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Electric-field-induced resonant spin polarization in a two-dimensional electron gas

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Electric response of spin polarization in two-dimensional electron gas with structural inversion asymmetry or Rashba spin-orbit coupling subjected to a magnetic field was studied by means of the linear response theory and numerical simulation with the disorder effect. It was found that an electric resonant response of spin polarization occurs when the Fermi surface is located near the crossing of two Landau levels, which is induced from the competition between the spin-orbit coupling and Zeeman splitting. The scaling behavior was investigated with a simplified two-level model, and the height of the resonant peak is reciprocally proportional to the electric field at low temperatures and to temperature for finite electric fields, respectively. Finally, numerical simulation illustrated that impurity potential opens an energy gap near the resonant point and suppresses the effect gradually with increasing strength of disorder. This resonant effect may provide an efficient way to control spin polarization by an external electric field.

I. INTRODUCTION

Recently, electric or nonmagnetic generation of spin polarization in semiconductors has attracted a lot of interest because of its potential application in spintronic devices of semiconductors.[1,2] In this attempt, the motion of electron interacts with its spin via the spin-orbit coupling, which provides a possible way to control electron spin by electric field instead of magnetic field. Several experiments were resolved to produce spin polarization in quantum wells,[3–6] strained semiconductors,[7] and hole-doped heterojunction.[8] In a two-dimensional electron gas (2DEG) with structural inversion asymmetry, it was understood that the spin-orbit coupling generates an effective momentum-dependent field to induce a net bulk spin polarization by electric fields or currents.[9–12] Recently, it was also proposed to generate spin polarization in a bulk 2DEG in the presence of in-plane magnetic fields and electric fields.[13] Electric-induced spin accumulation near the boundary of a sample was already observed experimentally in either *n*- or *p*-doped semiconductors as a consequence of spin Hall effect.[14–16] The ac field was also applied to induce and detect spin polarization. The spin orientation was achieved by the excitation of a high-frequency electric field.[17] Spin resonance by an ac field was also discussed extensively.[18,19]

Generally speaking, we may introduce the electric spin susceptibility $\chi_E^{\alpha\beta}$ to describe the response of spin polarization S_α to an external electric field E_β,[20]

$$S_\alpha(E) = S_\alpha(0) + \chi_E^{\alpha\beta}E_\beta, \tag{1}$$

where $S_\alpha(0)$ is the spin polarization in the absence of electric field. Usually, the electric spin susceptibility is a tensor, not a vector. The spin polarization is determined by the symmetry of spin-orbit coupling of the system. To generate spin polarization efficiently, a large electric spin susceptibility is expected. In this paper, we propose *an electric resonant response* of spin polarization in 2DEG with Rashba spin-orbit coupling. First, the linear response theory shows that the electric spin susceptibility becomes divergent when the crossing point of two Landau levels is accidentally located

near the Fermi surface. The additional degeneracy of two Landau levels is attributed to competition between the spin-orbit coupling and the Zeeman splitting. Then, a simplified two-level model was proposed to investigate the scaling behaviors of the resonant electric spin susceptibility. The resonant values of the electric spin susceptibility decay with either the applied electric field at low temperatures or with the temperatures in a weak electric field. Finally, we take into account the disorder effect and apply the truncation approximation to study the system numerically, which goes beyond the linear response theory. The dependence of the divergent behavior on the electric field and temperature was presented for finite disorder strengths. It is damped with the electric field and temperature when the energy scale of the electric field and temperature is larger than the impurity potential. The numerical consequence is consistent with the result of linear response theory.

II. MODEL HAMILTONIAN AND LINEAR RESPONSE THEORY

We start with a 2DEG with Rashba coupling confined in a two-dimensional plane $L_x \times L_y$. The model Hamiltonian in the presence of a perpendicular magnetic field B is given by

$$H = H_0 + H_R, \tag{2}$$

where

$$H_0 = \frac{1}{2m^*}\boldsymbol{\Pi}^2 - \frac{1}{2}g_s\mu_B B\sigma_z, \tag{3}$$

