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# A GLIMPSE OF A MATHEMATICAL ENCULTURATOR IN CHINESE MATHEMATICS CLASSROOMS: AN EXAMPLE FROM A SHANGHAI LESSON 

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#### Abstract

This paper has two aims. Firstly, it gives a brief summary of selected accounts of Chinese mathematics classroom teaching based on ongoing analysis of the Learner's Perspective Study. Secondly, it aims to present a description of an event of a Shanghai mathematics lesson to show how the teacher plays a role as a mathematics "enculturator" who plays an asymmetrical influential role in the shaping process. The theoretical point of departure is that classroom practice is a process of "mathematical enculturation". The process is a dynamic, intentional, shaping process which refers to what happens between the teacher and students within an institutionalised setting.


## Introduction

Within any particular educational system, the possibilities for experimentation and innovation are limited by many considerations. These limitations may be methodological or ethical. They may also be a consequence of our capacity to conceive possible alternatives and our own assumptions regarding acceptable classroom practice. These assumptions may show traits of local values and well entrenched practices developed in the history. The Learner's Perspective Study launched in 2000, aims to juxtapose the observable practices of the classroom and the meanings attributed to those practices by classroom participants, viz. the teachers and the learners. The project is guided by a belief that we need to learn from each other to get insights into the practices of mathematics classrooms in different countries (Clarke, et al. 2006).

With respect to Chinese mathematics teaching, LPS has already collected data from three cities: Hong Kong, Macau and Shanghai. Via analysis of the corpus of data either in parts or in whole for different research agenda from multiple perspectives, complementary accounts of mathematics classroom practices are being developed (for example, Mok and Lopez-Real, 2006, Huang, et al. 2006, Mok, 2006).

The aims of this paper are twofold. The first is to give a brief summary of selected accounts of the Chinese mathematics teaching based on the analysis of the LPS data in order to make a contribution to the understanding of Chinese mathematics classroom practice. In this preamble, I must make clear that there is inevitable ambiguity in using the phrase of "Chinese mathematics classroom practice" which suggests a kind of mathematics classroom practice bearing the label of Chinese. The question for how Chinese learn mathematics has been investigated by many international researchers (e.g., Fan, et al., 2004). As a result of the long history of Chinese culture, the large population, and the vast geographical and cultural variation between different regions within the nation, the rich practice of Chinese pedagogy by itself is a worthy item in the research agenda for mathematics education but not easy to answer in simple ways. Although it is in fact quite impossible to label a kind of practice as 'Chinese', reports of different studies and animated reflections suggest some traits of prominent characteristics emerged in these different studies of Chinese teaching. The selected accounts reported in this paper never intend to suggest any features as national characteristics. They are selected in order to give an abridged representation of the ongoing analysis of a corpus of empirical data in LPS and to enhance the understanding of Chinese mathematics classroom practice. This summary also provides a background for the second aim which is primary. The second aim is to present a snapshot of a Shanghai mathematics lesson in which the teacher demonstrated the role of an "enculturator" based the theoretical point of departure that classroom practice is a process of "mathematical enculturation" (Bishop, 1991). The process is a dynamic, intentional, shaping process which refers to what happens between the teacher and students within an institutionalised setting. Within the framework, the teacher is seen as a mathematics enculturator who plays an asymmetrical influential role in the shaping process.

## A Brief Note on Selected Accounts of Chinese Mathematics Teaching

According to Mok and Lopez-Real (2006), by a comparison of six teachers in Hong Kong and Shanghai, findings, show that analysis of a single 'snapshot' lesson is unlikely to reveal a national "script" in the sense claimed by Stigler and Hiebert (1999). However, interesting patterns of similarities and differences of the same region emerged in analysis of the two cities in terms of lesson organization (e.g., the use of group work) and teaching approaches (e.g., the use of exploratory activities). The results add an additional evidence to show that study in mathematics is not carried out in a rote-based learning environment.

