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Author(s)	Bai, CE; Lu, Y; Tao, Z
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Excludable Public Goods: Pricing and Social Welfare Maximization

Chong-En Bai^{a*}, Yi Lu^b, and Zhigang Tao^b

^aSchool of Economics and Management, Tsinghua University, Beijing, China

^bFaculty of Business and Economics, The University of Hong Kong, Hong Kong, China

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Abstract

The literature on excludable public goods has focused on the provision of those goods, but over-sighted the impacts of pricing strategies of those goods for social welfare maximization. In this paper, using a model of consumer heterogeneity, we compare two commonly used pricing strategies – per-unit usage pricing and buffet pricing – of excludable public goods in terms of social welfare maximization. We find that buffet pricing gives higher social welfare than usage pricing for the case of low consumer heterogeneity. We further find that, under uniform distribution, while buffet pricing is still preferred to usage pricing for the case of low consumer heterogeneity, the opposite holds for the case of high consumer heterogeneity. We also investigate the conditions under which simultaneous use of usage pricing and buffet pricing gives the highest social welfare.

Key Words: Excludable Public Goods, Per-unit Usage Pricing, Buffet Pricing, Social Welfare Maximization

JEL Codes: H41, D40

1 Introduction

The literature on excludable public goods has focused on the provision of those goods either by public agencies, or private firms, or public-private

*Corresponding author: Chong-En Bai, School of Economics and Management, Tsinghua University, Beijing, China. Telephone: (86)-10-6277-3183; email: baichn@sem.tsinghua.edu.cn.

partnerships, and concerned about the issues of information asymmetries and moral hazard involved.¹ It is arguable, however, that the ultimate objective for having those public goods is to maximize social welfare. The realization of social welfare with excludable public goods, in turn, depends on the specific pricing strategies used for the access to these goods. The case of the Japanese national highway system provides an excellent example of how poorly designed pricing strategy could lead to substantial losses in social welfare. As reported by Jason Singer (2003) in the *Asian Wall Street Journal* on September 15, 2003 that, due to hefty toll fees, Japanese drivers tried everything possible to avoid driving on the National highways, leaving the National highways empty but local routes congested. Meanwhile, the City of Chongqing in China has witnessed a significant increase in both car registration and usage since it changed its road charges from toll fees to annual passes on July 1, 2002.²

Despite the importance of pricing strategies for excludable public goods and their impacts on social welfare, there is limited work on this topic. This paper fills in the void by assuming away the issue of provision and focusing instead on the comparison of various pricing strategies in terms of social welfare generated. There are two commonly used pricing strategies for excludable public goods: per-unit *usage pricing*, and *buffet pricing* where consumers can enjoy any amount of excludable public goods for a certain period of time once paying a lump-sum fee in advance. Using a model where consumers differ in their willingness to pay, we find that buffet pricing gives higher social welfare than usage pricing for the case of low consumer heterogeneity. For a uniform distribution of consumer's willingness to pay, we further find that while buffet pricing is still preferred to usage pricing for the case of low consumer heterogeneity, the opposite holds for the case of high consumer heterogeneity. We also extend our analysis by investigating the conditions under which simultaneous use of usage pricing and buffet pricing gives the highest social welfare.

A paper related to ours is by Nahata, Ostaszewski and Sahoo (1999).³ They compare buffet pricing with usage pricing in terms of profit generated under the assumption that usage pricing involves an extra marginal cost than buffet pricing does. In contrast, we do not assume an extra marginal cost associated with usage pricing and furthermore our focus is on the comparison

¹See, for example, Breithut (1939), Vickrey (1963, 1969), Winston (1991), Lee (1991), Oum and Zhang (1990), Fraser (1996), and Cremer and Laffont (2003).

²We thank Professors Zhang Zongyi and Yang Jun of Chongqing University for providing us with this example.

³Other related studies include the pricing of public intermediate goods (e.g., Feldstein (1971) and, Yang (1991)), and the pricing of shared facilities (e.g., Scotchmer 1985).

of these two pricing strategies in terms of social welfare generated.