and the Rashba coupling

$$H_R = \frac{\alpha}{\hbar}(\Pi_x\sigma_y - \Pi_y\sigma_x). \tag{4}$$

m^* is the effective mass of the electron, $\boldsymbol{\Pi} = \mathbf{P} + \frac{e}{c}\mathbf{A}$ is the kinetic momentum, g_s is the Lande *g* factor, and μ_B is the Bohr magneton. We take the Landau gauge $A = yB\hat{\mathbf{x}}$ and consider the periodic boundary condition in the $\hat{\mathbf{x}}$ direction. The Hamiltonian can be solved analytically with the eigenvalues

$$\varepsilon_{n,s} = \hbar\omega_c\left[n + \frac{s}{2}\sqrt{(1-g)^2 + 8n\eta_R^2}\right], \quad (5)$$

where n is a non-negative integer and $s = \pm 1$.[21,22] Here, the cyclotron frequency $\omega_c = eB/m^*c$, the magnetic length $l_b = \sqrt{\hbar c/eB}$, the dimensionless g factor $g = g_s m^*/2m_e$, and the effective coupling $\eta_R = \alpha/l_b\hbar\omega_c$. The corresponding eigenvectors are expressed as

$$|nks\rangle = \begin{pmatrix} ic_{ns}^+\phi_{nk} \\ -sc_{ns}^-\phi_{n-1k} \end{pmatrix}, \quad (6)$$

where ϕ_{nk} is the eigenstate of the nth Landau level with $k = p_x$ the good quantum number because of the periodic boundary condition along the x direction. $c_{ns}^\pm$ are the coupling parameters of spins up and down. $c_{0,+1}^+ = 1$, $c_{0,-1}^- = 0$, and $c_{ns}^\pm = 1/\sqrt{1 + (u_n \mp s\sqrt{1 + u_n^2})^2}$ with $u_n = (1-g)/\sqrt{8n}\,\eta_R$ for $n \geq 1$.[22] Each Landau level has a degeneracy $N_k = L_xL_y/2\pi l_b^2$. One of the remarkable features of the spectra is the additional crossing of Landau levels, which is generated by the competition between Rashba coupling and Zeeman splitting such as $\varepsilon_{n,s=+1} = \varepsilon_{n+1,s=-1}$ if the integer n and the magnetic field B satisfy

$$\sqrt{(1-g)^2 + 8n\eta_R^2} + \sqrt{(1-g)^2 + 8(n+1)\eta_R^2} = 2. \quad (7)$$

One important factor in this system is the filling factor, i.e., the ratio of number of charge carriers to the Landau degeneracy $\nu = N_e/N_k = 2\pi l_b^2 n_e$ (n_e is the density of charge carriers). For a specific density, the filling factor is proportional to $1/B$, and for a specific field, it is proportional to the density n_e.

With this solution in mind, we are ready to study the electric response of spin polarization when the Fermi surface is located near the resonant point. We apply an *in-plane* weak electric field, say, E_y along the y direction. Then the electric spin susceptibility can be evaluated by means of the Kubo formula in the weak field limit,[23]

$$\chi_E^{\alpha y} = \frac{e\hbar}{L_xL_y}\,\mathrm{Im}\sum_{nn'kk'ss'}\frac{(f_{n's'} - f_{ns})}{(\varepsilon_{ns} - \varepsilon_{n's'})(\varepsilon_{ns} - \varepsilon_{n's'} + i\hbar/\tau)}$$
$$\times\langle nks|S_\alpha|n'k's'\rangle\langle n'k's'|v_y|nks\rangle, \quad (8)$$

where v_y is the velocity in the y direction, f_{ns} are the Fermi-Dirac distribution functions, and τ can be regarded as the lifetime of the quasiparticles. From this equation, the denominator becomes singular when two energy levels are degenerate near the Fermi surface. As a result, it is possible that the electric spin susceptibility may become singular for a long lifetime $\tau \to +\infty$, i.e., $\chi_E^{\alpha y}$ may become divergent. For a finite τ or finite temperature, $\chi_E^{\alpha y}$ still remains finite.

