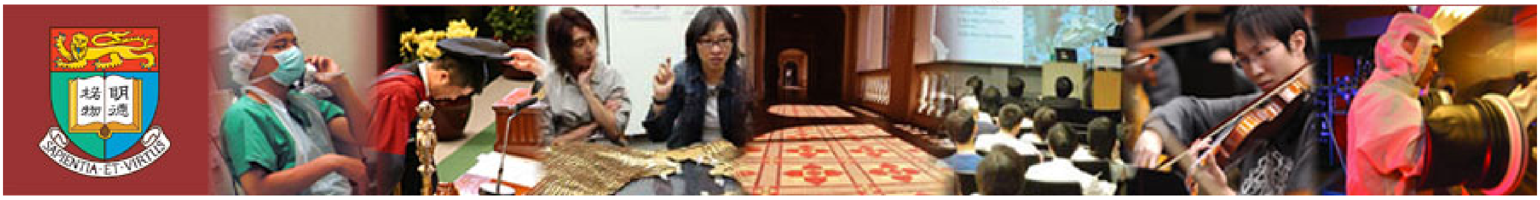

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Decentralized H_∞ Controller Design for Nonlinear Systems

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Consider a nonlinear system $\mathbf{\Sigma}$ described by equations of the form

$$\dot{x} = f(x) + g_1(x)w_0 + \sum_{i=1}^{q} g_{2i}(x)u_i \tag{1}$$

$$z = \left[h_1^T(x)\ u_1\ \cdots\ u_q\right]^T \tag{2}$$

$$y_i = h_{2i}(x) + w_i,\ i = 1,\cdots q \tag{3}$$

where $x \in X \subset R^n$ with $0 \in R^n$ is the state; $u_i \in R^{m_i}$ local control; $y_i \in R^{p_i}$ local measurement; w_0 and w_i's square-integrable disturbances; and z regulated regualted output. The functions $f(x)$, $g_1(x)$, $h_1(x)$, $g_{2i}(x)$ and $h_{2i}(x)$ $(i = 1,\cdots,q)$ are all known and smooth in X with $f(0) = 0$, $h_1(0) = 0$ and $h_{2i}(0) = 0$ $(i = 1,\cdots,q)$. For convenience, we denote

$$u = [u_1^T\ \cdots\ u_N^T]^T,\quad y = [y_1^T\ \cdots\ y_q^T]^T,$$
$$w_e = [w_0^T\ \cdots\ w_q^T]^T,\quad g_2(x) = [g_{21}(x)\ \cdots\ g_{2q}(x)]$$
$$h_2(x) = [h_{21}^T(x)\ \cdots\ h_{2q}^T(x)]^T.$$

Decentralized H_∞ Controller Design (*DHCD*) Problem: Given the system $\mathbf{\Sigma}$ of (1)-(3) and a positive constant γ, find controllers of the following form

$$\dot{\xi}_i = a_i(\xi_i) + b_i(\xi_i)y_i,\ \ \xi_i \in R^{v_i} \tag{4}$$

$$u_i = c_i(\xi_i),\quad i = 1,\cdots,q \tag{5}$$

such that the resulting closed-loop system is locally asymptotically stable, and has an L_2 gain $\leq \gamma$.

For some fundamental notions and results of nonlinear H_∞ control theory, the reader is referred to [4] or [12].

For $\mathbf{\Sigma}$ of (1)-(3) and smooth positive definite function V : $R^n \to R_+$ (with its Jacobian matrix $V_x(x)$), denote

$$\alpha_1^T(x) = 1/(2\gamma^2)g_1^T(x)V_x^T(x) \tag{6}$$

$$\alpha_2(x) = -(1/2)g_2^T(x)V_x^T(x) = \left[\alpha_{21}^T(x)\ \cdots\ \alpha_{2q}^T(x)\right]^T \tag{7}$$

$$\bar{\alpha}_2(\xi) = [\alpha_{21}^T(\xi_1)\ \alpha_{22}^T(\xi_2)\ \cdots\ \alpha_{2q}^T(\xi_q)]^T \in R^q \tag{8}$$

$$\bar{g}_1(\xi) = diag[g_1^T(\xi_1)\ \ g_1^T(\xi_2)\ \ \cdots\ \ g_1^T(\xi_q)]^T \tag{9}$$

$$\bar{h}_2(\xi) = diag[h_{21}^T(\xi_1)\ \ h_{22}^T(\xi_2)\ \ \cdots\ \ h_{2q}^T(\xi_q)]^T \tag{10}$$

$$\bar{f}(\xi) = \left[\bar{f}_1(\xi)\ \cdots\ \bar{f}_q(\xi)\right]^T \tag{11}$$

where $\xi = [\xi_1^T\ \xi_2^T\ \cdots\ \xi_q^T]^T$ with $\xi_i \in R^n$, and

$$\bar{f}_i(\xi) = f(\xi_i) + g_1(\xi_i)\alpha_1(\xi_i) + g_2(\xi_i)\alpha_2(\xi_i) - g_2(\xi_i)\bar{\alpha}_2(\xi). \tag{12}$$