The paper is organized as follows. The model setup for the analysis is laid out in Section 2, and the main analysis is presented in Section 3. The paper concludes with Section 4.

2 Model Setup

A government agency considers building an infrastructural facility (for example, highways, museums, and parks) at a fixed cost I . Once constructed, the facility provides an excludable-public good G to a community of consumers at zero marginal cost. Consumer demand for G is given by

$$q_i(p) = \theta_i - p, \quad (1)$$

where q_i is the quantity of consumption and θ_i represents consumer i 's highest willingness to pay. θ_i is private information, and the government agency only knows its cumulative distribution function $F(\theta)$ and the density distribution function $f(\theta)$. θ_i is assumed to lie in the support of $[\theta_0 - \varepsilon, \theta_0 + \varepsilon]$, where $\varepsilon \in [0, \theta_0]$ represents the degree of consumer heterogeneity.

Consider two commonly-used pricing strategies that the government agency can use for the excludable public good G . One is the per-unit *usage pricing*. Given the usage price p , only those consumers whose willingness to pay is higher than the price (i.e., $\theta_i > p$) will choose to enjoy good G . Let $\Omega_p(\varepsilon)$ denote the set of participating consumers under usage pricing. The associated revenue and the social welfare are

$$\begin{cases} \pi_p = \int_{\theta \in \Omega_p(\varepsilon)} p q_i(p) f(\theta) d\theta \\ SW_p = \int_{\theta \in \Omega_p(\varepsilon)} \left[\left(\int_p^\theta q_i(t) dt \right) + p q_i(p) \right] f(\theta) d\theta \end{cases} \quad (2)$$

The other pricing strategy is *buffet pricing* where consumers can enjoy any amount of G once paying a lump-sum fee T in advance. It can be shown that only those consumers with $\frac{\theta_i^2}{2} > T$ will choose to enjoy good G . Let $\Omega_T(\varepsilon)$ denote the set of participating consumers under buffet pricing. The associated revenue and the social welfare are

$$\begin{cases} \pi_T = \int_{\theta \in \Omega_T(\varepsilon)} T f(\theta) d\theta \\ SW_T = \int_{\theta \in \Omega_T(\varepsilon)} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta \end{cases} \quad (3)$$

When consumers are homogeneous ($\varepsilon = 0$), the highest buffet price chargeable is $\frac{\theta_0^2}{2}$ and the highest usage price chargeable is θ_0 . It can be shown that the maximum revenue under optimal buffet pricing is $\frac{\theta_0^2}{2}$ whereas

that under usage pricing is $\frac{\theta_0^2}{4}$. To make the analysis non-trivial, it is assumed that the investment cost can be recovered under this homogenous case, i.e., $I < \frac{\theta_0^2}{4}$.

The government agency's objective is to maximize social welfare subject to the constraint that the fixed cost I can be recovered (*investment recovery constraint*).

$$\begin{aligned} \max_j \quad & SW_j \\ \text{s.t.} \quad & \pi_j \geq I \end{aligned} \quad (4)$$

where $j \in \{T, p\}$. The government agency will choose the pricing strategy that gives a higher social welfare.

3 Usage Pricing versus Buffet Pricing

In comparing the social welfare achieved under usage pricing versus that under buffet pricing, it is useful to introduce the concept of *full-participation*, i.e., all consumers can enjoy good G while the fixed cost of investment can be recovered. In the presence of consumer heterogeneity, the maximum buffet price should be less than $\frac{(\theta_0 - \varepsilon)^2}{2}$ in order to have all consumers enjoy the good G . It can be shown that, under this case, the investment cost I can only be recovered when the degree of consumer heterogeneity is low enough (i.e., $\varepsilon < \varepsilon_T$). Similarly, the maximum usage price for ensuring all consumers to enjoy the good G is $\theta_0 - \varepsilon$; and the investment cost I can be recovered if $\varepsilon < \varepsilon_p$.