Huang et al. (2006), based on the analysis of the teaching of the particular procedural method of elimination in Hong Kong, Macau and Shanghai, shows that the
teachers in the three cities emphasized practicing with both explicit variation and implicit variation. These variations are embedded in both the design of the mathematical tasks and the teaching approaches. The analysis explains how such variations possibly enhance the learners' discernment of the critical features for an object of learning (Marton, et al., 2004). It also gives an explanation to the Chinese notion of the inseparable relationship between learning and practice. It further unfolds how practicing extensive problems can go beyond drilling by rote but help building up an interrelated knowledge structure of a mathematical object and developing flexible problem solving abilities.

Mok (2006) analysed the reflections by a Shanghai teacher and his students upon their lessons based on the video of the lessons and the interview data. The findings show that both parties see their lessons in a positive way and there is harmonious match between the expectations of the teacher and students.

## Methodology: The Learner’s Perspective Study

Since the launching of the Learner's Perspective Study Project (LPS) in 2000, the LPS team has engaged researchers in the investigation of mathematics classrooms of teachers in Australia, China, the Czech Republic, Germany, Israel, Japan, Korea, the Philippines, Singapore, South Africa, Sweden and the USA. LPS aims to juxtapose the observable practices of the classroom and the meanings attributed to those practices by classroom participants.

Essential features of the research method are (i) the on-site mixing of the images from two video cameras to provide a split-screen record of both teacher and student actions and (ii) the use of the technique of video-stimulated recall in interviews conducted immediately after the lesson to obtain participants' reconstructions of the lesson and the meanings which particular events held for them personally. Two students were interviewed after each lesson. Each teacher participated in three video-stimulated interviews and completed two substantial questionnaires before and after videotaping, as well as a shorter questionnaire after each videotaped lesson. Three competent teachers from different schools recommended by local researchers were chosen and their eighth grade lessons were recorded for a minimum of ten consecutive lessons for each class/teacher.

The event which will be described in this paper was taken from a grade-7 mathematics lesson by a teacher in Shanghai, China. As a result of the matching of curriculum topics, the students were one grade level lower than the students in other countries in LPS. The teacher is very competent with 17 years of teaching experience and
was awarded the title "Senior Lecturer in Secondary School" by the Shanghai Senior Academic Title Appraisal Group in 1997.

## A Theoretical Framework: The Process of Mathematical Enculturation

The theoretical point of departure is that classroom practice is a process of "mathematical enculturation" (Bishop, 1991). According to Bishop, there are two perspectives on mathematical enculturation. One perspective is that of the curriculum. The other is that of the process. The analysis presented in this paper puts focus on the process. The meaning and characteristics of the process developed by Bishop (ibid.) which are relevant in this paper are briefly explained in this section.

Mathematical enculturation is an interpersonal process and therefore it is an interactive process between people. In other words, the shaping process is a result of the interactive process between the teacher and the students. Within this process, concepts, meanings, processes and values are what are being shaped and eventually belong to the students.

The process of mathematical enculturation directly refers to what happens between the teacher and students within an institutionalised setting, i.e., the mathematics classroom in this case. The process is a dynamic, intentional, shaping process. The process is not a result of transmission, therefore, interpersonal and interactional, taking significant accountable of its social context. The process is formal, institutionalized intentional, accountable, therefore, it cannot be accidental. The process is understood within a knowledge frame, therefore, it is concerned with mathematical objects and processes, e.g., concepts and skills such as simultaneous equations in two unknowns in this paper.

There are three foci which help to clarify the nature of the mathematical enculturation process. The first is the asymmetrical nature of the enculturation relationship which describes the imbalance between teacher and learners' influence. Although the teacher's ideas cannot be unchanging in the process, it is the learners' ideas which are intended to be shaped not the teacher's. The second aspect is the intentional aspect. This attempts to answer what qualities and criteria which are to be striven for. The teacher's choices and decisions are hence pertinent during the process. The third aspect concerns with its ideational quality. For this, the focus of attention is on the communicability and sharing of mathematical ideas.

While seeing the enculturation asymmetrical, the teacher is inevitably in a more powerful and influential position. The mathematical culture as a result of the shaping
process requires the teacher to act as the mathematics enculturator. In a mathematics classroom, the teacher is granted with the power to determine the activities to be undertaken. The teacher should consequently always be able to justify the various learning activities chosen for a lesson and to what ends the process aims for.