For the purpose of numerical calculations, without losing generality, we take the model parameters such that the lowest energy level $|1\rangle = |n=0,k,s=+1\rangle$ and the first excited level $|2\rangle = |n=1,k,s=-1\rangle$ cross at a critical magnetic field $B = 9.8$ T. In the clean limit, i.e., $\tau \to +\infty$, we plot χ_E^{yy} versus the inverse of the magnetic field $1/B$ in Fig. 1 for a fixed electron density n_e. It is found that a resonant peak occurs at the critical magnetic field corresponding to a filling factor
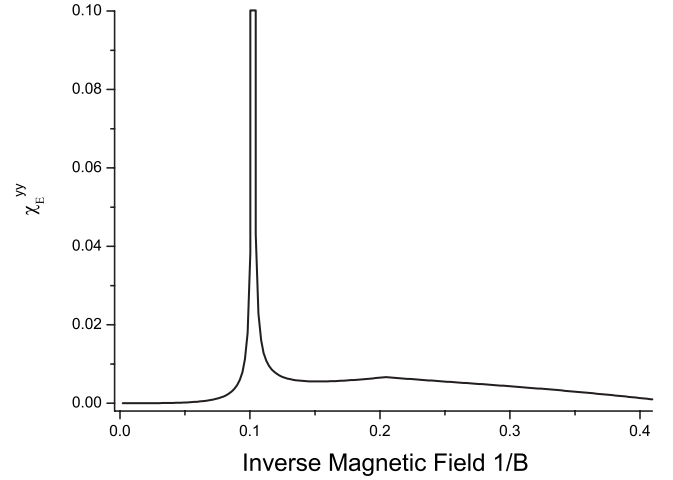

FIG. 1. χ_E^{yy} versus $1/B$ (B in units of T) at the temperature $T = 0$ for weak electric fields by means of the Kubo formula [Eq. (8)] when $\tau \to +\infty$. The parameters are taken as $\alpha = 4.16 \times 10^{-11}$ eV m, $n_e = 0.118 \times 10^{-16}$ m^{-2}, $m^* = 0.05 m_e$, and $g_s = 4$. The unit of χ_E^{yy} is $\hbar C/4\pi l_b^2 N$.

$\nu = 0.5$. The resonant peak indicates that it is possible for a weak electric field to induce a finite spin polarization at the resonant point.

It is worth stressing that the Kubo formula is a result of perturbation and is valid only when $E \to 0$ (but $E \neq 0$). At a first glance, the divergence of spin susceptibility in Fig. 1 may be unphysical because it might be caused by the approximation of perturbation in the linear response theory. To clarify the problem, we have to go beyond the linear response theory and investigate the problem by means of the nonperturbative approach.

III. NONLINEAR BEHAVIORS IN SIMPLIFIED TWO-LEVEL MODEL

In order to understand the physical origin of the resonance and to analyze the scaling behaviors of χ_E^{yy} at or near the resonant point, we consider a simplified two-level model around the resonant point. This is based on the fact that the dominant contribution to the electric-field-induced resonance is attributed to the energy crossing of the two levels. This truncation approximation reduces an infinite-dimension problem into a two-level one. As the two-level problem can be solved analytically, the effect of electric field can be taken into account. The method goes beyond the perturbation method and the nonlinear response of spin polarization to an external field can be revealed. For this purpose, we are only concerned with the two nearly degenerated levels $|1\rangle$ and $|2\rangle$ and ignore the contribution from other levels of higher energies. The Landau degeneracy of the two states for different k will be taken into account at the last step to calculate physical quantities since the Hamiltonian in an external field is diagonalized in block in the k space. Then the total Hamiltonian including the electric potential can be reduced to a 2×2 matrix:
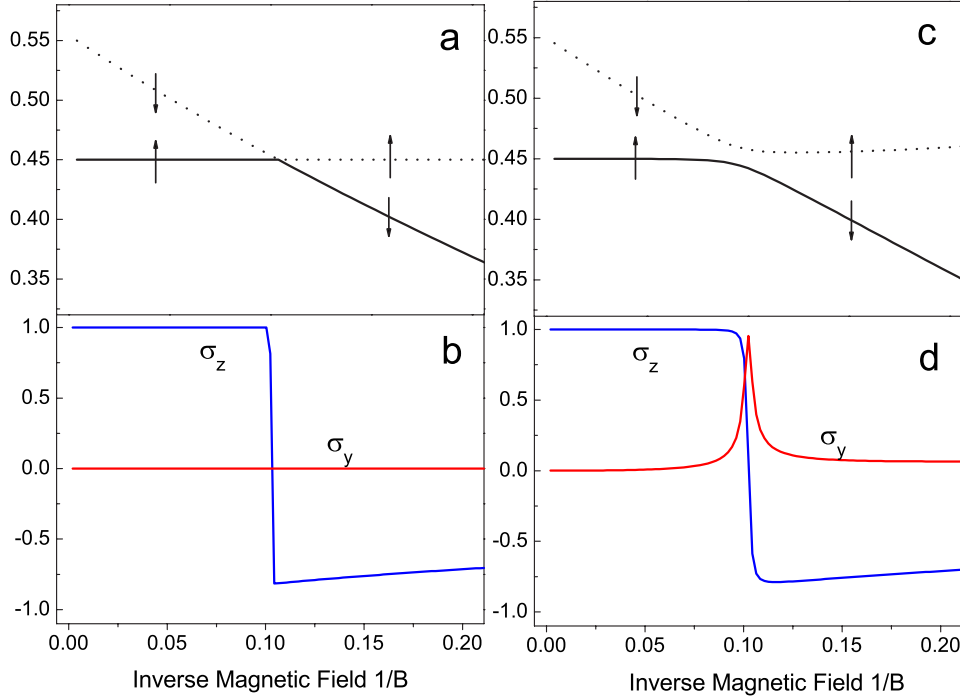
FIG. 2. (Color online) [(a) and (b)] Energy spectra in units of $\hbar\omega_c$ for the two degenerate levels in the absence of the electric field and the corresponding spin polarization in units of $\hbar/2$ of the lower-energy states. [(c) and (d)] Energies for the two anticrossing levels in the presence of the electric field and the corresponding spin polarization for the lower-energy level. The higher-energy states are denoted by the dotted line and the lower-energy states are denoted by the solid line. The arrows indicate the orientation of S_z. The spin polarizations S_z and S_y for the lower-energy states are denoted by the blue and red lines, respectively. The energy is in units of $\hbar\omega_c$ and B in unit of T. The parameters are taken to be the same as in Fig. 1: $\alpha=4.16\times10^{-11}$ eV m, $m^*=0.05m_e$, and $g_s=4$.