Define the *case of low consumer heterogeneity* as that where there is full participation under both pricing strategies, i.e., $\varepsilon \leq \min\{\varepsilon_T, \varepsilon_p\}$, and the *case of high consumer heterogeneity* as that where full participation is not satisfied in either pricing strategy, i.e., $\varepsilon > \max\{\varepsilon_T, \varepsilon_p\}$. We have:⁴

Proposition 1 (i) *For the case of low consumer heterogeneity, social welfare under optimal buffet pricing is higher than that under optimal usage pricing.* (ii) *For the case of high consumer heterogeneity, social welfare under optimal buffet pricing is higher than that under optimal usage pricing when the following condition holds:*

$$\int_{p^*}^{\theta_0 + \varepsilon} \left(\int_0^{p^*} q_i(t) dt - p^* q_i(p^*) \right) f(\theta) d\theta > \int_{p^*}^{m^*} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta \quad (5)$$

⁴All proofs are contained in the Appendix.

where p^* is the optimal usage price and $m^* = \sqrt{2T^*}$ is a transform of the optimal buffet price T^* .

The intuition for the results is as follows. There are two sources of loss in social welfare under either of these two pricing strategies. One is the loss occurred when not all consumers can enjoy the good G (called *Participation Loss*), and the other is the loss when the consumption level of a consumer is below the level at zero usage price (called *Consumption Loss*).

For the case of low consumer heterogeneity, there is no participation loss under either of the two pricing strategies. As for consumption loss, however, there exists under optimal usage pricing but not under optimal buffet pricing. Hence the result of Proposition 1 (i). For the case of high heterogeneity, there are both participation loss and consumption loss under optimal usage pricing, but there is only participation loss under optimal buffet pricing. Which one has higher social welfare hinges upon condition (5), where the left side is the consumption loss under optimal usage pricing and the right side is the difference in the participation loss between optimal buffet pricing and optimal usage pricing.

Next, using a uniform distribution of consumer's maximum willingness to pay, $f(\theta) = 1/2\varepsilon$, we can characterize the optimal pricing strategy for all types of consumer heterogeneity:

Proposition 2 *Under the uniform distribution of consumer's maximum willingness to pay, there exists a degree of consumer heterogeneity, $\varepsilon \in (\min\{\varepsilon_T, \varepsilon_p\}, \max\{\varepsilon_T, \varepsilon_p\})$ below which social welfare under optimal buffet pricing is higher than that under optimal usage pricing, but above which the opposite holds.*

In real-world situation, we observe the use of usage pricing and buffet pricing simultaneously. For example, parks often offer monthly passes as well as per entrance fees. Now we consider the choice of either pure usage pricing, or pure buffet pricing, or both, and find that simultaneous use of both pricing strategies could be optimal in some cases.

Proposition 3 *Under the uniform distribution of consumer's maximum willingness to pay, social welfare under optimal buffet pricing is the highest for the case of low consumer heterogeneity, but the simultaneous use of buffet pricing and usage pricing generates the highest social welfare for the case of high consumer heterogeneity.*

4 Conclusion

In this paper, we compare two commonly used pricing strategies – usage pricing and buffet pricing – of excludable public goods in terms of social welfare maximization. Using a model where consumers differ in their willingness to pay, we find that buffet pricing gives higher social welfare than usage pricing for the case of low consumer heterogeneity. For uniform distribution of consumer's willingness to pay, we further find that while buffet pricing is still preferred to usage pricing for the case of low consumer heterogeneity, the opposite holds for the case of high consumer heterogeneity. We also extend our analysis by investigating the conditions under which simultaneous use of usage pricing and buffet pricing gives the highest social welfare.