## A Glimpse of a Mathematics Enculturator in the Lesson

An episode from a grade-7 lesson (SH2-L03) was chosen below to help to see the asymmetrical teacher's role in the interaction. The teacher obviously did a lot to shape the activity, class discourse to guide the students to see what he wanted the students to understand about the mathematical object "a system of linear equations in two unknowns". In the episode, the analysis captured the following features:

1. The teacher gave strong guidance before the students started off their own discussion (line 1).
2. The teacher asked the questions which encouraged the students' reflection upon the mathematics (lines 4, 6), exploratory attempts (line 14) and different opinions (line 18).
3. The teacher affirmed answers both orally and in written form (line 12), by whole class (lines 9,11).
4. The teacher showed a demand and demonstrated the accuracy of expressing ideas mathematically (lines 9 to 22).
5. The students expressed their own answers and explanation under the teacher's invitation (lines 5, 7, 13, 15, 19).

## The Problem

"Guess it: How many chickens and rabbits are there? There are x rabbits and y chickens in a cage. There are altogether twelve heads, and forty legs. How many rabbits and chickens are there in the cage?"

The problem was very similar to the problem yesterday which was on two independent equations. Therefore the students were very familiar with the context. Without any difficulty, a student immediately suggested to write down two equations: $x+y=12$ and $4 x+2 y=40$. Next, the teacher asked the class to simplify $4 x+2 y=40$ to $2 x+y=20$. Then, the teacher asked the students to find the number of chickens and rabbits.

1. T: Good, so how many chickens and rabbits are there? ...Let me give you some pairs of numbers. [Speaking while writing on the board] The first pair: $x$ equals two, y equals four; the second pair: [CASH, CATHY making corrections] x
equals four, y equals twelve; the third pair: x equals eight, y equals four, there are many pairs like that, ...for now, discuss with your classmates how many chickens and rabbits there are? [Students discussing and the teacher walking around]
2. T: (...)
3. CATHY CATTY: [to S] (...)
4. T: Okay, stop now. How many chickens and rabbits are there in the question? ...Why? [The teacher resumed the attention of the whole class.
CATHY CATTY CASH putting up their hands]
5. Capella: There are eight rabbits, four chickens.
6. T: Why not ten and two, ten plus two is twelve!
7. Capella: Because when you substitute x equals two, y equals ten into two x plus y equals twenty (...)
8. T: Oh, it may not be suitable to substitute two and ten into the second equation, so is it suitable to substitute four and twelve into the second equation?
9. (Class): No.
10. T: Oh, no suitable, how about eight and four?
11. (Class): Suitable.
12. T: For they can satisfy both the first and second equations, in this question, the values of the pair of numbers have to satisfy the first equation and also the second equation. In mathematics, we use a pair of big brackets to join them together. [Writing on the board while speaking] For that, we can have a [showing a slide]...in mathematics, we call this a system of linear equations [Writing on the board]...a set of equations formed by the combination of linear equations is called a system of linear equations. So, according to the characteristics of the system of linear equations, what kind of system is it?... The second girl in the row.
13. Carry: A system of linear equations in two unknowns.
14. T: A system of linear equations in two unknowns. She said that this equation is [Writing on the board] a system of linear equations in two unknowns. Sit down, so students, what is a system of linear equations in two unknowns, ...she thinks that it is a system of linear equations in two unknowns, so what is system of linear equations in two unknown? Can you tell me? Try, that's alright even if you get it wrong, okay, this student is good today, you. [CATHY raising her hand]
15. Clean: There are two unknowns, and the power of the unknown (...) is one...we call this linear equations in two unknowns...
16. T: A system of linear equations, um.
17. Clean: A system of linear equations in two unknowns.
18. T: He said that equations that have two unknowns, and the power of the unknowns is one is called a system of linear equations in two unknowns. You seem to have very different opinions.
19. Cell: Two or above (...)
20. T: Two or above, um.
21. Cell: Equation of linear equations in two unknowns (...).
22. T: A system formed by linear equations in two unknowns, good, let me write it down [Writing on the board], that is a pair of linear equations in two unknowns, right? That is also linear equation in two unknowns, he says there should be two or above, so let me write one more [Writing on the board]

## Discussion

The teacher in this account is the same teacher analysed in Mok (2006). The analysis by Mok (2006) shows that the teacher takes up a strong influential role in his lessons which can be in some sense a feature of 'teacher-dominance'. The adverse effect of the teacher-dominance is significantly reduced by the teacher's clear philosophy for learning and expert pedagogical skills. The teacher has a clear understanding of the mathematical topic at a level of subtle detail and tries hard to help his students to see the same under his guidance. This style of teaching can be a kind of enculturation.