$$H^* = \begin{pmatrix} \varepsilon & v_E \\ v_E & -\varepsilon \end{pmatrix}, \qquad (9)$$

where $\varepsilon=\frac{1}{2}(\varepsilon_{1,-1}-\varepsilon_{0,+1})$ and $v_E=\langle 1|eE_y y|2\rangle$ $=\eta_R eEl_b c_{0,+1}^+ c_{1,-1}^-$, which is independent of k. The off-diagonal element by the electric potential lifts the degeneracy and opens an energy gap $2|v_E|$ at the resonant point of $\varepsilon=0$. In Fig. 2, the energies of the two levels and the spin polarizations for the lower-energy states are plotted with respect to the inverse of magnetic field. In the absence of electric fields the two levels cross at the resonant point $B=9.8$ T and each state has almost opposite spin polarization in the z direction. For a large B field, the state $|1\rangle=|n=0,k,s=+1\rangle$ is the lowest energy state with spin polarization $s_z=+\hbar/2$. By decreasing the magnetic field, the state $|2\rangle=|n=1,k,s=-1\rangle$ has the lower energy and becomes the lowest energy state after the two levels cross. The state $|2\rangle$ has spin polarization $S_z\approx-\hbar/2$. Thus, from the point of view of the lowest energy state, the spin polarization S_z has a jump near the crossing point, as shown in Fig. 2(b). In the presence of electric field, the two states of $|1\rangle$ and $|2\rangle$ will be admixed due to the off-diagonal term in H^*. The term will lift the degeneracy of the two levels at the resonant point. In the lowest energy state, the spin polarization rotates in the z-y plane from positive S_z to negative S_z when the magnetic field sweeps over the resonant point. At the resonant point, the system is almost polarized along the y direction in this two-level model. Comparing with the case in the absence of

electric field, we notice that nonzero S_y is induced by a weak electric field. Because there is a finite density of states at this point due to the Landau degeneracy, the total spin polarization in the y direction becomes finite. As a result, the electric spin susceptibility χ_E^{yy} becomes divergent at the point.