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Appendix to "Excludable Public Goods: Pricing and Social Welfare Maximization"

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Proof to Proposition 1:

(i) *Case of low consumer heterogeneity (i.e. $\varepsilon \leq \min\{\varepsilon_T, \varepsilon_p\}$)*

When $\varepsilon \leq \min\{\varepsilon_T, \varepsilon_p\}$, all the consumers can enjoy the good G , thus $\Omega_p(\varepsilon) = \Omega_T(\varepsilon) = [\theta_0 - \varepsilon, \theta_0 + \varepsilon]$ and the associated social welfare under usage pricing and buffet pricing are:

$$\begin{cases} SW_p = \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} \left[\left(\int_p^\theta q_i(t) dt \right) + pq_i(p) \right] f(\theta) d\theta \\ SW_T = \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta \end{cases} \quad (1)$$

where p is the minimum value to satisfy that $\pi_p = \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} pq_i(p) f(\theta) d\theta = I$ since $\partial SW_p / \partial p < 0$ and $p \in (0, \theta_0 - \varepsilon]$.

To compare the social welfare under two pricing strategies we simply get

$$\begin{aligned} SW_T - SW_p &= \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta - \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} \left[\left(\int_p^\theta q_i(t) dt \right) + pq_i(p) \right] f(\theta) d\theta \\ &= \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} \left(\int_0^\theta q_i(t) dt - \int_p^\theta q_i(t) dt \right) f(\theta) d\theta - \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} pq_i(p) f(\theta) d\theta \\ &= \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} \left(\int_0^p q_i(t) dt - \int_0^p q_i(p) dt \right) f(\theta) d\theta > 0 \end{aligned} \quad (2)$$

while the last inequality follows from the fact that $q_i(t) > q_i(p)$ when $t \in (0, p)$ for all i .

(ii) *Case of high consumer heterogeneity ($\varepsilon > \max\{\varepsilon_T, \varepsilon_p\}$).*

We start with the buffet pricing. Under current circumstance, only those consumers with high willingness to pay (i.e. $\frac{\theta_i^2}{2} \geq T$ where T is the buffet price) will choose to enjoy the good G . Thus $\Omega_T(\varepsilon) = [m, \theta_0 + \varepsilon]$, and the

associated revenue and social welfare are

$$\begin{cases} \pi_T = \int_m^{\theta_0+\varepsilon} \frac{m^2}{2} f(\theta) d\theta \\ SW_T = \int_m^{\theta_0+\varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta \end{cases} \quad (3)$$

where $m = \sqrt{2I}$ and $m > \theta_0 - \varepsilon$. Since $\partial SW_T / \partial m < 0$, it is easy to derive $m^* = m(\varepsilon)$, where m^* is the minimum value to satisfy $\pi_T = \int_m^{\theta_0+\varepsilon} \frac{m^2}{2} f(\theta) d\theta = I$. Thus the social welfare under the buffet pricing is $SW_T = \int_{m^*}^{\theta_0+\varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta$.

Similarly, it is easy to derive the social welfare under the usage pricing $SW_p = \int_{p^*}^{\theta_0+\varepsilon} \left[\int_{p^*}^\theta q_i(t) dt + p^* q_i(p^*) \right] f(\theta) d\theta$, where $p^* = p(\varepsilon)$ is the minimum value satisfying $\pi_p = \int_{p^*}^{\theta_0+\varepsilon} p q_i(p) f(\theta) d\theta = I$.

Thus the comparison of the social welfare under the buffet pricing and that under the usage pricing is

$$\begin{aligned} SW_T - SW_p &= \int_{m^*}^{\theta_0+\varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta - \int_{p^*}^{\theta_0+\varepsilon} \left[\int_{p^*}^\theta q_i(t) dt + p^* q_i(p^*) \right] f(\theta) d\theta \\ &= \int_{m^*}^{\theta_0+\varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta - \int_{p^*}^{\theta_0+\varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta \\ &\quad + \int_{p^*}^{\theta_0+\varepsilon} \left(\int_0^\theta q_i(t) dt - \int_{p^*}^\theta q_i(t) dt - p^* q_i(p^*) \right) f(\theta) d\theta \\ &= \int_{m^*}^{p^*} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta + \int_{p^*}^{\theta_0+\varepsilon} \left(\int_0^{p^*} q_i(t) dt - p^* q_i(p^*) \right) f(\theta) d\theta \end{aligned} \quad (4)$$

where the first term of the right hand in the last equation measures the difference in participation loss between buffet pricing and usage pricing, and the second term measures the consumption loss under usage pricing. Thus the social welfare under the buffet pricing is higher than that under the usage pricing is if and only if the following condition holds:

$$\int_{p^*}^{\theta_0+\varepsilon} \left(\int_0^{p^*} q_i(t) dt - p^* q_i(p^*) \right) f(\theta) d\theta > \int_{p^*}^{m^*} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta \quad (5)$$

Proof to Proposition 2:

We first will calculate the threshold points ε_p and ε_T .

