor dependent brane tension. Different  $l_0$  are plotted:  $l_0 = 10^{24}$  (dotted curve),  $l_0 = 10^{25}$  (solid curve), and  $l_0 = 10^{26}$  (dashed curve).

The time variations of the scale factor, and of the deceleration parameter of the Universe are presented, for different values of  $\beta$ , in Figs. 2 respectively. If we assume  $\lambda_0$  is fixed

by the vacuum energy scale, the value of  $\beta$  is well confined. Varying  $\beta$  would affect the observational constraint of  $a$ . From the graph of  $r$ , we can cross check that the value of  $\rho_{\max}$  indeed fulfills the requirement of Eq. (60). Besides  $\beta$ , we could examine the effect of adopting a different e-folding  $N$ . Since  $a_{\text{en}}$  is determined by  $\beta$  according to the condition Eq. (54), it follows that it is determined by observational constraints. Different e-foldings could be a result of differences in  $a_{\text{in}}$ . However, we find from Fig. 3 that  $a_{\text{in}}$  does not affect the post-inflationary epochs. Therefore  $a_{\text{in}}$  is not a robust parameters, and we cannot fully determine it.

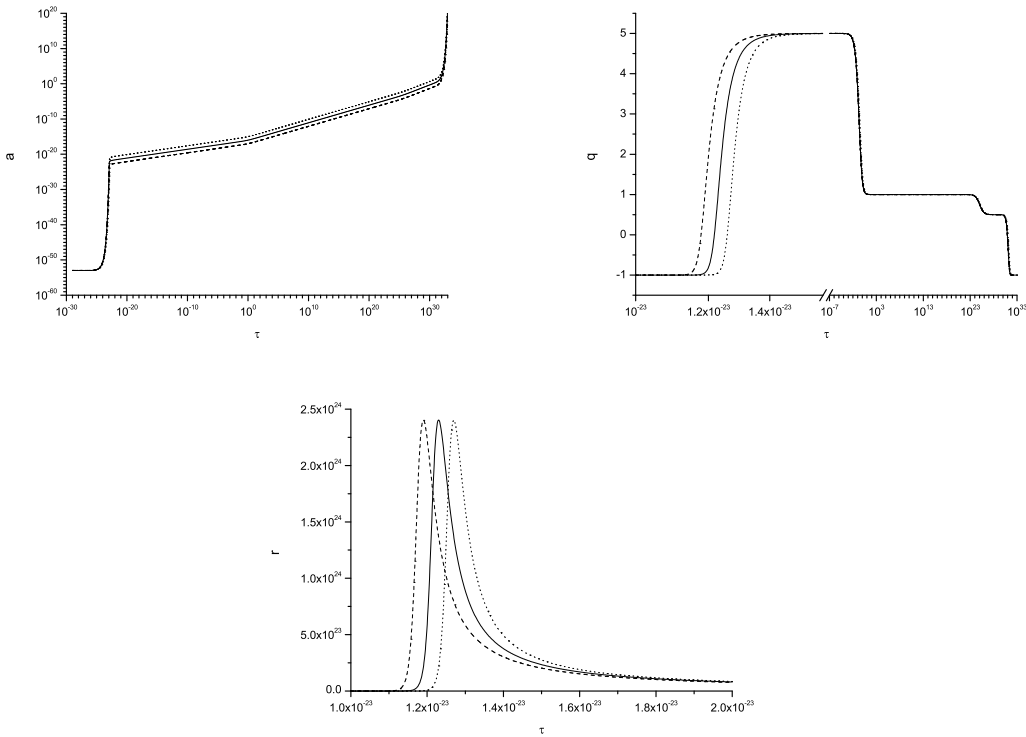


FIG. 2: Time variations of the scale factor  $a$ , deceleration parameter  $q$ , and energy density  $r$  of the Universe in braneworld models with varying scale factor dependent brane tension. Different  $\beta$  are plotted:  $\beta = 10^{43}$  (dotted curve),  $\beta = 10^{45}$  (solid curve), and  $\beta = 10^{47}$  (dashed curve).

## VIII. DISCUSSIONS AND FINAL REMARKS

In the present paper we have considered some cosmological implications of the braneworld models with variable tension. We have considered a thermodynamic interpretation of the

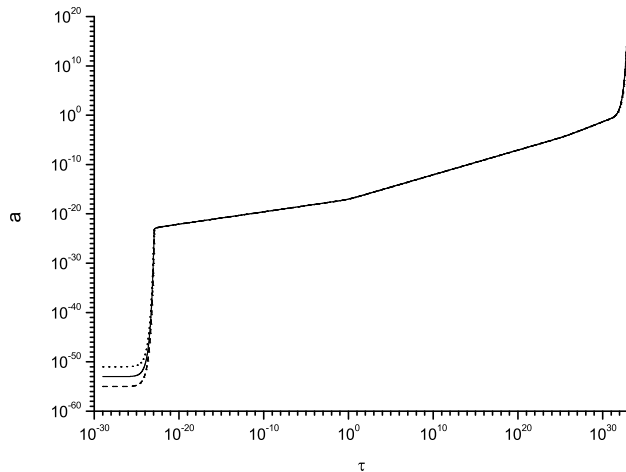


FIG. 3: Time variations of the scale factor  $a$  of the Universe in braneworld models with varying scale factor dependent brane tension. Different  $a_{\text{in}}$  are plotted:  $a_{\text{in}} = 10^{-51}$  (dotted curve),  $a_{\text{in}} = 10^{-53}$  (solid curve), and  $a_{\text{in}} = 10^{-55}$  (dashed curve).

model, and we have shown that from a thermodynamic point of view a variable brane tension can describe particle production processes in the early Universe, as well as the entropy production in the early stages of the cosmological evolution. A simple power-law inflationary cosmological model has also been obtained. By adopting a simple analytical form for  $\lambda$ , we have obtained a complete description of the dynamics and evolution of the Universe from the stage of the inflation to the phase of the late acceleration. Moreover, the differences between the energy scales of the theoretical vacuum energy at inflationary epoch and during late time acceleration are studied.

A variable brane tension can drive the inflationary evolution of the Universe, and also be responsible for matter creation in the post-inflationary phase (the reheating of the Universe). In the model we have adopted the brane tension converges to a constant that gives the gravitational constant, and the residue after cancelation with the  $\Lambda_5$  term would give the dark energy. It is interesting to note that, as opposed to the standard cosmological scenarios, in the present model matter creation takes place during the entire inflationary phase, but reaches a maximum after the exponential expansion of the Universe ends. Therefore in the braneworld models with varying brane tension there can be no clear distinction between inflation and reheating. By using numerical analysis as well as approximate analytical

methods, we have obtained the result that in this variable tension model there are 5 phases in the cosmological evolution of the Universe. At the beginning, there are the inflationary and the reheating phases. During these phases, the brane tension is the dominant energy in the Universe, and its magnitude is of the order of the vacuum energy at GUT scale. As the brane tension decays, the matter is created, and the Universe enters in the hot Universe phase of the Big Bang picture. The third phase is the quadratic density domination phase. This is a unique characteristic of the brane world scenario, during which the Universe is dominated by a quadratic term in the energy density of the radiation. The deceleration parameter increases from  $-1$  during inflation to  $5$  at this phase. To study the cosmological dynamics we have introduced a set of dimensionless quantities, which describe the evolution of the scaled densities in terms of a dimensionless time parameter  $\tau$ . By using the results of the numerical simulations for the rescaled variables, one can obtain some constraints on the physical parameters of the model.

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