Let $L_i(\xi_i) \in R^{n\times p_i}$ and the unknown observer gain be

$$\bar{L}(\xi) = diag[L_1(\xi_1)\ \ L_2(\xi_2)\ \ \cdots\ \ L_q(\xi_q)]. \tag{13}$$

Theorem 1: Consider the system $\mathbf{\Sigma}$ of (1)-(3) and a positive constant γ. Suppose the following conditions hold.

(i) The pair $\{f, h_1\}$ is locally detectable.

(ii) There exists a C^3 positive definite function $V(x)$, locally defined in a neighborhood of $x = 0$ and vanishing at $x = 0$, which satisfies the Hamilton-Jacobi inequality

$$H_s(x, V_x^T) \triangleq V_x f(x) + h_1^T(x)h_1(x) + \gamma^2\alpha_1^T(x)\alpha_1(x)$$
$$-\alpha_2^T(x)\alpha_2(x) \leq 0 \tag{14}$$

where V_x is the Jacobian matrix of $V(x)$.

(iii) There exist $n \times p_i$ matrix-valued functions $L_i(\xi_i)$ $(i = 1,\cdots,q)$ such that the following Hamilton-Jacobi inequality admits a C^3 positive definite solution $Q(\xi)$ that is locally defined in a neighborhood of $\xi = 0$ and vanishing at $\xi = 0$

$$H_{do}(\xi, Q_\xi^T) \triangleq Q_\xi(\bar{f}(\xi) - \bar{L}(\xi)\bar{h}_2(\xi)) + \bar{\alpha}_2^T(\xi)\bar{\alpha}_2(\xi)$$
$$+\frac{1}{4\gamma^2}Q_\xi\bar{g}_1(\xi)\bar{g}_1^T(\xi)Q_\xi^T + \frac{1}{4\gamma^2}Q_\xi\bar{L}(\xi)\bar{L}^T(\xi)Q_\xi^T \leq 0. \tag{15}$$

Furthermore, the Hessian matrix of $H_{do}(\xi, Q_\xi^T)$ is nonsingular at $\xi = 0$.

Then the following controller of order nq solves the DHCD problem for the system $\mathbf{\Sigma}$.

$$\dot{\xi}_i = f(\xi_i) + g_1(\xi_i)\alpha_1(\xi_i) + g_2(\xi_i)\alpha_2(\xi_i)$$
$$+L_i(\xi_i)(y_i - h_{2i}^T(\xi_i)) \tag{16}$$

$$u_i = \alpha_{2i}(\xi_i),\ i = 1,\cdots,q. \tag{17}$$

Remark 1: Theorem 1 shows how the *DHCD* problem for nonlinear systems be solved, which is an extension of the results for the centralized H_∞control problem in [3] and [4].

In Theorem 1, the observer gains $L_i(\xi_i)$'s are not given. Now we look at how to design these observer gains. Two methods will be presented. The first method makes use of the centralized observer design result in [4], and the idea is similar to that of Paz [8] for linear local observers. But our result here is for nonlinear systems.

Theorem 2: With all assumptions and the design otherwise the same as in Theorem 1, we further assume that the local observer gains $L_i(\xi_i)$, $i = 1, \cdots, q$, satisfy

$$S_x(\xi_i)L_i(\xi_i) = 2\gamma^2 h_{2i}^T(\xi_i),\ i = 1, \cdots, q. \quad (18)$$

Then the controller given by (16)-(17) with the above observer gains solves the *DHCD* problem for the nonlinear system $\mathbf{\Sigma}$.

Next, we present the second approach to the observer gain design. Since $f(x)$ and $h_{2i}(x)$ $(i = 1, \cdots, q)$ are smooth functions with $f(0) = 0$ and $h_{2i}(0) = 0$, there exist smooth matrix-valued functions $A(x)$ and $C_{2i}(x)$ $(i = 1, \cdots, q)$ such that

$$f(x) = A(x)x, \quad h_{2i}(x) = C_{2i}(x)x. \quad (19)$$

Theorem 3: Under conditions (i) and (ii) of Theorem 1, we assume that $V_x = 2x^T P(x)$ with $P(x)$ being a C^2 matrix-valued function. Furthermore, we assume that (in place of (iii) of Theorem 1) there exists a C^2 matrix-valued functions $T(\xi)$, locally defined and nonsingular in a neighborhood of $\xi = 0$, of the form