If we denote by $S_y^\pm$ the expectation value of the y-component spin polarization of the two split levels, the overall response of the spin polarization per unit area to a finite external field can be calculated by

$$\chi_E^{yy} = \frac{1}{2\pi l_b^2} \frac{(S_y^+ f_+ + S_y^- f_-)_E - (S_y^+ f_+ + S_y^- f_-)_{E=0}}{E}, \qquad (10)$$

with $f_\pm$ the Fermi-Dirac distribution of the two levels and $f_- + f_+ = \nu$. With the same parameters as in Fig. 1, electric spin susceptibility χ_E^{yy} near the resonant point is plotted as a function of $1/B$ in Fig. 3(a) for different electric fields. After some algebra, we can rewrite the spin susceptibility in Eq. (10) into the form

$$\chi_E^{yy} = \frac{\hbar}{2} \frac{f_-(1-f_+)}{\pi l_b^2 \nu E} \frac{|v_E|}{\sqrt{\varepsilon^2+|v_E|^2}} (1 - e^{-2\sqrt{\varepsilon^2+|v_E|^2}/kT}). \qquad (11)$$

Near the resonant point $\varepsilon\to0$, we have $\chi_E^{yy}\propto1/E$ at low temperatures $kT\ll|v_E|$; meanwhile, $\chi_E^{yy}\propto1/T$ for a weak electric field $kT\gg|v_E|$. It depends on the energy scales of the electric field energy ε and temperatures T. In Figs. 3(b) and 3(c), the dependence of the peak value of χ_E^{yy} on the electric field and temperature is plotted.
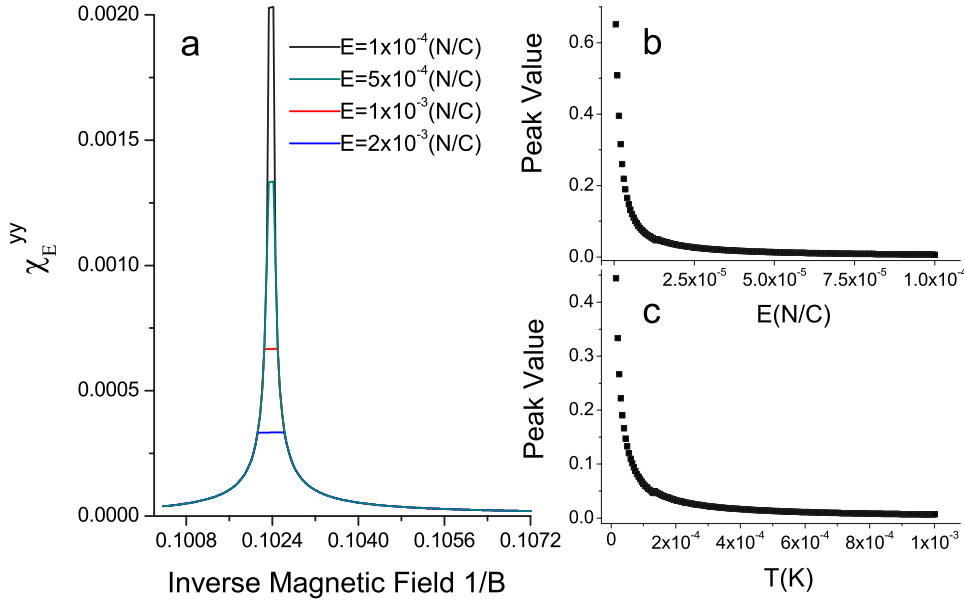
FIG. 3. (Color online) (a) The spin susceptibility χ_E^{yy} as a function of $1/B$ near the resonant point for different electric fields at $T=10^{-6}$ K. (b) The electric field E dependence of the peak value of χ_E^{yy} at a low temperature $T=10^{-6}$ K. (c) The temperature T dependence of the peak value for an electric field $E=10^{-6}$ N/C. The unit of χ_E^{yy} is $\hbar C/4\pi l_b^2 N$ and that of B is T.

From the point of view of spin polarization, the spin polarization per unit area is

$$S_y(E) = \frac{\hbar}{2} \frac{f_-(1-f_+)}{\pi l_b^2 \nu} \frac{|v_E|}{\sqrt{\varepsilon^2+|v_E|^2}} (1 - e^{-2\sqrt{\varepsilon^2+|v_E|^2}/kT}).$$

(12)

At the resonant point $\varepsilon=0$ and in low temperatures, $kT/|v_E|\rightarrow 0$, $S_y(E)\approx\hbar f_-(1-f_+)/(2\pi l_b^2\nu)$. Thus, a finite spin polarization can be induced by a weak electric field. In the absence of electric field,

$$S_y(0) = \frac{(S_y^+ f_+ + S_y^- f_-)_{E=0}}{2\pi l_b^2} \approx 0.$$

(13)