Lemma A.1.: There exists a threshold point $\varepsilon_T = \theta_0 - \sqrt{2I}$ under buffet pricing that when $\varepsilon \in [0, \varepsilon_T]$, all the consumers can enjoy the excludable

public good G ; when $\varepsilon \in (\varepsilon_T, \theta_0]$, only those consumers with high willingness to pay can enjoy the good G .

Proof: In order to let all the consumers enjoy the good G , the maximum chargeable buffet price is $\pi_T(\varepsilon) = \frac{(\theta_0 - \varepsilon)^2}{2}$ and so the maximum revenue collectable. $\pi_T(\varepsilon)$ is a monotonically decreasing function of ε . When $\varepsilon = 0$, $\pi_T(\varepsilon) = \frac{\theta_0^2}{2} > I$; and when $\varepsilon = \theta_0$, $\pi_T(\varepsilon) = 0 < I$. So there must exist a point ε_T at which $\pi_T = \frac{(\theta_0 - \varepsilon)^2}{2} = I$, and $\varepsilon_T = \theta_0 - \sqrt{2I}$.

Lemma A.2.: There exists a threshold point $\varepsilon_p = \frac{\theta_0 + \sqrt{\theta_0^2 - 4I}}{2}$ under usage pricing that when $\varepsilon \in [0, \varepsilon_p]$, all the consumers can enjoy the excludable public good G ; when $\varepsilon \in (\varepsilon_p, \theta_0]$, only those consumers with high willingness to pay can enjoy the good G .

Proof: Under full participation, the revenue under usage pricing is

$$\pi_p = \int_{\theta_0 - \varepsilon}^{\theta_0 + \varepsilon} pq_i(p) f(\theta) d\theta = p(\theta_0 - p) \quad (6)$$

There are two constraints needed to be satisfied: $\pi_p \geq I$ and $p \leq \theta_0 - \varepsilon$.

Let $h(p) = p(\theta_0 - p) - I = -p^2 + \theta_0 p - I$. It is easy to see that when $p \in [p_1, p_2]$, the fixed cost I can be recovered, where p_1, p_2 are the solutions to $p^2 - \theta_0 p + I = 0$ and $p_1 < p_2$. Next, to consider the constraint $p \leq \theta_0 - \varepsilon$, we insert $\theta_0 - \varepsilon$ into function $h(p)$ and get $h(\theta_0 - \varepsilon) = -\varepsilon^2 + \theta_0 \varepsilon - I$. It is easy to see that when $\varepsilon \in [\varepsilon_1, \varepsilon_2]$, $h(\theta_0 - \varepsilon) \geq 0$; when $\varepsilon \in [0, \varepsilon_1) \cup (\varepsilon_2, \theta_0]$, $h(\theta_0 - \varepsilon) < 0$, where $\varepsilon_1, \varepsilon_2$ are the solutions to $\varepsilon^2 - \theta_0 \varepsilon + I = 0$ and $\varepsilon_1 < \varepsilon_2$.

Combing the characteristics of $h(p)$ and $h(\theta_0 - \varepsilon)$, we show that:

If $\varepsilon \in [0, \varepsilon_1)$, $\theta_0 - \varepsilon > p_2$ as $\theta_0 - \varepsilon > \frac{\theta_0}{2}$ and $h(\theta_0 - \varepsilon) < 0$. Thus these two constraints can be satisfied, and p_1 is picked as the optimal usage price since $\partial SW_p / \partial p < 0$;

If $\varepsilon \in [\varepsilon_1, \varepsilon_2]$, $p_2 \geq \theta_0 - \varepsilon \geq p_1$ as $h(\theta_0 - \varepsilon) \geq 0$. Since p_1 is allowed under this circumstance, these two constraints can be satisfied;

If $\varepsilon \in (\varepsilon_2, \theta_0]$, $\theta_0 - \varepsilon < p_1$ as $\theta_0 - \varepsilon < \frac{\theta_0}{2}$ and $h(\theta_0 - \varepsilon) < 0$. Thus these two constraints can not be satisfied simultaneously.