$$T(\xi) = \begin{bmatrix} T_{11}(\xi_1) & T_{12}(\xi) & \cdots & T_{1q}(\xi) \\ T_{21}(\xi) & T_{22}(\xi_2) & \cdots & T_{2q}(\xi) \\ \vdots & \vdots & \ddots & \cdots \\ T_{q1}(\xi) & T_{q2}(\xi) & \cdots & T_{qq}(\xi_q) \end{bmatrix},\ T_{ii}(\xi_i) \in R^{n\times n} \quad (20)$$

that satisfies the matrix inequality

$$\begin{aligned} T(\xi)\bar{A}_c^T(\xi) + \bar{A}_c(\xi)T^T(\xi) + T(\xi)K_c^T(\xi)K_c(\xi)T^T(\xi) \\ -\gamma^2 T(\xi)\bar{C}^T(\xi)\bar{C}(\xi)T^T(\xi) + \frac{1}{\gamma^2}\bar{g}_1(\xi)\bar{g}_1^T(\xi) \\ +\gamma^2 [T(\xi) - T_D(\xi)]\bar{C}^T(\xi)\bar{C}(\xi)[T(\xi) - T_D(\xi)]^T < 0\ ; \quad (21) \end{aligned}$$

and there exists a positive definite function $Q^0(\xi)$ with $Q^0(0) = 0$ such that $(Q^0)_\xi = 2\xi^T T^{-1}(\xi)$ where

$$T_D(\xi) = diag[T_{11}(\xi)\ T_{22}(\xi)\ \cdots\ T_{qq}(\xi)] \quad (22)$$

$$\bar{A}_c(\xi) = diag[\bar{A}(\xi_1)\ \cdots\ \bar{A}(\xi_q)] - \begin{bmatrix} g_2(\xi_1) \\ \vdots \\ g_2(\xi_q) \end{bmatrix} diag[g_{21}^T(\xi_1)P^T(\xi_1)\ \cdots\ g_{2q}^T(\xi_q)P^T(\xi_q)] \quad (23)$$

$$\bar{A}(x) = A(x) + \frac{1}{\gamma^2}g_1(x)g_1^T(x)P^T(x) - g_2(x)g_2^T(x)P^T(x) \quad (24)$$

$$K_c(\xi) = diag[-g_{21}^T(\xi_1)P^T(\xi_1)\ \cdots\ -g_{2q}^T(\xi_q)P^T(\xi_q)] \quad (25)$$

$$\bar{C}(\xi) = diag[C_{21}(\xi_1)\ C_{22}(\xi_2)\ \cdots\ C_{2q}(\xi_q)]. \quad (26)$$

Denote

$$\bar{L} = \gamma^2 T_D(\xi)\bar{C}^T(\xi). \quad (27)$$

Then the controller given by (16)-(17) with the observer gain as in (27) solves the *DHCD* problem for the system $\mathbf{\Sigma}$.

With Theorem 3, the following result is immediate.

Corollary 4: Under conditions (i) and (ii) of Theorem 1, we assume that $V_x = 2x^T P(x)$ with $P(x)$ being a C^2 matrix-valued function. Furthermore, we assume that (in place of (iii) of Theorem 1) there exists a positive definite matrix $\bar{T}$ that satisfies the matrix inequality

$$\begin{aligned} \bar{T}\bar{A}_c^T(0) + \bar{A}_c^T(0)\bar{T} + \bar{T}K_c^T(0)K_c(0)\bar{T} - \gamma^2\bar{T}\bar{C}^T(0)\bar{C}(0)\bar{T} \\ + [\bar{T} - \bar{T}_D]\bar{C}^T(0)\bar{C}(0)[\bar{T} - \bar{T}_D] + \frac{1}{\gamma^2}\bar{g}_1(0)\bar{g}_1^T(0) < 0\ . \end{aligned} \quad (28)$$

where $\bar{T}_D$ is the diagonal blocks of $\bar{T}$. Denote

$$\bar{L}(\xi) = \gamma^2\bar{T}_D\bar{C}^T(0)\ . \quad (29)$$

Then the controller given by (16)-(17) with the above observer gain in (29) solves the DHCD problem for the system $\mathbf{\Sigma}$.

Remark 2: Theorem 3 presents a design method for local observer gains $L_i(\xi_i)$ $(i = 1, \cdots, q)$, which is based on the existence of solutions of the form (20) to the nonlinear matrix inequality (21). From Corollary 4, the linear local observer gains can be designed by solving a linear matrix inequality. These results are extensions to nonlinear decentralized control systems the results given in [13] for linear decentralized control systems. The results in [13] are in terms of solutions of modified algebraic Riccati equations.

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