It is worthy to notice that even in the weak field limit, $\lim_{E\rightarrow 0} S_y(E)\neq S_y(0)$ at the resonant point. This is the key point to understand this resonant effect. The finite spin polarization induced by a small electric field indicates the nonlinear behaviors of this effect. As a result, the spin susceptibility becomes singular in $1/E$ as $E\rightarrow 0$. These results are consistent with the prediction of the Kubo formula in Fig. 1. It indicates that the resonance is not caused by the perturbation approximation in the linear response approach but is attributed to the *removal of the degeneracy of two crossing Landau levels by the external field.*

IV. NUMERICAL SIMULATION

After establishing a physical picture for the electric resonance of spin polarization in a clean limit, we turn to study the effect of impurities. The effect of impurities can be described by introducing a finite lifetime τ phenomenologically in the Kubo formula. To go beyond the linear response theory, we do numerical simulation to study the impurity effect in the real space. For this purpose, we still take the periodic boundary condition in the x direction but an infinite potential wall in the y direction: $V(y)=0$ for $|y|<L_y/2$, and

$+\infty$ otherwise. The disorder potential $U(x,y)$ is taken to be the short-range impurities of strength u_i uniformly distributed at (x_i,y_i) in the plane:[24]

$$U(x,y) = \sum_i u_i \delta(x - x_i) \delta(y - y_i),$$

(14)

where $u_i \in (-u/2, u/2)$, $x_i \in (-L_x/2, L_x/2)$, and $y_i \in (-L_y/2, L_y/2)$. In the absence of impurity potential and Rashba coupling, the confined Landau levels $|\varphi_{nks}\rangle$ have been obtained analytically,[25]

$$|\varphi_{nks}\rangle = \exp(-iy_0 x/l_b^2) \phi_{\nu_n}(y - y_0)/\sqrt{2\pi l_b^2},$$

(15)

where $\phi_{\nu_n}(y-y_0)$ are the confluent hypergeometric functions and $y_0=2\pi l_b^2 k/L_x$. ν_n is determined numerically by the boundary condition $|\varphi_{nks}\rangle_{y=\pm L_y/2}=0$. The finite size effect removes the Landau degeneracy near the edge. All the confined Landau levels can be regarded as a complete set of basis. On this basis, the kinetic energy and the Zeeman term, H_0, has been diagonalized. The elements of the Rashba coupling and the disorder potential are $\langle\varphi_{n'k's'}|H_R|\varphi_{nks}\rangle$ and $\langle\varphi_{n'k's'}|U|\varphi_{nks}\rangle$, respectively. After taking into account the impurity potential and Rashba coupling, we perform the truncation approximation to reduce the whole Hamiltonian into an effective one with finite dimension. Furthermore, the effective Hamiltonian will be diagonalized numerically to calculate the eigenvalues and eigenfunctions. In the calculations, we take $n_{max}+1$ Landau levels, $n=0,1,\ldots,n_{max}$, and each Landau level has N_k discrete values of k ($n=0,1,2,\ldots,N_k-1$).[26] N_k can also be explained as the maximum number of electrons accommodated in each Landau level. Then the number of basis functions we retained in the truncation approximation is $N=2\times(n_{max}+1)\times N_k$ with double degeneracy of spin. By diagonalizing the $N\times N$ Hamiltonian numerically, one gets N-eigenvalues E_m^N and N-component wave vectors $|\Psi_m^N\rangle=\Sigma\alpha_{nks}^m|\varphi_{nks}\rangle$, which are the superposition of the basis $|\varphi_{nks}\rangle$. When a weak electric field $V=eE_y y$ is applied in the y direction, the expectation value of
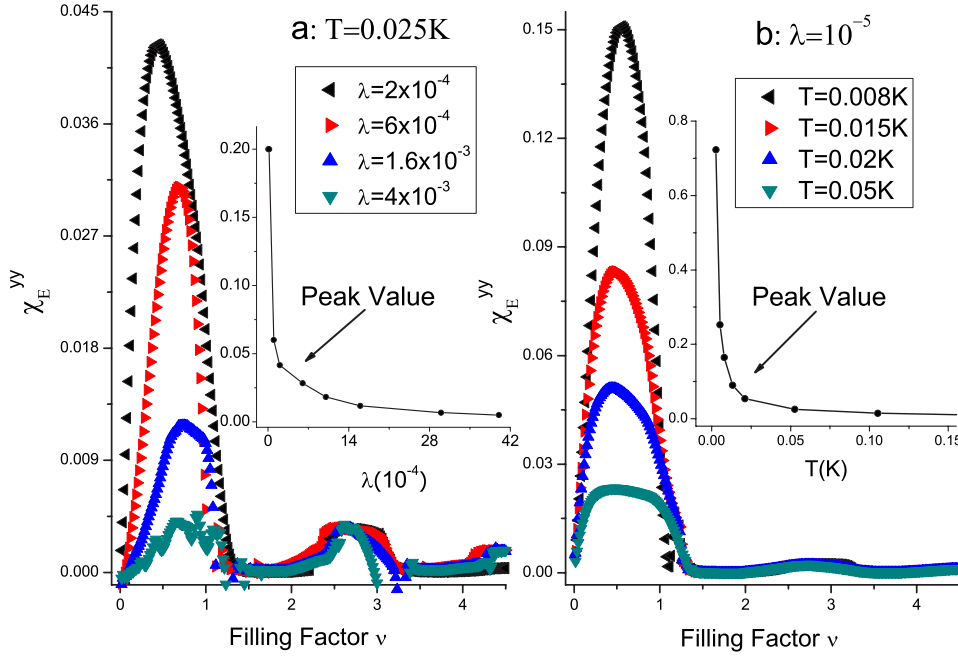
FIG. 4. (Color online) (a) χ_E^{yy} versus filling factor ν for four different disorder strengths λ for a fixed temperature $T=0.025$ K. The inset shows the disorder strength dependence of the peak value. (b) χ_E^{yy} versus the filling factor ν at different temperatures T for disorder strength $\lambda=10^{-5}$. The inset is for the temperature T dependence of the peak value. The electric field is taken to be as low as $E_y=10^{-8}$ N/C and the spin susceptibility is in units of $\hbar C/4\pi l_b^2 N$.