Thus in order to realize the full participation, we must have $\varepsilon \in [0, \varepsilon_2]$.

Next, if not all consumers can enjoy the excludable public good G , the revenue is then

$$\pi_p = \int_p^{\theta_0 + \varepsilon} pq_i(p) f(\theta) d\theta = \frac{p}{4\varepsilon} (\theta_0 + \varepsilon - p)^2 \quad (7)$$

and there are also two constraints related: $\pi_p \geq I$ and $p > \theta_0 - \varepsilon$.

Let $g(p) = p(\theta_0 + \varepsilon - p)^2 - 4\varepsilon I$. From the characteristics of $g(p)$ and $g(\theta_0 - \varepsilon)$, we can similarly derive that in order to satisfy the two constraints, we must have $\varepsilon \in [\varepsilon_1, \theta_0]$ and the optimal usage price charged is $p^* = p(\varepsilon) \in (0, \frac{1}{3}(\theta_0 + \varepsilon))$.

Finally, we consider the case when $\varepsilon \in [\varepsilon_1, \varepsilon_2]$, which is the overlap between the condition under full participation and that under not full participation. As shown in the above analysis, the government agency P charges either a low price $p = p_1$ for the case of full participation or a high price $p = p^*$ for the case of not full participation so as to maximize the social welfare and meet all the constraints. It is easy to show that $SW(p_1) > SW(p^*)$. So when $\varepsilon \in [\varepsilon_1, \varepsilon_2]$, the government agency P will charge a low price $p = p_1$ and all the consumers can enjoy the good G .

Thus the threshold point is $\varepsilon_p = \varepsilon_2 = \frac{\theta_0 + \sqrt{\theta_0^2 - 4I}}{2}$.

It is easy to see that $\varepsilon_p > \varepsilon_T$. Next, we compare the social welfare under the usage pricing and that under the buffet pricing in three cases: the case of low consumer heterogeneity, the case of middle consumer heterogeneity, and the case of high consumer heterogeneity.

Case of low consumer heterogeneity (i.e. $\varepsilon \leq \varepsilon_T$): there are full participation under both usage pricing and buffet pricing. According to *Proposition 1*, the social welfare under the buffet pricing is higher than that under the usage pricing.

Case of high consumer heterogeneity (i.e., $\varepsilon_p \leq \varepsilon$): a simple calculation shows that:

$$\begin{aligned}
SW_T - SW_p &= \int_{m^*}^{p^*} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta + \int_{p^*}^{\theta_0 + \varepsilon} \left(\int_0^{p^*} q_i(t) dt - p^* q_i(p^*) \right) f(\theta) d\theta \\
&= \frac{1}{12\varepsilon} [(\theta_0 + \varepsilon)^3 - m^{*3} - (\theta_0 + \varepsilon - p^*)^2(\theta_0 + \varepsilon + p^*)] \\
&= \frac{-x}{12\varepsilon} \left(p^{*2} - 2xp^* + 2m^{*2} - \frac{m^{*3}}{x} \right) \tag{8}
\end{aligned}$$

where $m^* = m(\varepsilon) \in (0, \frac{2}{3}(\theta_0 + \varepsilon))$ is a transform of the optimal buffet price T^* , $p^* = p(\varepsilon) \in (0, \frac{1}{3}(\theta_0 + \varepsilon))$ is the optimal usage price, and $x = \theta_0 + \varepsilon$.