In the episode shown, we can see that the teacher gives the students opportunities for discussion and guided exploration although the activities are limited by the nature of his design. The beginning of the episode (line 1 to 11) was close to the Socratic style. Till line 14, the teacher's question invited limited possible exploration based on they might have observed about equations while he clearly indicated that the students were invited to tell what they thought about the conceptual properties. This is a kind of true invitation as the teacher actually invited more than one student (Clean and Cell) to express their different opinions (lines $14 \& 18$ ). This kind of very directive focused reflection on a specific mathematical object happened often in this teacher's lessons. While welcoming his students to express their mathematical ideas in their own words, he makes an obvious demand in expressing ideas accurately in terms of content and language. His correction of Clean's answer noting the difference between "linear equations in two unknowns" and "a system of linear equations in two unknowns" is an example (lines 15-17). Many of these features are essential in shaping the students' understanding and appreciation of the mathematical objects as well as the culture of mathematics learning.

Recently, the educators in China affirm the need for further development in both the content of curriculum and pedagogy (Zhang and Dai, 2004). The teacher in this account is well aware of the fact that his teaching style is not the same as the traditional model of teaching which placed emphasis on practice and students imitating the teacher's work. He perceives his own model promoting students' understanding by their own apprehension. This is a kind of product of the teacher's own understanding of western models. However, meaningful changes involve the change in the values and the style of communication in the mathematical enculturation process. All teachers eventually have to develop and evaluate their own personalized pedagogy. Introducing the evaluation of the teacher's role from the perspective of an enculturator may give a new window for teachers' reflection in their professional development.

## Reference

Bishop, A.J. (1991). Mathematical enculturation: A cultural perspective on mathematics education. Dordrecht, the Netherlands: Kluwer Academic Publishers Groups.

Clarke, D., Keitel, C. and Shimizu, Y. (Eds.) (2006). Mathematics classrooms in twelve countries: The insiders' perspective. Rotterdam: the Netherlands: Sense Publishers.

Fan, L., Wong, N.Y., Cai, J., \& Li, S. (Eds.) (2004). How Chinese learn mathematics: perspectives from insiders. NJ: World Scientific publishing Co.

Huang, R., Mok, I.A.C., \& Leung, F.K.S. (2006) Repetition or Variation - "Practice" in the mathematics classrooms in China. In Clarke, D., Keitel, C. and Shimizu, Y. (Eds.) Mathematics classrooms in twelve countries: The insiders' perspective. Rotterdam: Sense Publishers B.V.

Marton, F. and Tsui, A.B.M. with Chik, P., Ko, P.Y., Lo, M.L., Mok, I.A.C., Ng, D., Pang, M.F., Pong, W.Y., \& Runesson, U. (2004). Classroom discourse and the space of learning. N.J.: Lawrence Erlbaum.

Mok, I.A.C. (2006). Teacher-dominating lessons in Shanghai - An insiders' story. In Clarke, D., Keitel, C. and Shimizu, Y. (Eds.) Mathematics classrooms in twelve countries: The insiders' perspective. Rotterdam: Sense Publishers B.V.

Mok, I.A.C., \& Lopez-Real, F. (2006). A tale of two cities: A comparison of six teachers in Hong Kong and Shanghai. In Clarke, D., Keitel, C. and Shimizu, Y. (Eds.) Mathematics classrooms in twelve countries: The insiders' perspective. Rotterdam: Sense Publishers B.V.

Stigler, J., \& Hiebert, J. (1999). The teaching gap. New York: Free Press.
Zhang D. and Dai, Z. (2004). "Two basics": Mathematics teaching approach and open-ended problem-solving in China. Regular lecture in the $10^{\text {th }}$ International Congress on Mathematics Education. July 4-11, Copenhagen, Denmark.