the total spin can be calculated numerically. As a result, χ_E^{yy} can be obtained,

$$\chi_E^{yy} = \frac{1}{L_x L_y} \sum_m \frac{\delta(S_y)_m}{E_y}, \qquad (16)$$

where

$$\delta(S_y)_m = f_m \langle \Psi_m^N | S_y | \Psi_m^N \rangle |_E - f_m \langle \Psi_m^N | S_y | \Psi_m^N \rangle |_{E=0}, \quad (17)$$

with f_m the Fermi-Dirac distribution.

In our calculations, the model parameters are taken as $L_x/N_1=L_y/N_2=\sqrt{2\pi}l_b$. In this paper, we take $N_1=10$ and $N_2=6$. Then the maximum number of electrons at each level is $N_k=L_y/\Delta y_0=N_1 \times N_2=60$. The number of Landau levels are truncated to $n_{\max}=5$; hence, $N=720$. In this truncation approximation, we are concerned only with the low energy physics. The magnetic field is chosen as $B_c=9.8$ T such that $E_{0,+1}=E_{1,-1}$ for the lowest and first excited level in the bulk region, with other parameters the same as in Fig. 1. 140 impurities of relative strength $|\lambda_i|=|u_i/(2\pi l_b^2 \hbar \omega)| \leq \lambda$ are randomly distributed over the sample. For each configuration of impurities, both the strength and position are generated randomly. The states are filled from lower to higher energy, and correspondingly, χ_E^{yy} can be calculated by the formula in Eq. (16) for each configuration.

After averaging over 10^4 different impurity configurations, we plot the average value of χ_E^{yy} versus the filling factor ν in Fig. 4(a) for different strengths of disorder, λ. The temperature is set to $T=0.025$ K, hence, the ratio $kT/\hbar \omega_c = 2 \times 10^{-4} \leq \lambda$ and the electric field $E_y=10^{-6}$ N/C such that $\tilde{e}=eE_y l_b/\hbar \omega_c \simeq 10^{-8} < \lambda$. When the electrons are filled up to $\nu=0.5$, χ_E^{yy} displays a resonant peak. In contrast, χ_E^{yy} is finite in the nondegenerate region with the filling number $\nu=2-4$ and tends to be suppressed when $\lambda=1.6 \times 10^{-3}$. The relative error due to impurity fluctuation is estimated to be about 0.01% and up to 5% at the resonant point due to the fact that