Having $\pi_T(m^*) = \pi_p(p^*) = I$, we can get $p^*(\theta_0 + \varepsilon - p^*)^2 = (\theta_0 + \varepsilon - m^*)m^{*2}$, which is equivalent to $[p^* + m^* - x][p^{*2} - (x + m^*)p^* + m^{*2}] = 0$. Since $p^* + m^* < x$, $p^{*2} - (x + m^*)p^* + m^{*2} = 0$, which leads to the solution $p^* = \frac{x + m^* - \sqrt{(x + m^*)^2 - 4m^{*2}}}{2}$ (the other solution is rejected as $p^* = \frac{x + m^* + \sqrt{(x + m^*)^2 - 4m^{*2}}}{2} > x$)

Insert $p^* = \frac{x+m^*-\sqrt{(x+m^*)^2-4m^{*2}}}{2}$ back into equation (8), we get

$$\begin{aligned} SW_T - SW_p &= \frac{1}{12\varepsilon} [(\theta_0 + \varepsilon)^3 - m^{*3} - (\theta_0 + \varepsilon - p^*)^2(\theta_0 + \varepsilon + p^*)] \\ &= \frac{-x}{12\varepsilon} \left(p^{*2} - 2xp^* + 2m^{*2} - \frac{m^{*3}}{x} \right) \end{aligned} \quad (9)$$

Let $l(p^*) = p^{*2} - 2xp^* + 2m^{*2} - \frac{m^{*3}}{x}$. It can be shown that when $p^* < \hat{p}$, $l(p^*) > 0$; when $p^* > \hat{p}$, $l(p^*) < 0$, where $\hat{p} = x - \sqrt{\frac{1}{x}(x^3 - 2m^{*2}x + m^{*3})}$. Furthermore, It is easy to prove that $p^* < \hat{p}$. Thus we get

$$SW_T - SW_p = \frac{-x}{12\varepsilon} l(p^*) < 0 \quad (10)$$

which means for the case of high consumer heterogeneity, the social welfare under the usage pricing is higher than that under the buffet pricing.

Case of middle consumer heterogeneity (i.e. $\varepsilon_T < \varepsilon < \varepsilon_p$), there is full participation under the usage pricing but not full participation under the buffet pricing. The social welfare function under the usage pricing and the buffet pricing are

$$\begin{cases} SW_p = \int_{\theta_0-\varepsilon}^{\theta_0+\varepsilon} \left[\left(\int_p^\theta q_i(t) dt \right) + pq_i(p) \right] f(\theta) d\theta = \frac{1}{6} [3\theta_0^2 + \varepsilon^2 - 3p^2] \\ SW_T = \int_m^{\theta_0+\varepsilon} \left(\int_0^\theta q_i(t) dt \right) f(\theta) d\theta = \frac{1}{12\varepsilon} [(\theta_0 + \varepsilon)^3 - m^3] \end{cases} \quad (11)$$

where $m = \sqrt{2T}$. According to the results in the previous two cases, we know that when $\varepsilon = \varepsilon_T$, $SW_T > SW_p$ and when $\varepsilon = \varepsilon_p$, $SW_T < SW_p$. Moreover, both SW_T and SW_p are monotonically increasing in ε . Thus there must exist a threshold point ε^* that when $\varepsilon \in (\varepsilon_T, \varepsilon^*)$, $SW_T > SW_p$ and when $\varepsilon \in (\varepsilon^*, \varepsilon_p]$, $SW_T < SW_p$.

Combined the results in these three cases, we can get that when when $\varepsilon \in (0, \varepsilon^*)$, $SW_T > SW_p$ and when $\varepsilon \in [\varepsilon^*, \theta_0]$, $SW_T < SW_p$.

Proof to Proposition 3:

For the case of low consumer heterogeneity, there are full participation under both the usage pricing and the buffet pricing. Since there is no social welfare loss under the buffet pricing and usage pricing may induce the consumption loss, the pure buffet pricing is the dominant pricing strategy.

For the case of high consumer heterogeneity, both the usage pricing and the buffet pricing exclude some consumers with low willingness to pay to enjoy the excludable public good G . As shown in the previous analysis, the social welfare under the usage pricing is higher than that under the buffet

pricing. Thus we only need to see if it is Pareto improvement to introduce the buffet pricing upon the optimal usage pricing (i.e. $p = p^* \in (0, \frac{1}{3}(\theta_0 + \varepsilon))$)

First, consider the choice of originally participating consumers. Now they have an additional choice that is whether to pay the buffet price and enjoy the maximum consumption or to pay the usage price. The choice depends on the comparison of consumer surplus under these two pricing strategies. Specifically, when $\frac{\theta_i^2}{2} - T' \geq \frac{(\theta_i - p^*)^2}{2}$, the consumer i will pay the buffet price, where p^* is the optimal usage price and T' is the lump-sum fee charged; otherwise, the consumer i will use the usage pricing.

Define $n = \sqrt{2T'}$. We now have three groups of consumers: when $\theta_i \in [\theta_0 - \varepsilon, p^*)$, consumers do not use the good G ; when $\theta_i \in [p^*, \frac{n^2 + (p^*)^2}{2p^*})$, consumers pay the usage price; when $\theta_i \in [\frac{n^2 + (p^*)^2}{2p^*}, \theta_0 + \varepsilon]$, consumers pay the buffet pricing.¹ And the associated revenue is

$$\begin{aligned} \pi(n, p^*) &= \int_{p^*}^{\frac{n^2 + (p^*)^2}{2p^*}} p^*(\theta - p^*)f(\theta)d\theta + \int_{\frac{n^2 + (p^*)^2}{2p^*}}^{\theta_0 + \varepsilon} \frac{n^2}{2}f(\theta)d\theta \\ &= \frac{1}{16p^*\varepsilon}[n^2 - (p^*)^2]^2 + \frac{1}{8p^*\varepsilon}[2p^*(\theta_0 + \varepsilon) - n^2 - (p^*)^2] \end{aligned} \quad (12)$$

where n is set to just recover the fixed cost I , i.e. $\pi(n, p^*) = I$. Since $\pi_{p^*} = \frac{p^*}{4\varepsilon}(\theta_0 + \varepsilon - p^*)^2 = I = \pi(n, p^*)$, we can have the following equation:²

$$n = [2p^*(\theta_0 + \varepsilon) - 3(p^*)^2]^{1/2} \quad (13)$$

Given n and p^* , it is easy to see that:

$$\begin{aligned} SW(n, p^*) &= \int_{p^*}^{\frac{n^2 + (p^*)^2}{2p^*}} \frac{\theta^2 - (p^*)^2}{2}f(\theta)d\theta + \int_{\frac{n^2 + (p^*)^2}{2p^*}}^{\theta_0 + \varepsilon} \frac{\theta^2}{2}f(\theta)d\theta \\ &= \frac{1}{12\varepsilon} \left[(\theta_0 + \varepsilon)^3 - \frac{3}{2}p^*n^2 + \frac{1}{2}(p^*)^3 \right] \\ &> SW_{p^*} = \frac{1}{12\varepsilon}(\theta_0 + \varepsilon - p^*)^2(\theta_0 + \varepsilon + 2p^*) \end{aligned} \quad (14)$$

However, the original pricing issue is that the government agency P chooses a optimal pricing structure (n, p) to maximize the social welfare

¹To make the analysis interesting, the transformed buffet price, n should be higher than the optimal usage price, p^* . Otherwise, all the consumers will choose to pay the buffet fee and the situation is reduced to the pure buffet pricing situation, which is dominated by the pure usage pricing.

²Another solution, $n = [2p^*(\theta_0 + \varepsilon) - (p^*)^2]^{1/2}$, is deleted since the social welfare is a decreasing function of n .

$SW(n, p)$, given the constraint that the fixed cost I can be recovered, i.e., $\pi(n, p) = I$. Denote the optimal choice as (\tilde{n}, \tilde{p}) , we have:

$$SW(\tilde{n}, \tilde{p}) \geq SW(n, p^*) > SW_{p^*} \quad (15)$$

So for the case of high consumer heterogeneity, the simultaneous use of buffet pricing and usage pricing generates the highest social welfare.