the susceptibility is very sensitive to the energy gap opened by impurities. The peak height decreases with the disorder strength λ and the peak value versus λ is demonstrated in the inset. When we extrapolate to the limit $\lambda \to \infty$, the peak value tends to be suppressed completely. The dependence of the resonant χ_E^{yy} on the disorder strength is similar to that on the electric field, thus, the impurity scattering opens a gap between the degenerate levels just like the electric field. Inversely, we fix the disorder strength at $\lambda=10^{-5}$ and plot the electric spin susceptibility χ_E^{yy} in Fig. 4(b) at different temperatures with $kT/\hbar \omega_c > \lambda$. The peak height of χ_E^{yy} decays with temperatures. The scaling behavior of the peak value versus T is shown in the inset. In the presence of disorder, the dependence of the resonant peak of χ_E^{yy} on the electric field is also simulated and plotted in Fig. 5, denoted by the black and red dots, respectively for the disorder strengths
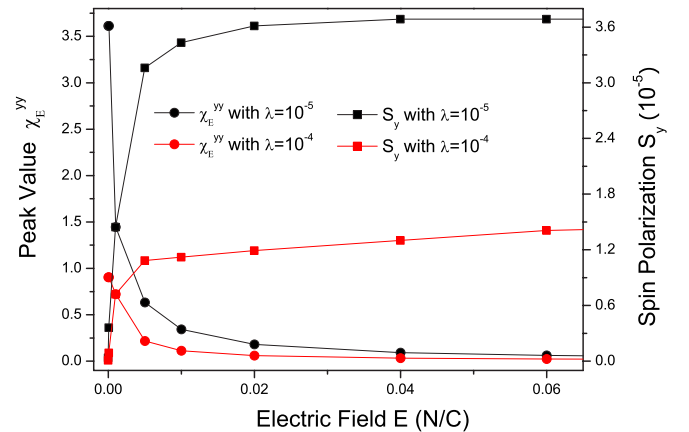

FIG. 5. (Color online) The electric field E dependence of χ_E^{yy} and the spin polarization S_y are denoted, respectively, by the dots and diamonds in the presence of disorder. The temperature is set to zero. The electric spin susceptibility χ_E^{yy} is in units of $\hbar C/4\pi l_b^2 N$ and S_y is in unit of $\hbar/4\pi l_b^2$.

$\lambda = 10^{-5}$ and 10^{-4}. At low temperatures, when $\tilde{e} = eE_y l_b / \hbar \omega_c \ll \lambda$, χ_E^{yy} is independent of the electric field E but diverges as $1/E$ when it is comparable with or greater than the disorder strength λ. The spin polarization $S_y(E) = S_y(0) + \chi_E^{\alpha\beta} E_\beta$ increases linearly with a weaker electric field but saturates at higher electric fields, as plotted in Fig. 5 by the black and red diamonds for the disorder strengths $\lambda = 10^{-5}$ and 10^{-4}, respectively.

V. SUMMARY AND DISCUSSION

At last, the occurrence of this resonance is not limited only in the Rashba system. The electric spin susceptibility depends on the symmetry of spin-orbit coupling explicitly. The present work can be generalized to a system with the Dresselhaus coupling $H_D = \beta(p_x\sigma_x - p_y\sigma_y)$. Because the Rashba coupling can be transformed to the Dresselhaus coupling under the transformations of $\sigma_x \to \sigma_y$, $\sigma_y \to \sigma_x$, and $\sigma_z \to -\sigma_z$,[27] we conclude that it is χ_E^{xy} instead of χ_E^{yy} which would become divergent at the resonant point.

In conclusion, a tiny electric field may generate a finite spin polarization in a disordered Rashba system in the presence of a magnetic field. As a result, the electric spin susceptibility exhibits a resonant peak when the Fermi surface goes through the crossing point of two Landau levels. Numerical results demonstrate that the result goes beyond the linear response theory. This provides a mechanism to control spin polarization efficiently by an electric field in semiconductors. As the spin polarization can be measured very accurately, it is believed that the effect can be verified in the samples of 2DEG, such as the heterojunction of InGaAs/InAlAs.

ACKNOWLEDGMENTS

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